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Cyclopedia

of

Civil Engineering

A General Reference Work

ON SURVEYING, RAILROAD ENGINEERING, STRUCTURAL ENGINEERING, ROOFS
AND BRIDGES, MASONRY AND REINFORCED CONCRETE, HIGHWAY
CONSTRUCTION, HYDRAULIC ENGINEERING, IRRIGATION,
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THE editors have freely consulted the standard technical literature of America and Europe in the preparation of these volumes. They desire to express their indebtedness, particularly, to the following eminent authorities, whose well-known treatises should be in the library of everyone interested in Civil Engineering.

Grateful acknowledgment is here made also for the invaluable co-operation of the foremost Civil, Structural, Railroad, Hydraulic, and Sanitary Engineers in making these volumes thoroughly representative of the very best and latest practice in every branch of the broad field of Civil Engineering; also for the valuable drawings and data, illustrations, suggestions, criticisms, and other courtesies.

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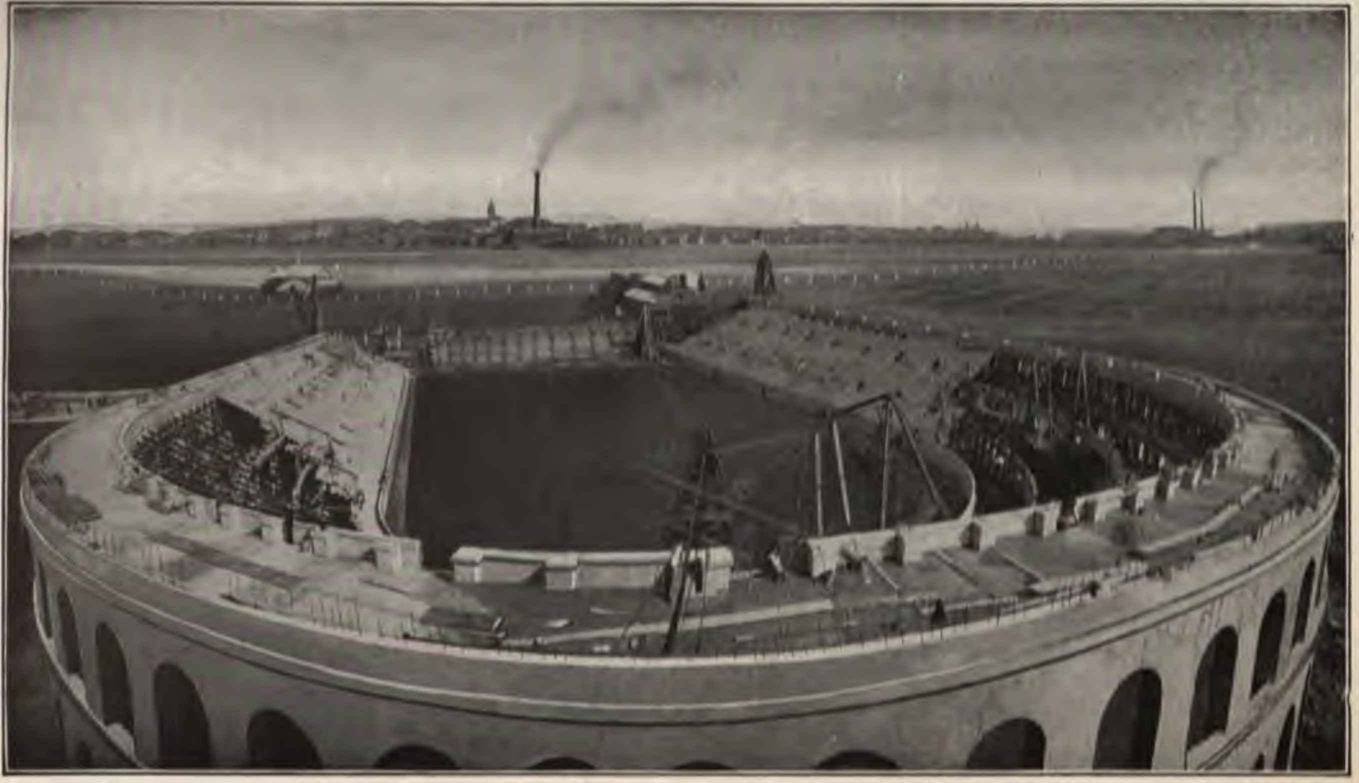

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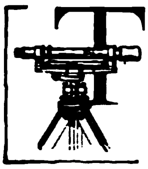
Foreword



THE marvelous developments of the present day in the field of Civil Engineering, as seen in the extension of railroad lines, the improvement of highways and waterways, the increasing application of steel and reinforced concrete to construction work, the development of water power and irrigation projects, etc., have created a distinct necessity for an authoritative work of general reference embodying the results and methods of the latest engineering achievement. The Cyclopedia of Civil Engineering is designed to fill this acknowledged need.

¶ The aim of the publishers has been to create a work which, while adequate to meet all demands of the technically trained expert, will appeal equally to the self-taught practical man, who, as a result of the unavoidable conditions of his environment, may be denied the advantages of training at a resident technical school. The Cyclopedia covers not only the fundamentals that underlie all civil engineering, but their application to all types of engineering problems; and, by placing the reader in direct contact with the experience of teachers fresh from practical work, furnishes him that adjustment to advanced modern needs and conditions which is a necessity even to the technical graduate.

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Therefore, while these volumes are a marked innovation in technical literature — representing, as they do, the best ideas and methods of a large number of *different* authors, each an acknowledged authority in his work — they are by no means an experiment, but are in fact based on what long experience has demonstrated to be the best method yet devised for the education of the busy workingman. They have been prepared only after the most careful study of modern needs as developed under conditions of actual practice at engineering headquarters and in the field.

Grateful acknowledgment is due the corps of authors and collaborators — engineers of wide practical experience, and teachers of well-recognized ability — without whose co-operation this work would have been impossible.

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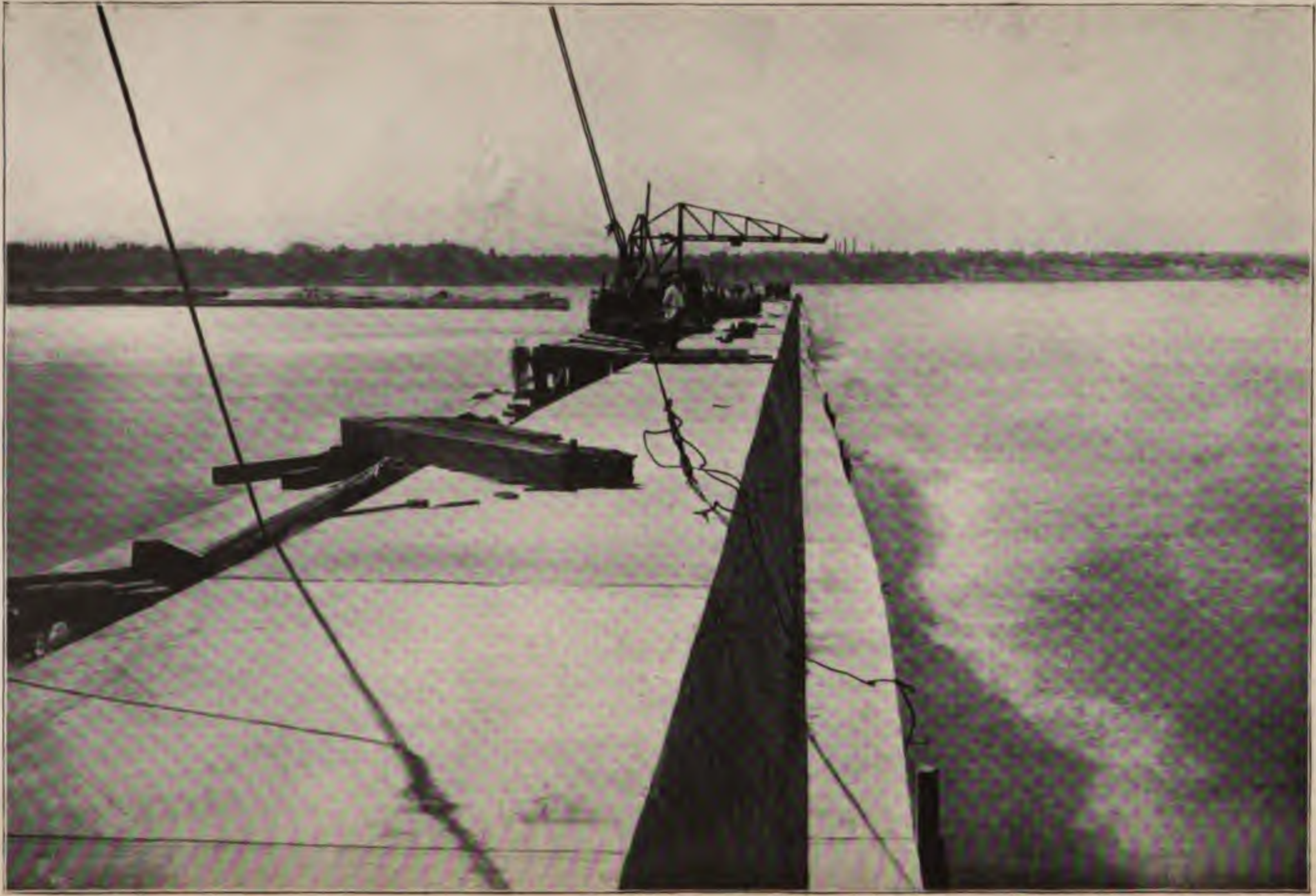

LAKE ARM OF WEST BREAKWATER, HARBOR OF CLEVELAND, OHIO

Outer face and top looking west. Shows irregularity of base, which conforms to face of cribs; also the straightening of the parapet, which

MASONRY AND REINFORCED CONCRETE

PART I

MASONRY MATERIALS

Masonry may be defined as construction in which the chief constructive material is stone or an artificial mineral product such as brick, terra-cotta, or cemented blocks. Under this broad definition, even Reinforced Concrete may be considered as a specialized form of masonry construction.

NATURAL STONE AND ITS CHARACTERISTICS

1. From the constructor's standpoint, any stone is good which will fulfil certain desired characteristics. These various characteristics are not found combined in the highest degree in any one kind of stone. It is essential to learn to what extent these various desirable characteristics are combined in the various types of stone which are quarried. At the same time, it should not be forgotten that stones of the same nominal classification vary greatly in the extent of their desirability. The chief characteristics to be considered by the constructor are *Cost*, *Durability*, *Strength*, and *Appearance*. Although in some cases this represents the order in which these qualifications are desired, in other cases the order is indefinitely varied. For example, in a high-grade public building or monument, a good appearance is considered essential, regardless of cost. In a subsurface foundation, appearance is of absolutely no importance.

2. **Cost.** The cost of any stone depends on its intrinsic valuation in the quarry, the cost of quarrying and dressing, and the cost of transportation from the quarry to the site of the structure. The cost of transportation is often the most important, and this consideration frequently decides not only the choice of stone but even the type of construction—whether stone masonry or concrete. To give a rough idea of the cost of stone quarrying, a few values are

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quoted from Gillette's "Handbook of Cost Data." In one instance the cost of quarrying granite, exclusive of rental of plant, rental of quarry, and cost of stripping off the upper soil, averaged about $4.50 per cubic yard. In another instance the cost of quarrying rubble amounted to $1.80 per cubic yard. The cost of explosives was not included in this estimate, but it should not have increased the cost to over $2.00 per cubic yard. In another instance the cost of quarrying gneiss amounted to $3.55, not including explosives and teaming. Even these items should not have made the total cost more than $4.25 per cubic yard.

3. **Durability.** Under many conditions the most important qualification is durability. The lack of it is also the most seriously disappointing quality. Rocks which have remained hard and tough for unnumbered ages while covered by earth from air and frost, will disintegrate after a comparatively few years' exposure.

Atmospheric Influences. A very porous stone will absorb water, which may freeze and cause crystals near the surface to flake off. Even though such action during a single winter may be hardly perceptible, the continued exposure of fresh surfaces to such action may sooner or later cause a serious loss and disintegration. Even rain water which has absorbed carbonic acid from the atmosphere will soak into the stone, and the acid will have a greater or less effect on nearly all stones. Quartz is the only constituent which is absolutely unaffected by acid. The sulphuric acid gas given off by coal will also affect building stone very seriously.

Fire. Natural stones are far less able to withstand a conflagration than the artificial compounds such as brick, concrete, and terra-cotta. Granite, so frequently considered the type of durability, is especially affected. Limestone and marble will be utterly spoiled, at least in appearance, if not structurally, by a hot fire. Sandstone is the least affected of the natural stones.

Hardness. The hardness of a stone is essential to its resistance to abrasive action when used for floors, treads, and similar uses. The value of trap rocks for macadam and ballast is largely due to this quality.

4. **Strength.** [illegible] in arch the [illegible] tion. The average crushing strength of various kinds of stone will be

quoted later. The tensile strength should never be depended on, except to a very limited extent as a function of the transverse strength. Even this is only applicable to such cases as the lintels over doors and windows, the footing stones for foundations, and the cover stones for box culverts. It is usually true that a stone which is free from cracks and which has a high crushing strength also, has as much transverse strength as should be required of any stone.

5. **Appearance.** It is seldom that an engineer need concern himself with the appearance of a stone, provided it is satisfactory in the respects previously mentioned. The presence of iron oxide in a stone will sometimes cause a deterioration in appearance by the formation of a reddish stain on the outer surface. It usually happens, however, that a stone whose strength and durability are satisfactory will have a sufficiently good appearance, unless in high-grade architectural work, where it is considered essential that a certain color or appearance shall be obtained.

TESTING STONE

6. Of the above four qualities, only two—durability and strength—are susceptible of laboratory testing, and even for these qualities the best known laboratory tests are not conclusive. The deterioration and partial failure of the masonry in some of the best known cathedrals of Europe, which commanded the best available talent in their construction, are startling illustrations of the impracticability of determining from laboratory tests the effect on stone of long-continued stress, combined perhaps with other destructive influences. Although the best technical advice was obtained in selecting the stone for the Parliament House in London, and the stone selected was undoubtedly subjected to the best known tests, it was apparently impossible to foresee the effect of the London atmosphere, which is now so seriously affecting the stone. Several of the tests to be described below should be considered as being *negative tests*. If the stones fail under these tests, they are probably inferior; if they do not fail, they are perhaps safe, but there is no certainty. A long experience, based on a knowledge of the characteristics of stones which have proven successful, is of far greater value than a dependence on the results of laboratory tests. The tests attempt to simulate the actual destructive agencies as far as possible, but since

a great deal of stonework which was apparently satisfactory when constructed and for a few years after, has failed for a variety of reasons, attempts are made to use *accelerated tests*, which are supposed by their concentration to affect the stone in a few minutes or hours as much as the milder causes acting through a long period of years.

7. **Absorption.** It is generally said that stones having the least absorption are the best. The absorptive power is measured by first drying the stone for many hours in an oven, weighing it, then soaking it for, say, 24 hours, and again weighing it. The *increase* in the weight of the soaked stone (due to the weight of water absorbed), divided by the weight of the dry stone, equals the *ratio of absorption.* The granites will absorb as an average value a weight of water equal to about $\frac{1}{150}$ of the weight of the stone. For sandstone the ratio is about $\frac{1}{24}$.

The test for absorption has but little value except to indicate a closeness of grain (or the lack of it), which *probably* indicates something about the strength of the stone, as well as its liability to some kinds of disintegration.

8. **Test for Frost.** The only real test is to wash, dry, and weigh test specimens, very carefully; then soak them in water, and expose them to intensely cold and intensely warm temperatures alternately. Finally wash, dry, and weigh them. If the freezing has resulted in breaking off small pieces, or possibly in fracturing the stone, the loss in weight or the breakage will give a measure of the effect of cold winters. However, as such low temperatures cannot be produced artificially except at considerable expense, and as a sufficient degree of cold is ordinarily unobtainable when desired, such a test is usually impracticable.

An attempt to simulate such an effect by boiling the specimen in a concentrated solution of sulphate of soda and observing the subsequent disintegration of the stone, if any, is known as *Brard's test.* Although this method is much used for lack of a better, its value is doubtful and perhaps deceptive, since the effect is largely chemical rather than mechanical. The destructive effect on the stone is usually greater than that of freezing, and might result in condemning a really *good* stone.

9. **Chemical Test.** The most difficult and uncertain matter to determine is the probable effect of the acids in the atmosphere.

These acids, dissolved in rain water, soak into the stone and combine with any earthy matter in the stone, which then leaches out, leaving small cavities. This not only results in a partial disintegration of the stone, but also facilitates destruction by freezing. If the stone specimen, after being carefully washed, is soaked for several days in a one per cent solution of sulphuric and hydrochloric acid, the liquid being frequently shaken, the water will become somewhat muddy if there is an appreciable amount of earthy matter in the stone. Such an effect is supposed to indicate the probable action of a vitiated atmosphere. Of course it should be remembered that such a consideration is important only for a structure in a crowded city where the atmosphere is vitiated by poisonous gases discharged from factories and from all chimneys.

10. **Physical Tests.** A test made by crushing a block of stone in a testing machine is apparently a very simple and conclusive test, but in reality the results are apt to be inconclusive and even deceptive. This is due to the following reasons, among others:

(*a*) The crushing strength of a cube per square inch is far less than that of a slab having considerably greater length and width than height.

(*b*) The result of a test depends very largely on the preparation of the specimen. If sawed, the strength will be greater than if cut by chipping. If the upper and lower faces are not truly parallel, so that there is a concentration of pressure on one corner, the apparent result will be less.

(*c*) The result depends on the imbedment. Specimens which are rubbed and ground with machines that will insure truly parallel and plane surfaces, will give higher results than when wood, lead, leather, or plaster-of-paris cushions are employed.

(*d*) The strength of masonry depends largely on the crushing strength of the mortar used and the thickness of the joints. Other things being equal, an increase in the crushing strength of the stone (or brick) which is used does not add proportionately to the strength of the masonry as a whole; and if the mortar joints are very thick, it adds little or nothing. Since the strength of the masonry is the only real criterion, the strength of a cube of the stone is of comparatively little importance.

In short, tests of two-inch cubes (the size usually employed) are valuable chiefly in comparing the strength of two or more different kinds of stones, all of which are tested under precisely similar conditions. A comparison of such figures with the figures obtained by others will have but little value unless the precise conditions of the other tests are accurately known. Under any conditions, the results of the tests will bear but little relation to the actual strength of the masonry to be built.

11. **Quarry Examinations.** These are generally the surest tests, and should never be neglected if the choice of stone is a matter of great importance. *Field stone* and outcropping rock which have withstood the weather for indefinite periods of years, can usually be relied on as being durable against all deterioration except that due to acids in the atmosphere, to which they probably have not been subjected in the country as they might be in a city. On the other hand, however, large blocks of stone can seldom be obtained from field stones. If a quarry has been opened for several years, a comparison of the other surfaces with those just exposed may indicate the possible disintegrating or discoloring effects of the atmosphere. A stone which is dense and of uniform structure, and which will not disintegrate, may be relied on to withstand any physical stress to which masonry should be subjected.

BUILDING STONE

12. **Limestone.** Carbonate of lime forms the principal ingredient of limestone. A pure limestone should consist only of carbonate of lime. However, none of our natural stones are chemically pure, but all contain a greater or less amount of foreign material. To these impurities are due the beautiful and variegated coloring which makes limestone valuable as a building material.

Limestone occurs in stratified beds, and ordinarily is regarded as originating as a chemical deposit. It effervesces freely when an acid is applied; its texture is destroyed by fire; the fire drives off its carbonic acid and water, and forms quicklime. Limestone varies greatly in its physical properties. Some limestones are very durable, hard, and strong, while others are very soft and easily broken.

There are two principal classes of limestone—*granular* and *compact*. In each of these classes are found both marble and ordinary building stone. The granular stone is generally best for building purposes, and the finer-grained stones are usually better for either marble or fine cut-stone. The coarse-grained varieties often disintegrate rapidly when exposed to the weather. All varieties work freely, and can be obtained in blocks of any desired dimensions.

13. **Marble.** When limestone is wholly crystalline and suitable

for ornamental purposes, it is called *marble;* or, in other words, any limestone that can be polished is called marble. There are a great many varieties of marble, and they vary greatly in color and appearance. Owing to the cost of polishing marble, it is used chiefly for ornamental purposes.

14. **Dolomite.** When the carbonate of magnesia occurring in limestone rises to about 45 per cent, the stone is then called *dolomite.* It is usually whitish or yellowish in color, and is a crystalline granular aggregate. It is harder than the ordinary limestones, and also less soluble, being scarcely at all acted upon by dilute hydrochloric acid. There is no essential difference between limestone and dolomite with respect to color and texture.

15. **Sandstone.** Sandstones are composed of grains of sand that have been cemented together through the aid of heat and pressure, forming a solid rock. The cementing material usually is either silica, carbonate of lime, or an iron oxide. Upon the character of this cementing material is dependent, to a considerable extent, the color of the rock and its adaptability to architectural purposes. If silica alone is present, the rock is of a light color and frequently so hard that it can be worked only with great difficulty. Such stones are among the most durable of all rock, but their light color and poor working qualities are a drawback to their extensive use. Rocks in which carbonate of lime is the cementing material are frequently too soft, crumbling and disintegrating rapidly when exposed to the weather. For many reasons the rocks containing ferruginous cement (iron oxide) are preferable. They are neither too hard to work readily, nor liable to unfavorable alteration when exposed to atmospheric agencies. These rocks usually have a brown or reddish color.

Sandstones are of a great variety of colors, which, as has already been stated, is largely due to the iron contained in them. In texture, sandstones vary widely—from a stone of very fine grain, to one in which the individual grains are the size of a pea. Nearly all sandstones are more or less porous, and hence permeable to a certain extent by water and moisture. Sandstones absorb water most readily in the direction of their lamination or grain. The strength and hardness of sandstones vary between wide limits. Most of the varieties are easily worked, and split evenly. The formations of sandstone in the United States are very extensive. The crushing strength of sandstone varies widely, being from 2.500 pounds to 13.500 pounds per square inch, and specimens have been obtained that require a load of 29.270 pounds per square inch to crush them.

16. **Conglomerates.** Conglomerates differ from sandstone only in structure, being coarser and of a more uneven texture. The grains are usually an inch or more in diameter.

17. **Granite.** The essential components of the true granites are quartz and potash feldspar. Granites are rendered complex, although the essential minerals are but two in number, by the presence of numerous accessories which essentially modify the appearance of the rocks; and these properties render them important as building stone. The prevailing color is some shade of gray, though greenish, yellowish, pink, and deep red are not uncommon. These various hues are due to the color of the prevailing feldspar and the amount and kind of the accessory minerals. The hardness of granite is due largely to the condition of the feldspathic constituent, which is valuable. Granites of the same constituents differ in hardness.

Granites do not effervesce with acids, but emit sparks when struck with steel. They possess the properties of strength, hardness, and durability, although they vary in these properties as well as in their structure. They furnish an extensive variety of the best stone for the various purposes of the engineer and architect. The crushing strength of granite is variable, but usually is between 15.000 and 20.000 pounds per square inch.

18. **Trap Rock.** Trap rock, or diabase, is a crystalline, granular rock, composed essentially of feldspar and augite; but nearly all contains magnetite and frequently olivine. They are basic in composition and in structure; they are, as a rule, massive. The texture, as a general thing, is fine, compact, and homogeneous. The colors are somber, varying from greenish, through dark gray, to nearly black. Owing to its lack of rift, its hardness, and its compact texture, trap rock is generally very hard to work. It has been used to some extent for building and monumental work, but is more generally used for paving purposes. Within the last few years, on account of its great strength and wear-resisting qualities, it has been extensively used in concrete work. The crushing strength of trap rock or diabase is usually between 20.000 and 25.000 pounds per square inch.

19. Seasoning of Stone. Stone, to weather well, should be laid with its bedding (lamination) horizontal, as it was first laid down by nature in the quarry. The stone, moreover, will offer greater resistance to pressure if laid in this manner, and, it is said, will stand a greater amount of heat without disintegrating. This is important in cities where any building is liable to have its walls highly heated by neighboring burning structures.

Some stones that are liable to be destroyed by the effects of frost on first being taken from the quarries, are no longer so after being exposed for some time to the air, having lost their quarry water through evaporation. This difference is very manifest between stones quarried in summer and those quarried in winter. It has frequently happened that stones of good quality have been entirely ruined by hard freezing immediately after being taken from the quarry; while, if they are quarried during the warm season of the year and have an opportunity to lose their quarry water by evaporation prior to cold weather, they withstand freezing very well. This particularly applies to some marbles and limestones. This change is accounted for by the claim put forward, that the quarry water of the stones carries in solution carbonate of lime and silica, which is deposited in the cavities of the rock as evaporation proceeds. Thus additional cementing material is added, rendering the rock more compact. This also will account for the hardening of some stones after being quarried a short time. When first quarried they are soft, easily sawed and worked into any desirable shape; but after the evaporation of their quarry water, they become hard and very durable.

Table I gives the physical properties of many of the most important varieties and grades of building stone found in the United States.

BRICK

20. Definition and Characteristics. The term *brick* is usually applied to the product resulting from burning moulded prisms of clay in a kiln at a high temperature.

Common brick is not extensively used in engineering structures, except in the construction of sewers and the lining of tunnels. Brick is easily worked into structures of any desirable shape, easily handled or transported, and comparatively cheap. When well constructed,

TABLE I

Physical Properties of Building Stones

From Merrill's "Stones for Buildings and Decoration."

Kind of Stone	Locality	Position	Strength per Square Inch	Specific Gravity	Weight per Cubic Foot	Ratio of Absorption
			lbs		lbs	
Granite	Grape Creek, Colo	Bed Edge	14.492 17.352	2.603	163	.048
Granite	Stony Creek, Conn.	Bed Edge	15.000 16.750	2.645	165	1/201
Granite	[illegible]		22.610	. .	. . .	. . .
Granite	City Point, Me.	Bed	15.046	2.65	166	. . .
Granite	East St. Cloud, Minn.	Bed Edge	28.000 26.250	2.609	163	. . . .
Diabase	New Duluth, Minn.	Bed Edge	26.250 26.250	3.005	188	1/338
Limestone	Bedford, Ind.		6.500	. . . .	147	1/24
Limestone	" "		10.125	. . . .	152	1/32
Limestone	Greensburgh, Ind.		16.875	. . . . .	170	1/117
Limestone	Conshohocken, Pa.		15.150	. . . .	. . .	. . . .
Limestone	Stillwater, Minn.		25.000	2.762	173	1/251
Limestone	" "	Bed Edge	10.750 12.750	2.567	161	1/40
Sandstone	Buckhorn, Larimer Co., Colo	Bed Edge	18.573 17.261	2.379	168	.040
Sandstone	Fort Collins, Larimer Co. Colo	Bed Edge	11.707 10.784	2.252	141	.072
Sandstone	Brandford, Fremont Co., Colo.	Bed Edge	3.308 2.894	2.004	125	. . . .
Sandstone	Marquette, Mich.	Bed	6.323	2.166	135	1/20
Sandstone	Kasota, Minn.	Bed	10.700	2.630	164	1/56
Sandstone	Albion, N. Y.	Bed	13.500	2.420	151	1/44
Sandstone	Cleveland, O.	Bed	6.800	2.240	140	1/37
Sandstone	Seneca, O.	Bed	9.687	2.390	149	1/32

brick masonry compares very well in strength with stone masonry, but is not so heavy as stone. Brickwork is but slightly affected by changes of temperature or humidity.

Brick is made of common clay (silicate of alumina), which usually contains compounds of lime, magnesia, and iron. Good brick clay is often found in a natural state. The quality of the brick depends greatly on the quality of the clay used, and equally as much on the care taken in its manufacture.

Oxide of iron gives brick hardness and strength. The red color of brick is also due to the presence of iron. The presence of carbonate of lime in the clay of which brick is made, is injurious, since the carbonate is decomposed during the burning, forming caustic potash, which, by the absorption of water, will cause the brick to disintegrate. An excess of silicate of lime makes the clay fusible, which softens the brick and thereby causes distortion during the burning process. Magnesia in small quantities has but little influence on brick. Sand, in quantities not in excess of about 25 per cent, will help to preserve the form of the brick, and is beneficial to that extent; but in greater quantities than 25 per cent, it makes the brick brittle and weak.

21. **Requisites for Good Brick.** Good brick should be of regular shape, with plane faces, parallel surfaces, and sharp edges and angles. It should show a fine, uniform, compact texture; should be hard, and, when struck a sharp blow, should ring clearly; and should not absorb more water than one-tenth of its weight. The specific gravity should be 2 or more. Good brick will bear a compressive load of 6,000 pounds per square inch when the sides are ground flat and pressed between plates. The modulus of rupture should be at least 800 pounds per square inch.

22. **Absorptive Power.** The amount of water that a brick absorbs is very important in indicating the durability of brick, particularly its resistance to frost. Very soft brick will absorb 25 to 30 per cent of their weight of water. Weak, light-red ones will absorb 20 to 25 per cent; this grade of brick is used commonly for filling interior walls. The best brick will absorb only 4 to 5 per cent, but brick that will absorb 10 per cent is called good.

23. **Color of Bricks.** The color of brick depends greatly upon the ingredients of the clay; but the temperature of the burn-

ing, the moulding sand, and the amount of air admitted to the kiln also have their influence. Pure clay or clay mixed with chalk will produce white brick. Iron oxide and pure clay will produce a bright red brick when burned at a moderate heat. Magnesia will produce brown brick; and when it is mixed with iron, produces yellow brick. Lime and iron in small quantities produce a cream color; an increase of lime produces brown, and an increase of iron red.

24. **Size and Weight.** The standard size for common brick is 8¼ by 4 by 2¼ inches; and for face brick, 8⅜ by 4⅛ by 2¼ inches. There are numerous small variations from these figures; and also, since the shrinkage during burning is very considerable and not closely controlled, there is always some uncertainty and variation in the dimensions. Bricks will weigh from 100 to 150 pounds per cubic foot according to their density and hardness, the harder bricks being of course the heavier per unit of volume.

25. **Classification of Common Bricks.** Bricks are usually classified in three ways: (*a*) Manner of moulding; (*b*) position in kiln; (*c*) their shape or use.

(*a*) The manner in which brick is moulded has produced the following terms:

Soft-mud Brick. A brick moulded either by hand or by machine, in which the clay is reduced to mud by adding water.

Stiff-mud Brick. A brick moulded from dry or semi-dry clay. It is moulded by machinery.

Pressed Brick. A brick moulded with semi-dry or dry clay.

Re-pressed Brick. A brick made of soft mud, which, after being partly dried, is subjected to great pressure.

(*b*) The classification with regard to their position in the kiln applies only to the old method of burning. With the new methods, the quality is nearly uniform throughout the kiln. The three grades taken from the old-style kiln were:

Arch Brick. Bricks forming the sides and top of the arches in which the fire is built are called arch bricks. They are hard, brittle, and weak from being over-burnt.

Body, Cherry, or Hard Brick. Bricks from the interior are called body, cherry, or hard brick, and are of the best quality.

Pale, Salmon, or Soft Brick. Bricks forming the exterior of the kiln are under-burnt, and are called soft, salmon, or pale brick. They are used only for filling, being too weak for ordinary use.

(*c*) The classification of brick in regard to their use or shape has given rise to the following terms:

Face Brick. Brick that are uniform in size and color and are suitable for the exposed places of buildings.

Sewer Brick. Common hard brick, smooth and regular in form.

Paving Brick. Very hard common vitrified brick, often made of shale. They are larger than the ordinary brick, and are often called *paving blocks.*

Compass Brick. Brick having four short edges which run radially to an axis. They are used to build circular chimneys.

Voussoir Brick. Brick having four long edges running radially to an axis. They are used in building arches.

26. **Crushing Strength.** The results of crushing tests of brick vary greatly, depending on the details of the tests made. Many reports fail to give the details under which these tests are made, and in that case the real value of the results of the test as announced is greatly reduced.

The following results were obtained at the U. S. Arsenal at Watertown, Mass., by F. E. Kidder. The specimens were rubbed on a revolving bed until the top and bottom faces were perfectly true and parallel.

MAKE OF BRICK	NO. OF SPECIMENS TESTED	PRESSURE AT WHICH SPECIMENS BEGAN TO FAIL	COMPRESSION (per sq. in.)
Philadelphia Face Brick	3	3, 527 lbs.	5, 918 lbs.
Cambridge Brick	4	4, 655 "	12, 186 "
Boston Brick	3	7, 880 "	11, 670 "
New England Pressed	4	4, 764 "	12, 490 "

The following results were obtained by C. Y. Davis, the tests being made at the Watertown Arsenal:

KIND OF BRICK	COMPRESSION (per sq. in.)	KIND OF BRICK	COMPRESSION (per sq. in.)	KIND OF BRICK	COMPRESSION (per sq. in.)
Red	9,540 lbs.	Pressed	6,470 lbs.	Arch	7,600 lbs.
"	*8,530 "	"	*9,190 "	"	*10,290 "
"	6,050 "	"	5,960 "	"	6,800 "
"	6,700 "	"	6,750 "		

These specimens were tested to select brick for the U. S. Pension Office at Washington, D. C. The specimens tested were submitted by manufacturers.

*Indicates the brick selected.

27. **Fire Brick.** Furnaces must be lined with a material which

is even more refractory than ordinary brick. The oxide and sulphide of iron, which are so common and comparatively harmless in ordinary brick, will ruin a fire brick if they are present to a greater extent than a very few per cent. Fire brick should be made from nearly pure sand and clay. There is comparatively little need for mechanical strength, but the chief requirement is their infusibility, and pure clay and silica fulfil this requirement very perfectly.

28. **Sand-Lime Brick.** Within the last few years, the sand-lime brick industry has been developed to some extent. The materials for making this brick consist of sand and lime; and they were first made by moulding ordinary lime mortar in the shape of a clay brick, and were hardened by the carbon dioxide of the atmosphere.

There are two general methods of manufacturing these bricks:

1. Brick made of sand and lime, and hardened in the atmosphere. This hardening may be hastened by placing the brick in an atmosphere rich in carbon dioxide, or still less time will be required if the hardening is done with carbon dioxide under pressure.

2. Brick made of sand and lime and hardened by steam under atmospheric pressure. This process may be hastened by having the steam under pressure.

When sand-lime bricks are made by the first process, it requires several weeks for the bricks to harden; and by the second method it requires only a few hours; the latter method is the one generally used in this country. The advantages claimed for these bricks are that they improve with age; are more uniform in size, shape, and color; have a low porosity and no efflorescence; and do not disintegrate by freezing. The compressive strength of sand-lime brick of a good quality ranges from 2,500 to 4,500 pounds per square inch.

CONCRETE BUILDING BLOCKS

29. The growth of the concrete block industry has been rapid. The blocks are taking the place of wood, brick, and stone for ordinary wall construction. They are strong, durable, and cheap. The blocks are made at a factory or on the site of the work where they are to be used, and are placed in the wall in the same manner as brick or stone. There are two general types of blocks made—the *one-piece block*, and the *two-piece block*. The one-piece type consists of a single block, with hollow cores, making the whole thickness of the wall. In the two-piece type, the front and back of the blocks are

made in two separate pieces, and bonded when laid up in the wall. The one-piece blocks are more generally used than the two-piece blocks.

30. **Size of Blocks.** Various shapes and sizes of blocks are made. Builders of some of the standard machines have adopted a standard length of 32 inches and a height of 9 inches for the full-sized blocks, with width of 8, 10, and 12 inches. Lengths of 8, 12, 16, 20, and 24 inches are made from the same machine, by the use of parting plates and suitably divided face-plates. Most machines are constructed so that any length between 4 and 32 inches, and any desired height, can be obtained.

The size of the openings (the cores) varies from one-third to one-half of the surface of the top or bottom of the block. The building laws of many cities state that the openings shall amount to only one-third of the surface. For any ordinary purpose, blocks with 50 per cent open space are stronger than necessary.

31. **Material.** The material for making concrete blocks consists of Portland cement, sand, and crushed stone or gravel. Owing to the narrow space to be filled with concrete, the stone and gravel are limited to one-half or three-quarters of an inch in size. At least one-third of the material, by weight, should be coarser than ⅛ inch. A block made with gravel or screenings (sand to ¾-inch stone), with proportions of 1 part Portland cement to 5 parts screenings, will be as good as a block with 1 part Portland cement and 3 parts sand. These materials will be further treated under the headings of "Portland Cement," "Sand," and "Stone."

32. **Proportions.** The proportions generally used in the making of concrete blocks, vary from a mixture of 1 part cement, 2 parts sand, and 4 parts stone, to a mixture of 1 part cement, 3 parts sand, and 6 parts stone. A very common mixture consists of 1 part cement, 2½ parts sand, and 5 parts stone. A denser mixture may be secured by varying these proportions somewhat; that is, the maker may find that he secures a more compact block by using 2¾ parts sand and 4¾ parts stone; but a leaner mixture than 1 : 2½ : 5 is not to be recommended. In strength this mixture will have a crushing resistance far beyond any load that it will ever have to support. Even a mixture of 1:3:6 or 1:3½:7 will be stronger than necessary to sustain any ordinary load. Such a mixture, however, would be

porous and unsatisfactory in the wall of a building. Blocks, in being handled at the factory, carted to the building site, and in being placed in the wall, will necessarily receive more or less rough handling; and safety in this respect calls for a stronger block than is needed to bear the weight of a wall of a building. For a high-grade water-tight block, a 1 : 2 : 4 or a 1 : 2½ : 4 mixture is generally used.

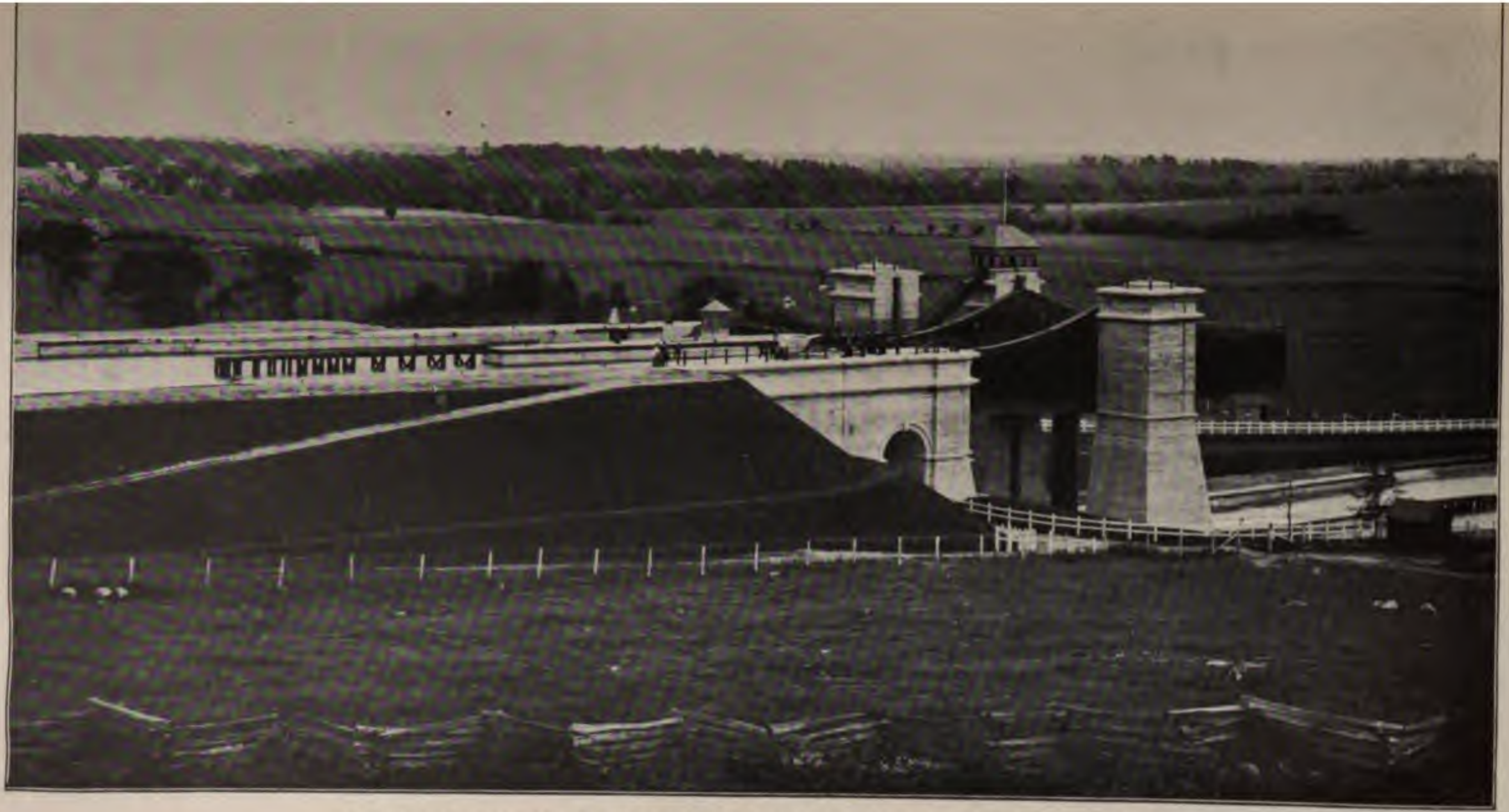
33. **Proportion of Water.** Blocks made with dry concrete will be soft and weak, even if they are well sprinkled after being taken out of the forms. Blocks that are to be removed from the machine as soon as they are made will stick to the plates and sag out of shape, if the concrete is mixed too wet. Therefore there should be as much water as possible used, without causing the block to stick or sag out of shape when being removed from the moulds. This amount of water is generally 8 to 9 per cent of the weight of the dry mixture. To secure uniform blocks in strength and color, the same amount of water should be used for each batch.

34. **Mixing and Tamping.** The concrete should be mixed in a batch mixer, although good results are obtained in hand-mixed concrete. The tamping is generally done with hand-rammers. Pneumatic tampers, operated by an air-compressor, are used successfully. Moulding concrete by *pressure* is not successful unless the concrete is laid in comparatively thin layers.

35. **Curing of Blocks.** The blocks are removed from the machine on a steel plate, on which they should remain for 24 hours. The blocks should be protected from the sun and dry winds for at least a week, and thoroughly sprinkled frequently. They should be at least four weeks old before they are placed in a wall. If they are built up in a wall while green, skrinkage cracks will be apt to occur in the joints.

36. **Mixture for Facing.** For appearance, a facing of a richer mixture is often used, generally consisting of 1 part cement to 2 parts sand. The penetration of water may be effectively prevented by this rich coat. Care must be taken to avoid a seam between the two mixtures.

Blocks are made with either a plane face, or of various ornamental patterns, as tool-faced, paneled, rock-faced, etc. Coloring of the face is often desired. Mineral coloring, rather than chemical, should be used, as the chemical color may injure the concrete or fade.



HYDRAULIC LIFT LOCK ON TRENT VALLEY CANAL NEAR PETERBOROUGH, ONTARIO

The only example in America of this unique type of lock. The trussed chamber shown between the towers is provided with gates at each end, forming a receptacle in which floating vessels are lowered or raised to the level of the water below or above the lock.

PUBLIC
ASTOR
TILDEN

37. Cost of Making. The following is quoted from a paper by N. F. Palmer, C. E.:

Blocks 8 by 9 by 32 inches; gang consisted of five workmen, and foreman; record for one hour, 30 blocks; general average for 10 hours, 200 blocks. The itemized cost was as follows:

1 foreman	@ $2.50	$2.50
5 helpers	@ 2.00	10.00
13 bbls. cement	@ 2.00	26.00
10 cu. yds. sand and gravel	@ 1.00	10.00
Interest and depreciation on machine		2.00
		$50.50

This is the equivalent of $50.50 ÷ 200, or 25¼ cents per block; or, since the face of the block was 9 by 32 inches, or exactly 2 square feet, the equivalent of 12.6 cents per square foot of an 8-inch wall.

Another illustration, quoted from Gillette, for a 10-inch wall, was itemized as follows, *for each square foot* of wall:

Sand	2.0 cents
Cement @ $1.60 per barrel	4.5 "
Labor @ $1.83 per day	3.8 "
Total per square foot	10.3 "

This is apparently considerably cheaper than the first case, even after allowing for the fact that the second case does not provide for interest, depreciation on plant, etc., which in the first case is only 4 per cent of the total. This allowance of 4 per cent is probably too small.

CEMENTING MATERIALS

38. The principal cementing materials are *Common Lime, Hydraulic Lime, Pozzuolana, Natural Cement,* and *Portland Cement.* There are a few other varieties, but their use is so limited that they need not be considered here.

39. Common Lime. This is produced by burning "limestone" whose chief ingredient is carbonate of lime. Except in the form of marble, a limestone usually contains other substances—perhaps up to 10 per cent of silica, alumina, magnesia, etc. The process of burning drives off the carbonic acid, and leaves the protoxide of calcium. This is the lime of commerce; and to preserve it from deterioration, it must be kept dry and even protected from a free circulation of air. When exposed freely to the air for a long period, it will become *air-slaked*; that is, it will absorb both moisture and carbonic acid from the air, and will lose its ability to harden. The first step in using common lime is to combine it with water, which it absorbs readily so that its volume is increased to 2½ or 3½ times what it was before. Its weight is at the same time increased about one-fourth; and the mass, which consisted originally of large lumps with some powder, is reduced to an unctuous mass of smooth paste. The lime is then called *slaked lime,* the process of slaking being accompanied by the development of great heat. The purer the lime, the greater the development of heat and the greater the expansion in volume. It is soluble in water which is not already "hard," or which does not already contain considerable lime in solution. A good lime will make a smooth paste with only a very small percentage (less than 10 per cent) of foreign matter or clinker. By such simple means a lime may be readily tested.

The hardening of common lime mortar is due to the formation of a carbonate of lime (substantially the original condition of the stone) by the absorption from the atmosphere of carbonic oxide. This will penetrate for a considerable depth in course of time; but instances are common in which masonry has been torn down after having been erected many years, and the lime mortar in the interior of the mass has been found still soft and unset, since it was hermetically cut off from the carbonic oxide of the atmosphere. For the same reason, common lime mortar will not harden under water, and therefore it is utterly useless to employ it for work under water or for large masses of masonry.

When the qualities of slaking and expansion are not realized or are obtained only very imperfectly, the lime is called *lean* or *poor* (rather than *fat*) and its value is less and less, until it is perhaps worthless for use in making mortar, or for any other use except as fertilizer. The cost of lime is about 60 cents per barrel of 230 pounds net.

40. **Hydraulic Lime.** This is derived from limestones containing about 10 to 20 per cent of clay or silica, which is intimately mixed with the carbonate of lime in the structure of the stone. During the process of burning, some of the lime combines with the clay (or the silica) so as to form the aluminate or silicate of lime. The excess of lime becomes quicklime as before. During the process of slaking, which should be done by mere *sprinkling,* the lime having been intimately mixed with the clay or silica, the expansion of the

lime completely disintegrates the whole mass. This slaking is done by the manufacturer. The lime having a much greater avidity for the water than the aluminate or the silicate, the small amount of water used in the slaking is absorbed entirely by the lime, and the aluminate or the silicate is not affected. The setting of hydraulic lime appears to be due to the crystallizing of the aluminate and silicate; and since this will be accomplished even when the masonry is under water, it receives from this property its name of *hydraulic lime.* It is used but little in this country, and is all imported.

41. **Pozzuolana or Slag Cement.** Pozzuolana is a form of cementing material which has been somewhat in use since very ancient times. Apparently it was first made from the lava from the volcano Vesuvius, the lava being picked up at Pozzuoli, a village near the base of the volcano. It consists of a combination of silica and alumina, which is mixed with common lime. Its chemical composition is therefore not very unlike that of hydraulic lime. It also possesses the ability to harden under water. Its use is very limited, and its strength and hardness comparatively small, compared with that of Portland cement. It should never be used where it will be exposed for a long time to dry air, even after it has thoroughly set. It appears to withstand the action of sea water somewhat better than Portland cement; and hence it is sometimes used instead of Portland cement as the cementing material for large masses of masonry or concrete which are to be deposited in sea water, when the strength of the cement is a comparatively minor consideration. Artificial pozzuolana is sometimes made by grinding up blast-furnace slag which has been found by chemical analysis to have the correct chemical composition.

42. **Natural Cement.** Natural cement is obtained by burning an argillaceous or a magnesian limestone which happens to have the proper chemical composition. The resulting clinker is then finely ground and is at once ready for use. Such cement was formerly and is still commonly called *Rosendale cement,* owing to its having been produced first in Rosendale, Ulster County, New York. A very large part of the natural cement now produced in this country comes from Ulster County, New York, or from near Louisville, Kentucky. Cement rock from which

natural cement can be made, is now found widely scattered over the country.

In Europe the name *Roman cement* is applied to substantially the same kind of product. Since the cement is made wholly from the rock just as it is taken out of the quarry, and also since it is calcined at a much lower temperature than that employed in making Portland cement, it is considerably cheaper than Portland cement. On the other hand, its strength is considerably less than that of Portland cement, and the time of setting is much quicker. Sometimes this quickness of setting is a very important point—as, for instance, when it is desired to obtain a concrete which shall attain considerable hardness very quickly. On the other hand, the quickness of setting may be a serious disadvantage, because it may not allow sufficient time to finish the concrete work satisfactorily and prevent the disturbance of mortar which has already taken an initial set. Natural cement is still largely used, on account of its cheapness, especially when the cement is not required to have very great strength. The disadvantage due to its quick setting (when it is a disadvantage) may be somewhat overcome by the use of a small percentage of lime when mixing up the mortar.

It is not always admitted, at least in the advertisements, that a given brand of cement is a natural cement; and the engineer must therefore be on his guard, in buying a cement, to know whether it is a quick-setting natural cement of comparatively low strength or a true Portland cement.

43. **Portland Cement.** Portland cement consists of the product of burning and grinding an artificial mixture of carbonate of lime and clay or slag, the mixture being very carefully proportioned so that the ingredients shall have very nearly the fixed ratio which experience has demonstrated to give the best results.

> "If a deposit of stone containing exactly the right amount of clay, and of exactly uniform composition, could be found, Portland cement could be made from it, simply by burning and grinding. For good results, however, the composition of the raw material must be *exact*, and the proportion of carbonate of lime in it must not vary even by one per cent. No natural deposit of rock of exactly this correct and unvarying composition is known or likely ever to be found; therefore Portland cement is always made from an artificial mixture, usually, if free from organic matter, containing about 75 per cent carbonate of lime and 25 per cent clay."—S. B. NEWBERRY, in Taylor and Thompson's "Concrete Plain and Reinforced."

As before stated, Portland cement is stronger than natural cement; it sets more slowly, which is frequently a matter of great advantage, and yet its rate of setting is seldom so slow that it is a disadvantage. Although the cost is usually greater than that of natural cement, yet improved methods of manufacture have reduced its cost so that it is now usually employed for all high-grade work where high ultimate strength is an important consideration.

In a general way, it may be said that the characteristics of Portland cement on which its value as a material to be used in construction work chiefly depends may be briefly indicated as follows :

When the cement is mixed with water and allowed to set, it should harden in a few hours, and should develop a considerable proportion of its ultimate strength in a few days. It should also possess the quality of permanency so that no material change in form or volume will take place on account of its inherent qualities or as the result of exterior agencies. There is always found to be more or less of shrinkage in the volume of cement and concrete during the process of setting and hardening; but with any cement of really good quality, this shrinkage is not so great as to prove objectionable. Another very important characteristic is that the cement shall not lose its strength with age. Although some long-time tests of cement have apparently indicated a slight decrease in the strength of cement after the first year or so, this decrease is nevertheless so slight that it need not affect the design of concrete, even assuming the accuracy of the general statement.

To insure absolute dependence on the strength and durability of any cement which it is proposed to use in important structural work, it is essential that the qualities of the cement be determined by thorough tests.

CEMENT TESTING

44. The thorough testing of cement, as it is done for the largest public works, should properly be done in a professional testing laboratory. A textbook of several hundred pages has recently been written on this subject. The ultimate analysis and testing of cement, both chemically and physically, is beyond the province of the ordinary engineer. But the ordinary engineer does have frequent occasion

to obtain cement in small quantities when testing in professional laboratories is inconvenient or unduly expensive. Fortunately it is possible to make some simple tests without elaborate apparatus which will at least show whether the cement is radically defective and unfit for use. It is unfortunately true that an occasional barrel of even the best brand of cement will prove to be very inferior to the standard output of that brand. This practically means that in any important work, using a large quantity of cement, it is not sufficient to choose a brand, as the result of preliminary favorable tests, and then accept all shipments without further test. Several barrels in every carload should be sampled for testing. It is not too much to prescribe that *every* barrel should be tested by at least a few of the simpler forms of testing given below. The following methods of testing are condensed from the progress report of the Committee on Uniform Tests of Cement, as selected by the American Society of Civil Engineers. The statements may therefore be considered as having the highest authority obtainable on this subject.

45. **Sampling.** The number of samples that should be taken depends on the importance of the work but it is chiefly important that the sample should represent a fair average of the contents. The sample should be passed through a sieve having twenty meshes per linear inch, in order to break up lumps and remove any foreign material. If several small amounts are taken from different parts of the package, this also insures that the samples will be mixed so that the result will be a fair average. When it is only desired to determine the average characteristic of a shipment, the samples taken from different parts of the shipment may be mixed, but it will give a better idea of the uniformity of the product to analyze the different samples separately. Cement should be taken from a barrel by boring a hole through the center of one of the staves, midway between the heads, or through the head. A portion of the cement can then be withdrawn, even from the center, by means of a sampling iron similar to that used by sugar inspectors.

46. **Chemical Analysis.** Ordinarily, it is impracticable for an engineer to make a chemical analysis of cement which will furnish reliable information regarding its desirability, but the engineer should understand something regarding the desirable chemical constituents of the cement. It should be realized that the fineness

of the grinding and the thoroughness of the burning may have a far greater influence on the value of the cement than slight variations from the recognized standard proportions of the various chemical constituents. Too high a proportion of lime will cause failure in the test for soundness or constancy of volume, although a cement may fail on such a test owing to improper preparation of the raw material or defective burning. On the other hand, if the cement is made from very finely ground material and is thoroughly burned, it may contain a considerable excess of lime and still prove perfectly sound. The permissible amount of magnesia in Portland cement is the subject of considerable controversy. Some authorities say that anything in excess of 8 per cent is harmful, others declare that the amount should not exceed 4 per cent or 5 per cent. The proportion of sulphuric-anhydride should not exceed 1.75 per cent. It may be considered that the other tests of cement are a far more reliable indication of its quality than any small variation in the chemical constituents from the proportions usually considered standard.

47. **Specific Gravity.** The specific gravity of cement is lowered by *under-burning*, *adulteration*, and *hydration*, but the adulteration must be in considerable quantities to affect the results. Since the differences in specific gravity are usually very small, great care must be exercised in making the tests. When properly made, the tests afford a quick check for under-burning or adulteration. The determination of specific gravity is conveniently made with Le Chatelier's apparatus. This consists of a flask D, Fig. 1, of 120-cu. cm. (7.32-cu. in.) capacity, the neck of which is about 20 cm. (7.87 in.) long; in the middle of this neck is a ball C, above and below which are two marks F and E; the volume between these marks is 20 cu. cm. (1.22 cu. in.). The neck has a diameter of about 9 mm. (0.35 in.), and is graduated into tenths of cu. cm. above the mark F. Benzine (62° Baumé naphtha), or kerosene free from water, should be used in making the determination.

The specific gravity may be determined in two ways:

First. The flask is filled with either of these liquids to the lower mark E, and 64 gr. (2.25 oz.) of powder, previously dried at 100° Cent. (212° Fahr.) and cooled to the temperature of the liquid, is gradually introduced through the funnel B (the stem of which extends into the flask to the top of the bulb C) until the proper mark F is reached. The difference in weight between the cement remaining and the original quantity (64 gr.) is the weight which has displaced 20 cu. cm.

Second. The whole quantity of powder is introduced, and the level of the liquid rises to some division of the graduated neck. This reading plus 20 cu. cm. is the volume displaced by 64 gr. of the powder. The specific gravity is then obtained from the formula:

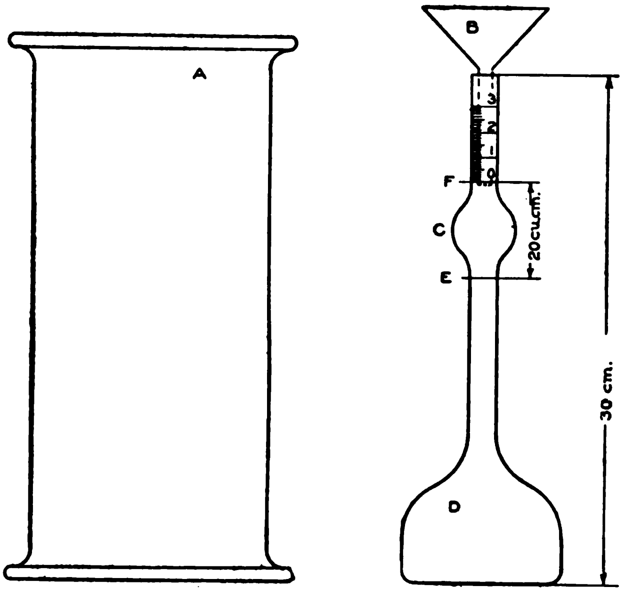


Fig. 1. Le Chatelier's Apparatus for Determining Specific Gravity.

$$\text{Specific Gravity} = \frac{\text{Weight of cement}}{\text{Displaced volume}}.$$

The flask during the operation is kept in water in a jar A in order to avoid variation in the temperature of the liquid. The results should agree within 0.01.

48. **Fineness.** It is generally accepted that the coarser materials in cement are practically inert, and it is only the extremely fine powder that possesses adhesive cementing qualities. The more finely

cement is pulverized, all other conditions being the same, the more sand it will carry and produce a mortar of a given strength. The degree of pulverization which the cement receives at the place of manufacture is ascertained by measuring the residue retained on certain sieves. Those known as No. 100 and No. 200 sieves are recommended for this purpose. The sieve should be circular, about 20 cm. (7.87 inches) in diameter, 6 cm. (2.36 inches) high, and provided with a pan 5 cm. (1.97 inches) deep, and a cover. The wire cloth should be woven from brass wire having the following diameters: No. 100, 0.0045 inches; No. 200, 0.0024 inches. This cloth should be mounted on the frame without distortion. The mesh should be regular in spacing and be within the following limits:

No. 100, 96 to 100 meshes to the linear inch.
No. 200, 188 to 200 meshes to the linear inch.

50 grams (1.76 oz.) or 100 gr. (3.52 oz.) should be used for the test and dried at a temperature of 100° Cent. or 212° Fahr., prior to sieving.

The thoroughly dried and coarsely screened sample is weighed and placed on the No. 200 sieve, which, with pan and cover attached, is held in one hand in a slightly inclined position, and moved forward and backward, at the same time striking the side gently with the palm of the other hand, at the rate of about 200 strokes per minute. The operation is continued until not more than $\frac{1}{10}$ of 1 per cent passes through after one minute of continuous sieving. The residue is weighed, then placed on the No. 100 sieve and the operation repeated. The work may be expedited by placing in the sieve a small quantity of large shot. The results should be reported to the nearest tenth of 1 per cent.

49. **Normal Consistency.** The use of a proper percentage of water in making the pastes, cement and water, from which pats, tests of setting, and briquettes are made, is exceedingly important, and affects vitally the results obtained. The determination consists in measuring the amount of water required to reduce the cement to a given state of plasticity, or to what is usually designated the normal *consist-*

ency. Various methods have been proposed for making this determination, none of which has been found entirely satisfactory. The Committee recommends the following:

The apparatus for this test consists of a frame K, Fig. 2, bearing a movable rod L, with the cap A at one end, and at the other the cylinder B, 1 cm. (0.39 in.) in diameter, the cap, rod, and cylinder weighing 300 gr. (10.58 oz.). The rod, which can be held in any desired position by a screw F, carries an indicator, which moves over a scale (graduated to centimeters) attached to the frame K. The paste is held by a conical, hard-rubber ring I, 7 cm. (2.76 in.) in diameter at the base, 4 cm. (1.57 in.) high, resting on a glass plate J about 10 cm. (3.94 in. square).

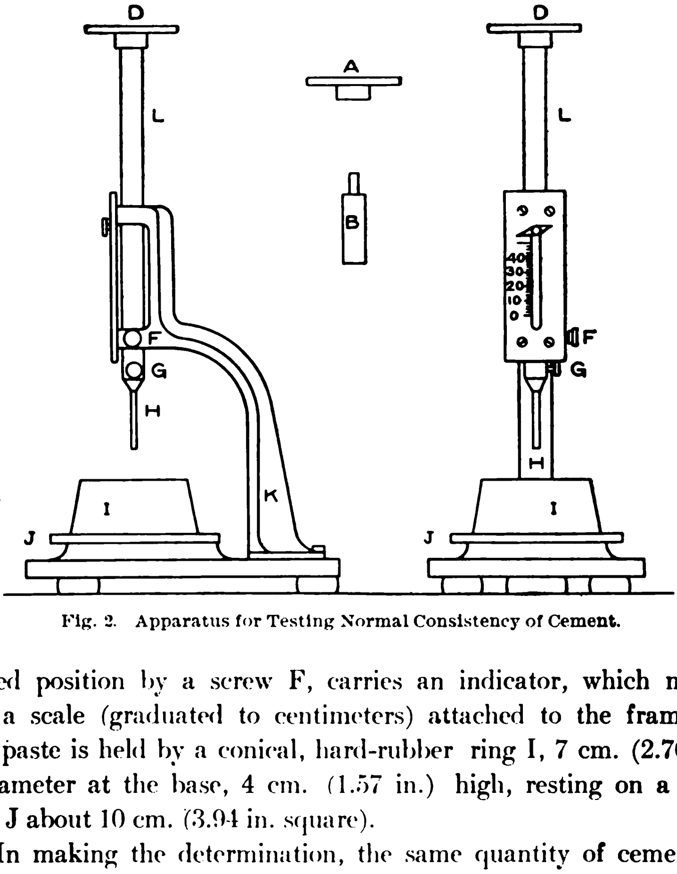


Fig. 2. Apparatus for Testing Normal Consistency of Cement.

In making the determination, the same quantity of cement as will be subsequently used for each batch in making the briquettes (but not less than 500 grams) is kneaded into a paste, as described later in paragraph on "Mixing," and quickly formed into a ball with the hands, completing the operation by tossing it six times from one hand to the other, maintained 6 inches apart; the ball is then pressed into the rubber ring, through the larger opening, smoothed off, and placed (on its large end) on a glass plate and the smaller end smoothed off with a trowel; the paste confined in the ring, resting on the plate, is placed under the rod bearing the cylinder, which is brought in contact with the surface and quickly released.

The paste is of normal consistency when the cylinder penetrates to a point in the mass 10 mm. (0.39 in.) below the top of the ring. Great care must be taken to fill the ring exactly to the top. The trial pastes are made with varying percentages of water until the correct consistency is obtained. The Committee has recommended, as normal, a paste the consistency of which is rather wet, because it believes that variations in the amount of compression to which the briquette is subjected in moulding are likely to be less with such a paste. Having determined in this manner the proper percentage of water required to produce a paste of normal consistency, the proper percentage required for the mortars is obtained from an empirical formula. The Committee hopes to devise a formula. The subject proves to be a very difficult one, and, although the Committee has given it much study, it is not yet prepared to make a definite recommendation.

Note. The Committee on Standard Specifications for Cement inserts the following table for temporary use to be replaced by one to be devised by the Committee of the American Society of Civil Engineers.

TABLE II

Percentage of Water for Standard Sand Mortars

PERCENTAGE OF WATER FOR NEAT CEMENT	ONE CEMENT THREE STANDARD OTTAWA SAND	PERCENTAGE OF WATER FOR NEAT CEMENT	ONE CEMENT THREE STANDARD OTTAWA SAND	PERCENTAGE OF WATER FOR NEAT CEMENT	ONE CEMENT THREE STANDARD OTTAWA SAND
15	8.0	23	9.3	31	10.7
16	8.2	24	9.5	32	10.8
17	8.3	25	9.7	33	11.0
18	8.5	26	9.8	34	11.2
19	8.7	27	10.0	35	11.5
20	8.8	28	10.2	36	11.5
21	9.0	29	10.3	37	11.7
22	9.2	30	10.5	38	11.8

	1 to 1	1 to 2	1 to 3	1 to 4	1 to 5
Cement.....	500	333	250	200	167
Sand	500	666	750	800	833

50. **Time of Setting.** The object of this test is to determine the time which elapsed from the moment water is added until the paste

ceases to be fluid and plastic (called the "initial set"), and also the time required for it to acquire a certain degree of hardness (called the "final" or "hard set"). The former of these is the more important, since, with the commencement of setting, the process of crystallization or hardening is said to begin. As a disturbance of this process may produce a loss of strength, it is desirable to complete the operation of mixing and moulding or incorporating the mortar into the work before the cement begins to set. It is usual to measure arbitrarily the beginning and end of the setting by the penetration of weighted wires of given diameters.

For this purpose the Vicat Needle, which has already been described, should be used. In making the test, a paste of normal consistency is moulded and placed under the rod L, Fig. 2, as described in a previous paragraph. This rod bears the cap D at one end and the needle H, 1 mm. (0.039 in.) in diameter, at the other, and weighs 300 gr. (10.58 oz.). The needle is then carefully brought in contact with the surface of the paste and quickly released. The setting is said to have commenced when the needle ceases to pass a point 5 mm. (0.20 in.) above the upper surface of the glass plate, and is said to have terminated the moment the needle does not sink visibly into the mass.

The test pieces should be stored in moist air during the test; this is accomplished by placing them on a rack over water contained in a pan and covered with a damp cloth, the cloth to be kept away from them by means of a wire screen; or they may be stored in a moist box or closet. Care should be taken to keep the needle clean, as the collection of cement on the sides of the needle retards the penetration, while cement on the point reduces the area and tends to increase the penetration. The determination of the time of setting is only approximate, being materially affected by the temperature of the mixing water, the temperature and humidity of the air during the test, the percentage of water used, and the amount of moulding the paste receives.

The following approximate method, not requiring the use of apparatus, is sometimes used, although not referred to by the Committee. Spread cement paste of the proper consistency on a piece of glass, having the cement cake about three inches in diameter and about one inch thick at the center, thinning towards the edges. When the cake is hard enough to bear a gentle pressure of the finger nail, the cement has begun to set, and when it is not indented by a considerable pressure of the thumb nail, it is said to have set.

51. **Standard Sand.** The Committee recognizes the grave objections to the standard quartz now generally used, especially on account of its high percentage of voids, the difficulty of compacting in the moulds, and its lack of uniformity; it has spent much time in investigating the various natural sands which appeared to be available and suitable for use. For the present, the Committee recommends the natural sand from Ottawa, Ill., screened to pass a sieve having 20 meshes per linear inch and retained on a sieve having 30 meshes per linear inch; the wires to have diameters of 0.0165 and 0.0112 inches, respectively, i.e., half the width of the opening in each case. Sand having passed the No. 20 sieve shall be considered standard when

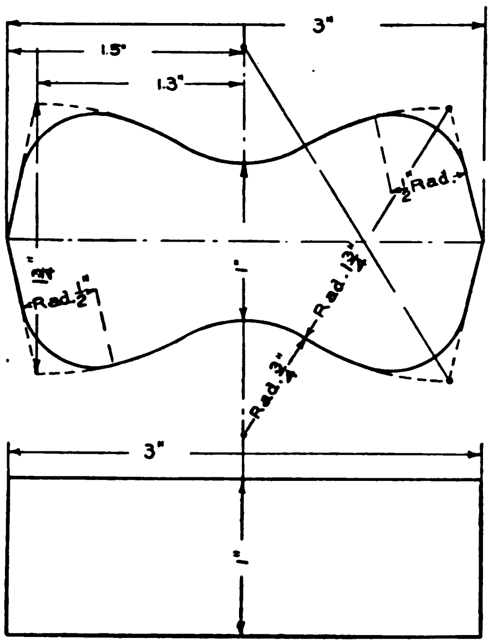


Fig. 3. Form of Briquette.

not more than one per cent passes a No. 30 sieve after one minute continuous sifting of a 500-gram sample.

52. **Form of Briquette.** While the form of the briquette recommended by a former Committee of the Society is not wholly satisfactory, this Committee is not prepared to suggest any change, other than rounding off the corners by curves of ½-inch radius, Fig. 3.

53. **Moulds.** The moulds should be made of brass, bronze, or some equally non-corrodible material, having sufficient metal in the sides to prevent spreading during moulding.

Gang moulds, which permit moulding a number of briquettes at one time, are preferred by many to single moulds; since the greater quantity of mortar that can be mixed tends to produce greater uniformity in the results. The type shown in Fig. 4 is recommended. The moulds should be wiped with an oily cloth before using.

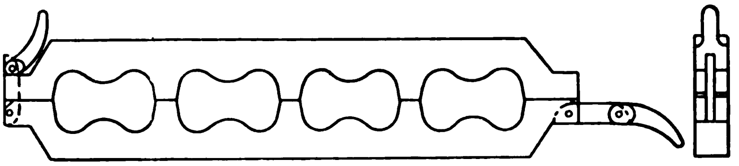


Fig. 4. Gang Moulds.

54. **Mixing.** All proportions should be stated by weight; the quantity of water to be used should be stated as a percentage of the dry material. The metric system is recommended because of the convenient relation of the gram and the cubic centimeter. The temperature of the room and the mixing water should be as near 21° Cent. (70° Fahr.) as it is practicable to maintain it. The sand and cement should be thoroughly mixed dry. The mixing should be done on some non-absorbing surface, preferably plate glass. If the mixing must be done on an absorbing surface it should be thoroughly dampened prior to use. The quantity of material to be mixed at one time depends on the number of test pieces to be made; about 1000 gr. (35.28 oz.) makes a convenient quantity to mix, especially by hand methods.

The material is weighed and placed on the mixing table, and a crater formed in the center, into which the proper percentage of clean water is poured; the material on the outer edge is turned into the crater by the aid of a trowel. As soon as the water has been absorbed, which should not require more than one minute, the operation is completed by vigorously kneading with the hands for an additional 1½ minutes, the process being similar to that used in kneading dough. A sand-glass affords a convenient guide for the time of kneading. During the operation of mixing the hands should be protected by gloves, preferably of rubber.

55. **Moulding.** Having worked the paste or mortar to the proper consistency, it is at once placed in the moulds by hand. The moulds should be filled at once, the material pressed in firmly with the fingers and smoothed off with a trowel without ramming; the material should be heaped up on the upper surface of the mould, and, in smoothing off, the trowel should be drawn over the mould in such a manner as to exert a moderate pressure on the excess material. The mould should be turned over and the operation repeated. A check upon the uniformity of the mixing and moulding is afforded by weighing the briquettes just prior to immersion, or upon removal from the moist closet. Briquettes which vary in weight more than 3 per cent from the average should not be tested.

56. **Storage of the Test Pieces.** During the first 24 hours after moulding, the test pieces should be kept in moist air to prevent them from drying out. A moist closet or chamber is so easily devised that the use of the damp cloth should be abandoned if possible. Covering the test pieces with a damp cloth is objectionable, as commonly used, because the cloth may dry out unequally, and, in consequence, the test pieces are not all maintained under the same condition. Where a moist closet is not available, a cloth may be used and kept uniformly wet by immersing the ends in water. It should be kept from direct contact with the test pieces by means of a wire screen or some similar arrangement.

A moist closet consists of a soapstone or slate box, or a metal-lined wooden box: the metal lining being covered with felt and this felt kept wet. The bottom of the box is so constructed as to hold water, and the sides are provided with cleats for holding glass shelves on which to place the briquettes. Care should be taken to keep the air in the closet uniformly moist. After 24 hours in moist air the test pieces for longer periods of time should be immersed in water maintained as near 21° Cent. (70° Fahr.) as practicable; they may be stored in tanks or pans, which should be of non-corrodible material.

57. Tensile Strength. The tests may be made on any standard machine. A solid metal clip, as shown in Fig. 5, is recommended. This clip is to be used without cushioning at the points of contact with the test specimen. The bearing at each point of contact should be ¼-inch wide, and the distance between the center of contact on the same clip should be 1¼ inches. Test pieces should be broken as soon as they are removed from the water. Care should be observed in centering the briquettes in the testing machine, as cross-strains, produced by improper centering, tend to lower the breaking strength. The load should not be applied too suddenly, as it may produce vibration, the shock from which often breaks the briquette before the ultimate strength is reached. Care must be taken that the clips and the sides of the briquette be clean and free from grains of sand or dirt, which would prevent a good bearing. The load should be applied at the rate of 600 lbs. per minute. The average of the briquettes of each sample tested should be taken as the test, excluding any results which are manifestly faulty.

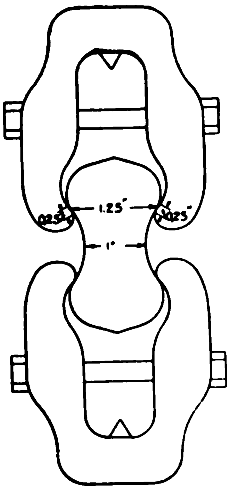


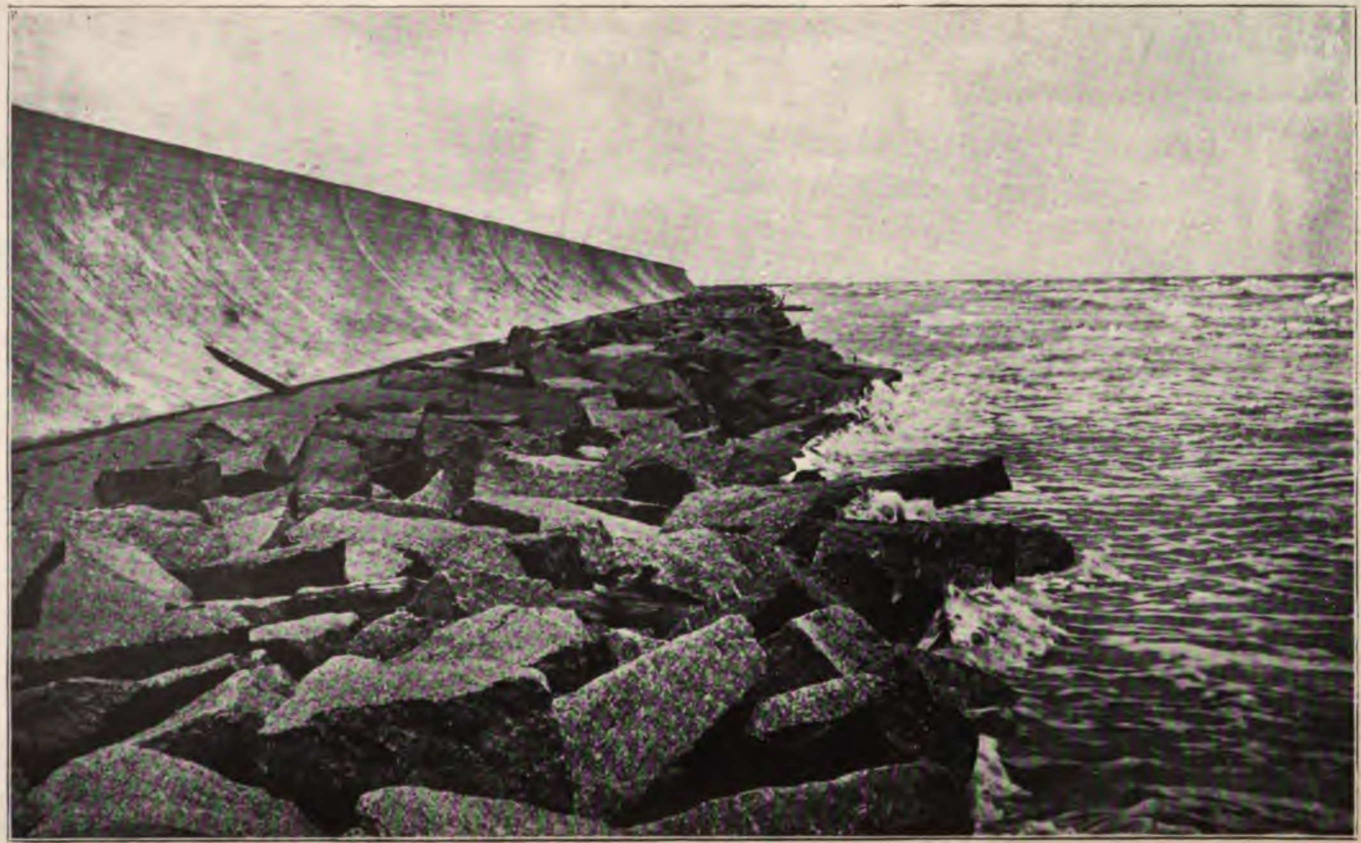
Fig. 5. Metal Clip for Testing Tensile Strength.

58. Constancy of Volume. The object is to develop those qualities which tend to destroy the strength and durability of a cement. As it is highly essential to determine such qualities at once, tests of this character are for the most part made in a very short time, and are known, therefore, as accelerated tests. Failure is revealed by cracking, checking, swelling, or disintegration, or all of these phenomena. A cement which remains perfectly sound is said to be of constant volume.

Methods. Tests for constancy of volume are divided into two classes:

(1) Normal tests, or those made in either air or water maintained at about 21° Cent. (70° Fahr.).

(2) Accelerated tests, or those made in air, steam, or water at a temperature of 45° Cent. (115° Fahr.) and upward. The test pieces should be allowed to remain 24 hours in moist air before immersion in water or steam, or preservation in air. For these tests, pats,



PART OF THE SEA WALL OF GALVESTON, TEXAS

Built of concrete resting on piles protected by sheet piling and riprap. Wall tapers from 16 ft. thick at base to 5 ft. at top, 17 ft. above sea-level. Total length of wall, 17,593 ft., or about 3⅓ miles. Cost, $1,500,000. Designed to protect city from recurrence of disastrous overflows due to such hurricanes as that of September, 1900. The work includes also a raising of the grade of the entire city, costing about $2,000,000.

about 7½ cm. (2.95 in.) in diameter, 1¼ cm. (0.49 in.) thick at the center, and tapering to a thin edge, should be made, upon a clean glass plate [about 10 cm. (3.94 in.) square], from cement paste of normal consistency.

Normal Test. A pat is immersed in water maintained as near 21° Cent. (70° Fahr.) as possible for 28 days, and observed at intervals. A similar pat is maintained in air at ordinary temperature and observed at intervals.

Accelerated Test. A pat is exposed in any convenient way in an atmosphere of steam, above boiling water, in a loosely closed vessel, for 3 hours.

To pass these tests satisfactorily, the pats should remain firm and hard, and show no signs of cracking, distortion, or disintegration. Should the pat leave the plate, distortion may be detected best with a straight-edge applied to the surface which was in contact with the plate. In the present state of our knowledge it cannot be said that cement should necessarily be condemned simply for failure to pass the accelerated tests; nor can a cement be considered entirely satisfactory, simply because it has passed these tests.

59. General Conditions. The committee recommends that:

All cement shall be inspected.

Cement may be inspected either at the place of manufacture or on the work.

In order to allow ample time for inspecting and testing, the cement should be stored in a suitable weather-tight building having the floor properly blocked or raised from the ground.

The cement shall be stored in such a manner as to permit easy access for proper inspection and identification of each shipment.

Every facility shall be provided by the contractor, and a period of at least twelve days allowed for the inspection and necessary tests.

Cement shall be delivered in suitable packages, with the brand and name of manufacturer plainly marked thereon.

A bag of cement shall contain 94 pounds of cement, net. Each barrel of Portland cement shall contain 4 bags, and each barrel of natural cement shall contain 3 bags of the above net weight.

Cement failing to meet the 7-day requirements may be held awaiting the results of the 28-day tests, before rejection.

All tests shall be made in accordance with the methods proposed by the Committee on Uniform Tests of Cement of the American Society of Civil Engineers, presented to the Society January 21, 1903, and amended January 20, 1904, with all subsequent amendments thereto.

The acceptance or rejection shall be based on the following requirements:

NATURAL CEMENT

60. **Definition.** This term shall be applied to the finely pulverized product resulting from the calcination of an argillaceous limestone at a temperature only sufficient to drive off the carbonic acid gas.

61. **Specific Gravity.** The specific gravity of the cement thoroughly dried at 100° C., shall be not less than 2.8.

62. **Fineness.** It shall leave by weight a residue of not more than 10 per cent on the No. 100, and 30 per cent on the No. 200 sieve.

63. **Time of Setting.** It shall develop initial set in not less than ten minutes, and hard set in not less than thirty minutes, nor more than three hours.

64. **Tensile Strength.** The minimum requirements for tensile strength for briquettes one inch square in cross-section, shall be within the following limits, and shall show no retrogression in strength within the periods specified:

NEAT CEMENT

Age	Strength
24 hours in moist air	50–100 lbs.
7 days (1 day in moist air, 6 days in water)	100–200 "
28 days (1 day in moist air, 27 days in water)	200–300 "
ONE PART CEMENT, THREE PARTS STANDARD SAND	
7 days (1 day in moist air, 6 days in water)	25– 75 "
28 days (1 day in moist air, 27 days in water)	75–150 "

65. **Constancy of Volume.** Pats of neat cement about three inches in diameter, one-half inch thick at the center, and tapering to a thin edge, shall be kept in moist air for a period of twenty-four hours.

(*a*) A pat is then kept in air at normal temperature.

(*b*) Another is kept in water maintained as near 70° F. as practicable.

These pats are observed at intervals for at least 28 days, and, to pass the tests satisfactorily, should remain firm and hard and show no signs of distortion, cracking, or disintegrating.

PORTLAND CEMENT

66. **Definition.** This term is applied to the finely pulverized product resulting from the calcination to incipient fusion of an intimate mixture of properly proportioned argillaceous and cal-

careous materials, to which no addition greater than 3 per cent has been made subsequent to calcination.

67. **Specific Gravity.** The specific gravity of the cement, thoroughly dried at 100° C., shall be not less than 3.10.

68. **Fineness.** It shall leave by weight a residue of not more than 8 per cent on the No. 100 sieve, and not more than 25 per cent on the No. 200 sieve.

69. **Time of Setting.** It shall develop initial set in not less than thirty minutes, and must develop hard set in not less than one hour nor more than ten hours.

70. **Tensile Strength.** The minimum requirements for tensile strength for briquettes one inch square in section, shall be within the following limits, and shall show no retrogression in strength within the periods specified:

Age	Strength
NEAT CEMENT	
24 hours in moist air	150-200 lbs.
7 days (1 day in moist air, 6 days in water)	450-550 "
28 days (1 day in moist air, 27 days in water)	550-650 "
ONE PART CEMENT, THREE PARTS SAND	
7 days (1 day in moist air, 6 days in water)	150-200 "
28 days (1 day in moist air, 27 days in water)	200-300 "

Constancy of Volume. Pats of neat cement about three inches in diameter, one-half inch thick at the center, and tapering to a thin edge, shall be kept in moist air for a period of twenty-four hours.

(*a*) A pat is then kept in air at normal temperature and observed at intervals for at least 28 days.

(*b*) Another pat is kept in water maintained as near 70° F. as practicable, and observed at intervals for at least 28 days.

(*c*) A third pat is exposed in any convenient way in an atmosphere of steam, above boiling water, in a loosely closed vessel, for five hours.

These pats, to pass the requirements satisfactorily, shall remain firm and hard, and show no signs of distortion, checking, cracking, or disintegrating.

71. **Sulphuric Acid and Magnesia.** The cement shall not contain more than 1.75 per cent of anhydrous sulphuric acid (SO_3), and not more than 4 per cent of magnesia (MgO).

72. **Testing Machines.** There are many varieties of testing

machines on the market. Many engineers have constructed "home-made" machines which serve their purpose with sufficient accuracy. One very common type of machine is illustrated in Fig. 6. *B* is a reservoir containing shot, which falls through the pipe *I*, which is closed with a valve at the bottom. The briquette is carefully placed between the clips, as shown in the figure, and the wheel *P* is turned until the indicators are in line. The hook lever *Y* is moved so that a screw worm is engaged with its gear Then open the automatic valve *J* so as to allow the shot to run into the cup *F*. By means of a small valve, the flow of shot into the cup may be regulated. Better results will be obtained by allowing the shot to run slowly into the cup. The crank is then turned with just sufficient speed so that the scale beam is held in position until the briquette is broken. Upon the breaking of the briquette, the scale beam falls, and automatically closes the valve *J*. The weight of the shot in the cup *F* then indicates, according to some definite ratio, the stress required to break the briquette.

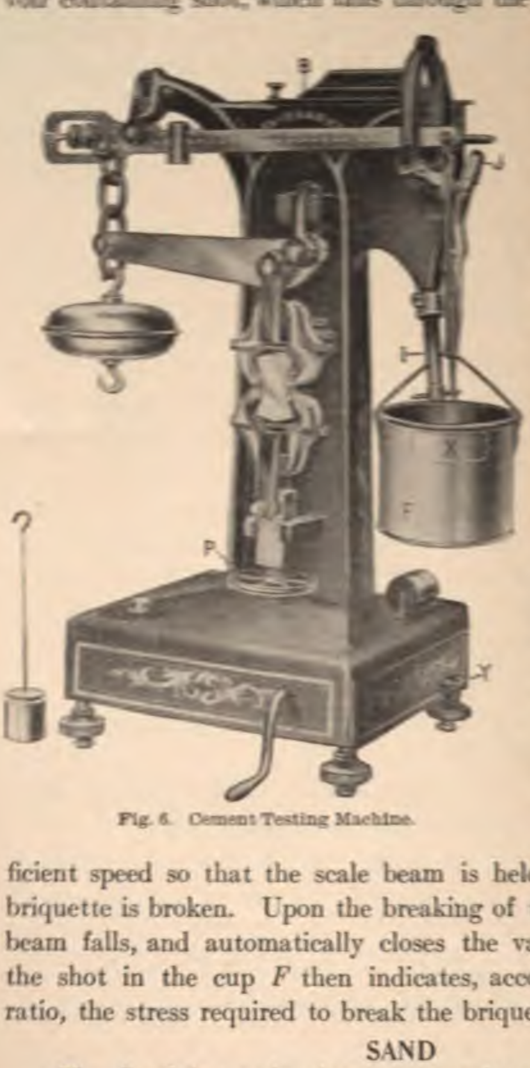


Fig. 6. Cement Testing Machine.

SAND

73. Sand is nearly always a constituent part of mortar and concrete. The strength of the masonry is dependent to a considerable extent on the qualities of the sand, and it is therefore important that the desirable and the defective qualities should be understood.

74. **Object.** The chief object of the sand is economy. If the joints between stones, especially in rubble masonry, were filled with a paste of neat cement, the cost would be excessive, and the increase in the strength of the masonry, if any, would be utterly disproportionate to the great increase in cost. Secondly, the use of sand is a practical necessity in lime mortar, since neat lime will contract and crack very badly when it hardens.

75. **Essential Qualities.** The word "sand" as used above is intended as a generic term to apply to any finely divided material which will not injuriously affect the cement or lime, and which is not subject to disintegration or decay. Sand is almost the only material that is sufficiently cheap, which will fulfil these requirements, although stone screenings (the finest material coming from a stone crusher), powdered slag, and even coal dust have occasionally been used as substitutes. Specifications usually demand that the sand shall be "sharp, clean, and coarse," and such terms have been repeated so often that they are accepted as standard notwithstanding the frequent demonstration that modifications of these terms are not only desirable but also economical. These words also ignore other qualities which should be considered, especially when deciding between two or more different sources of sand supply.

76. **Geological Character.** Quartz sand is the most durable and unchangeable Sands which consist largely of grains of feldspar, mica, hornblende, etc., which will decompose upon prolonged exposure to the atmosphere, are less desirable than quartz, although, after being made up into the mortar, they are virtually protected against further decomposition.

77. **Coarseness.** A mixture of coarse and fine grains, with the coarse grains predominating, is found very satisfactory, as it makes a denser and stronger concrete with a less amount of cement than when coarse-grained sand is used with the same proportion of cement. The small grains of sand fill the voids caused by the coarse grains so that there is not so great a volume of voids to be filled by the cement. The sharpness of sand can be determined approximately by rubbing a few grains in the hand or by crushing it near the ear and noting if a grating sound is produced; but an examination through a small lens is better.

78. **Sharpness.** Experiments have shown that round grains of

sand have less voids than angular ones, and that water-worn sands have from 3 per cent to 5 per cent less voids than corresponding sharp grains. In many parts of the country where it is impossible, except at a great expense, to obtain the sharp sand, the round grain is used with very good results. Laboratory tests made under conditions as nearly as possible identical, show that the rounded-grain sand gives as good results as the sharp sand. In consequence of such tests, the requirement that sand shall be *sharp* is now considered useless by many engineers, especially when it leads to additional cost.

79. **Cleanness.** In all specifications for concrete work, is found the clause: "The sand shall be clean." This requirement is sometimes questioned, as experimenters have found that a small percentage of clay or loam often gives better results than when clean sand is used. "Lean" mortar may be improved by a small percentage of clay or loam, or by using dirty sand, for the fine material increases the density. In rich mortars, this fine material is not needed, as the cement furnishes all the fine material necessary, and if clay or loam or dirty sand were used, it might prove detrimental. Whether it is really a benefit or not, depends chiefly upon the richness of the concrete and the coarseness of the sand. Some idea of the cleanliness of sand may be obtained by placing it in the palm of one hand and rubbing it with the fingers of the other. If the sand is dirty, it will badly discolor the palm of the hand. When it is found necessary to use dirty sand, the strength of the concrete should be tested.

Sand containing loam or earthy material is cleansed by washing with water, either in a machine specially designed for the purpose, or by agitating the sand with water in boxes provided with holes to permit the dirty water to flow away.

Very fine sand may be used alone, but it makes a weaker concrete than either coarse sand or coarse and fine sand mixed. A mortar consisting of very fine sand and cement will not be so dense as one of coarse sand and the same cement, although, when measured or weighed dry, both contain the same proportion of voids and solid matter. In a unit measure of fine sand, there are more grains than in a unit measure of coarse sand, and therefore more points of contact. More water is required in gauging a mixture of fine sand and cement than in a mixture of coarse sand and the same cement.

The water forms a film and separates the grains, thus producing a larger volume having less density.

The screenings of broken stone are sometimes used instead of sand. Tests frequently show a stronger concrete when screenings are used than when sand is used. This is perhaps due to the variable sizes of the screenings, which would have a less percentage of voids.

80. **Percentage of Voids.** As before stated, a mortar is strongest when composed of fine and coarse grains mixed in such proportion that the percentage of voids shall be the least. The simplest method of comparing two sands is to weigh a certain gross volume of each, the sand having been thoroughly shaken down. Assuming that the stone itself of each kind of sand has the same density, then the heavier volume of sand will have the least percentage of voids. The actual percentage of voids in packed sand may be approximately determined by measuring the volume of water which can be added to a given volume of packed sand. If the water is poured into the sand, it is quite certain that air will remain in the voids in the sand, which will not be dislodged by the water, and the apparent volume of voids will be *less* than the actual. The precise determination involves the measurement of the specific gravity of the stone of which the sand is composed, and the percentage of moisture in the sand, all of which is done with elaborate precautions. Ordinarily such precise determinations are of little practical value, since the product of any one sandbank is quite variable. While it would be theoretically possible to mix fine and coarse sand, varying the ratios according to the varying coarseness of the grains as obtained from the sand-pit, it is quite probable that an over-refinement in this particular would cost more than the possible saving is worth. Ordinarily sand has from 28 to 40 per cent of voids. An experimental test of sand of various degrees of fineness, 12½ per cent of it passing a No. 100 sieve, showed only 22 per cent of voids; but such a value is of only theoretical interest.

BROKEN STONE

81. This term ordinarily signifies the product of a stone crusher or the result of hand-breaking by hammering large blocks of stone; but the term may also include *gravel*, described below.

82. **Classification of Stones.** The best, hardest, and most

durable broken stone comes from the *trap rocks*, which are dark, heavy, close-grained rocks of igneous origin. The term *granite* is usually made to include not only true granite, but also gneiss, mica schist, syenite, etc. These are just as good for concrete work, and are usually less expensive. *Limestone* is suitable for some kinds of concrete work; but its strength is not so great as that of granite or trap rock, and it is more affected by a conflagration. *Conglomerate*, often called *pudding stone*, makes a very good concrete stone. The value of *sandstone* for concrete is very variable according to its texture. Some grades are very compact, hard, and tough, and make a good concrete; other grades are friable, and, like *shale* and *slate*, are practically unfit for use. *Gravel* consists of pebbles of various sizes, produced from stones which have been broken up and then worn smooth with rounded corners. The very fact that they have been exposed for indefinite periods to atmospheric disintegration and mechanical wear, is a proof of the durabilty and mechanical strength of the stone.

83. **Size of Stone and its Uniformity.** There is hardly any limitation to the size of stone which may be used in large blocks of massive concrete, since it is now frequently the custom to insert these large blocks and fill the spaces between them with a concrete of smaller stone. But the term *broken stone* should be confined to those pieces of a size which may be readily mixed up in a mass, as is done when mixing concrete; and this virtually limits the size to stones which will pass through a 2½-inch ring. The lower limit in size is very indefinite, since the product of a stone crusher includes all sizes down to stone dust screenings, such as are substituted partially or entirely for sand, as previously noted. Practically the only use of broken stone in masonry construction is in the making of concrete; and, since one of the most essential features of good concrete construction is that the concrete shall have the greatest possible density, it is important to reduce the percentage of voids in the stone as much as possible. This percentage can be determined with sufficient accuracy for ordinary unimportant work, by the very simple method previously described for obtaining that percentage with sand—namely, by measuring how much water will be required to fill up the cavities in a given volume of dry stone. As before, such a simple determination is somewhat inexact, owing to the probability that

bubbles of air will be retained in the stone which will reduce the percentage somewhat, and also because of the uncertainty involved as to whether the stone is previously dry or is saturated with water. Some engineers drop the stone slowly into the vessel containing the water, rather than pour the water into the vessel containing the stone, with the idea that the error due to the formation of air bubbles will be decreased by this method. The percentage of error, however, due to such causes, is far less than it is in a similar test of sand, and the error for ordinary work is too small to have any practical effect on the result.

84. *Example.* A pail having a mean inside diameter of 10 inches and a height of 14 inches is filled with broken stone well shaken down; a similar pail filled with water to a depth of 8 inches is poured into the pail of stone until the water fills up all the cavities and is level with the top of the stone; there is still $2\frac{1}{4}$ inches depth of water in the pail. This means that a depth of $5\frac{3}{4}$ inches has been used to fill up the voids. The area of a 10-inch circle is 78.54 square inches and therefore the volume of the broken stone was $78.54 \times 14 = 1{,}099.56$ cubic inches. The volume of the water used to fill the pail was 78.54×5.75, or 451.6 cubic inches. This is 41 per cent of the volume of the stone, and is in this case the percentage of voids. The accuracy of the above computation depends largely on the accuracy of the measurement of the *mean inside diameter* of the pail. If the pail were truly cylindrical, there would be no inaccuracy. If the pail is flaring, the inaccuracy might be considerable; and if a precise value is desired, more accurate methods should be chosen to measure the volume of the stone and of the water.

It is invariably found that unscreened stone or *the run of the crusher* has a far less percentage of voids than screened stone, and it is therefore not only an extra expense, but also an injury to the concrete, to specify that broken stone shall be screened before being used in concrete, unless, as described later, it is intended to mix definite proportions of several sizes of carefully screened broken stone. Since the proportion of large and small particles in the run of the crusher depends considerably upon the character of the stone which is being broken up, and perhaps to some extent on the crusher itself, these proportions should be tested at frequent intervals during the progress of the work; and the amount of sand to be added to make a good concrete should be determined by trial tests, so that the resulting percentage of voids shall be as small as it is practicable to make it. It is usually found that the percentage of voids in crusher-run granite is a little larger than in limestone or gravel. This gives a slight advantage to the limestone and gravel, which tends to compensate for the weakness of the limestone and the rounded corners of the gravel.

85. Broken stone is frequently sold by the ton, instead of by the cubic yard; but as its weight varies from 2,200 to 3,200 pounds per cubic yard, an engineer or contractor is uncertain as to how many cubic yards he is buying or how much it costs him per cubic yard, unless he is able to test the particular stone and obtain an average figure as to its weight per unit of volume.

86. **Cinders.** Cinders for concrete should be free from coal or soot. Usually a better mixture can be obtained by screening the fine stuff from the cinders and then mixing in a larger proportion of sand, than by using unscreened material, although, if the fine stuff is uniformly distributed through the mass, it may be used without screening, and a less proportion of sand used.

As shown later, the strength of cinder concrete is far less than that of stone concrete; and on this account it cannot be used where high compressive values are necessary. But on account of its very low cost compared with broken stone, especially under some conditions, it is used quite commonly for roofs, etc., on which the loads are comparatively small.

One possible objection to the use of cinders lies in the fact that they frequently contain sulphur and other chemicals which may produce corrosion of the reinforcing steel. In any structure where the strength of the concrete is a matter of importance, cinders should not be used without a thorough inspection, and even then the unit compressive values allowed should be at a very low figure.

MORTAR

87. The term *mortar* is usually applied to the mixture of sand and cementing material which is placed between the large stones of a stone structure, although the term might also be properly applied to the matrix of the concrete in which broken stone is embedded. The object of the mortar is to furnish a cushion for the stones above it, which, as far as possible, distributes the pressure uniformly and

relieves the stones of transverse stresses and also from the concentrated crushing pressures to which the projecting points of the stone would be subjected.

88. **Common Lime Mortar.** The first step in the preparation of common lime mortar is the slaking of the lime. This should be done by putting the lime into a water-tight box, or at least on a platform which is substantially water-tight, and on which a sort of pond is formed by a ring of sand. The amount of water to be used should be from 2½ to 3 times the volume of the unslaked lime.

The "volume" of unslaked lime is a very uncertain quantity, varying with the amount of settlement caused by mere shaking which it may receive during transit. A *barrel* of lime means 230 pounds. If the barrel has a volume of 3.75 cubic feet, it would be just filled by 230 pounds of lime when this lime weighed about 61 pounds per cubic foot. This same lime, however, *may* be so shaken that it will weigh 75 pounds per cubic foot, in which case its volume is reduced to 81 per cent, or 3.05 cubic feet. Combining this with 2½ to 3 times its volume of water, will require about 8½ cubic feet of water to one barrel of lime. On the other hand, if the lime has absorbed moisture from the atmosphere, and has become more or less *air-slaked*, its volume may become very materially increased.

Although close accuracy is not necessary, the lime paste will be injured if the amount of water is too much or too little. In short, the amount of water should be as near as possible that which is chemically required to hydrate the lime, so that on the one hand it shall be completely hydrated, and on the other hand it shall not be drowned in an excess of water which will injure its action in ultimate hardening. About three volumes of sand should be used to one volume of lime paste. Owing to the fact that the paste will, to a considerable extent, nearly fill the voids in the sand, the volume obtained from one barrel of unslaked lime made up into a mortar consisting of one part of lime paste to three parts of sand, will make about 6.75 barrels of mortar, or a little less than one cubic yard.

89. **Natural Cement Mortar.** This is largely used, especially when mixed with lime to retard the setting, in the construction of walls of buildings, cellar foundations, and, in general, in masonry where the unit-stresses are so low that strength is a minor consideration, but where a lime mortar would not harden because it is to be

under water or in a solid mass where the carbonic acid of the atmosphere could not penetrate to the interior. When natural cement is dumped loosely in a pile, the apparent volume is increased one-third or even one-half. This must be allowed for in mixing. A *barrel* averages 3.3 cubic feet. Therefore a 1:4 mortar of natural cement would require one barrel of cement to 13.2 cubic feet (about one-half a cubic yard) of sand. A bottomless box similar to that illustrated in Fig. 7, and with inside dimensions of 3 feet × 2 feet 6 inches × 1 foot 9 inches, contains 13.2 cubic feet. It is preferable to use even charges of one barrel of cement in mixing up a batch of mortar, rather than to dump it out and measure it loosely. If the size of the barrel varies from the average value given above, the size of the sand box should be varied accordingly. The barrels coming from any one cement mill may usually be considered as of uniform capacity. Since it is practically somewhat difficult to measure accurately the volume of a barrel, owing to its swelling form, it is best to fill a sample barrel with loose, dry sand, and then to measure the volume of that sand by emptying it into a rectangular box whose inside area, together with the height of sand in it, can be readily measured.

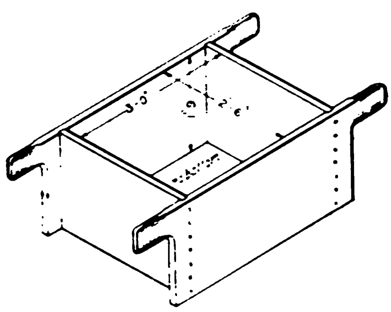


Fig. 7. Bottomless Box for Measuring Sand.

90. **Portland Cement Mortar.** A barrel of Portland cement will contain 370 to 380 pounds, net, of cement. Its capacity averages about 3.3 cubic feet, although with some brands the capacity may reach 3.75. The expansion when the cement is thrown loosely in a pile or into a measuring box, varies from 10 to 40 per cent. The subject will be discussed further under the head of "Concrete."

91. **Lime in Cement Mortar.** Lime is frequently employed in the cement mortar used for buildings, for a combination of reasons:

(*a*) It is unquestionably more economical; but if the percentage added (or that which replaces the cement) is more than about 5 per cent, the strength of the mortar is sacrificed. The percentage of loss of strength depends on the richness of the mortar.

(*b*) When used with a mortar leaner than 1:2, the substitution of about 10 per cent of lime for an equal weight of cement will render concrete more water-tight, although at some sacrifice in strength.

(c) It always makes the mortar *work* more easily and smoothly. In fact, a rich cement mortar is very *brash*; it will not stick to the bricks or stones when striking a joint. It actually increases the output of the masons to use a mortar which is rendered smoother by the addition of lime.

The substitution of more than 20 per cent of lime decreases the strength faster than the decrease in cost, and therefore should not be permitted unless strength is a secondary consideration and the combination is considered more as an addition of cement to a lime mortar in order to render it hydraulic.

92. **Effect of Re-gauging or Re-mixing Mortar.** Specifications and textbooks have repeatedly copied from one another a requirement that all mortar which is not used immediately after being mixed and before it has taken an *initial set* must be rejected and thrown away. This specification is evidently based on the idea that after the initial set has been disturbed and destroyed, the cement no longer has the power of hardening, or at least that such power is very materially and seriously reduced. Repeated experiments, however, have shown that under some conditions the ultimate strength of the mortar (or concrete) is actually increased, and that it is not seriously injured even when the mortar is re-gauged several hours after being originally mixed with water.

Such a specification against re-mixing is never applied to lime paste, since it is well known that a lime paste is considerably improved by being left for several days (or even months) before being used. This is evidently due to the fact that even during such a period the carbonic acid of the atmosphere cannot penetrate appreciably into the mass of the paste, while the greater length of time merely insures a more perfect slaking of the lime. The presence of free, unslaked lime in either lime or cement mortar is always injurious, because it generally results in expansion and disruption and possibly in injurious chemical reaction.

Tests with Portland cement have shown that if it is re-mixed two hours after being combined with water, its strength, both tensile and compressive, is greater after six months' hardening, although it will be less after seven days' hardening, than in similar specimens which are moulded immediately after mixing. It is also found that the re-mixing makes the cement much slower in its setting. The adhesion, moreover, is reduced by re-mixing, which is an important consideration in the use of reinforced concrete.

The effects of tests with natural cement are somewhat contradictory, and this is perhaps the reason for the original writing of such a specification. The result of an elaborate series of tests made by Mr. Thomas F. Richardson showed that quick-setting cements which had been re-mixed showed a considerable falling off in strength in specimens broken after 7 days and 28 days of hardening, yet the ultimate strength after six months of hardening was invariably increased. It is also found that for both Portland and natural cements there is a very considerable increase in the strength of the mortar when it is worked continuously for two hours before moulding or placing in the masonry. Such an increase is probably due to the more perfect mixing of the constituents of the mortar.

The conclusion of the whole matter appears to be, that when it is desirable that considerable strength shall be attained within a few days or weeks (as is generally the case, and especially so with reinforced-concrete work), the specification against re-mixing should be rigidly enforced. For the comparatively few cases where a slow acquirement of the ultimate strength is permissible, re-mixing might be tolerated, although there is still the question whether the expected gain in ultimate strength would pay for the extra work. It would be seldom, if ever, that this claimed property of cement mortar could be relied on to save a batch of mortar which would otherwise be rejected because it had been allowed to stand after being mixed until it had taken an initial set.

93. **Proportions of Materials for Mortar.** (1) *Lime Mortar.* As previously stated in section 88, a barrel of unslaked lime should be mixed with about 8½ cubic feet of water. This will make about 9 cubic feet of lime paste. Mixing this with a cubic yard of sand will make about 1 cubic yard of 1:3 lime mortar. This means approximately 1 volume of unslaked lime to 8 volumes of sand.

(2) *Cement Mortars.* The volume of cement depends very largely on whether it is loosely dropped in a pile, shaken together, or packed. The practical commercial methods of obtaining a mixture of definite proportions will be given later under "Concrete," section 94. Natural cement mortars are usually mixed in the 1:2 ratio, although a 1:1 mixture would probably be used for tunnel

work or bridge abutments where natural cement would be used at all. Portland cement will be used to make 1:3 mortar for ordinary work, and 1:2 mortar for very high-grade work. As previously stated, a small percentage of lime is sometimes substituted for an equal volume of cement in order to make the mortar work better.

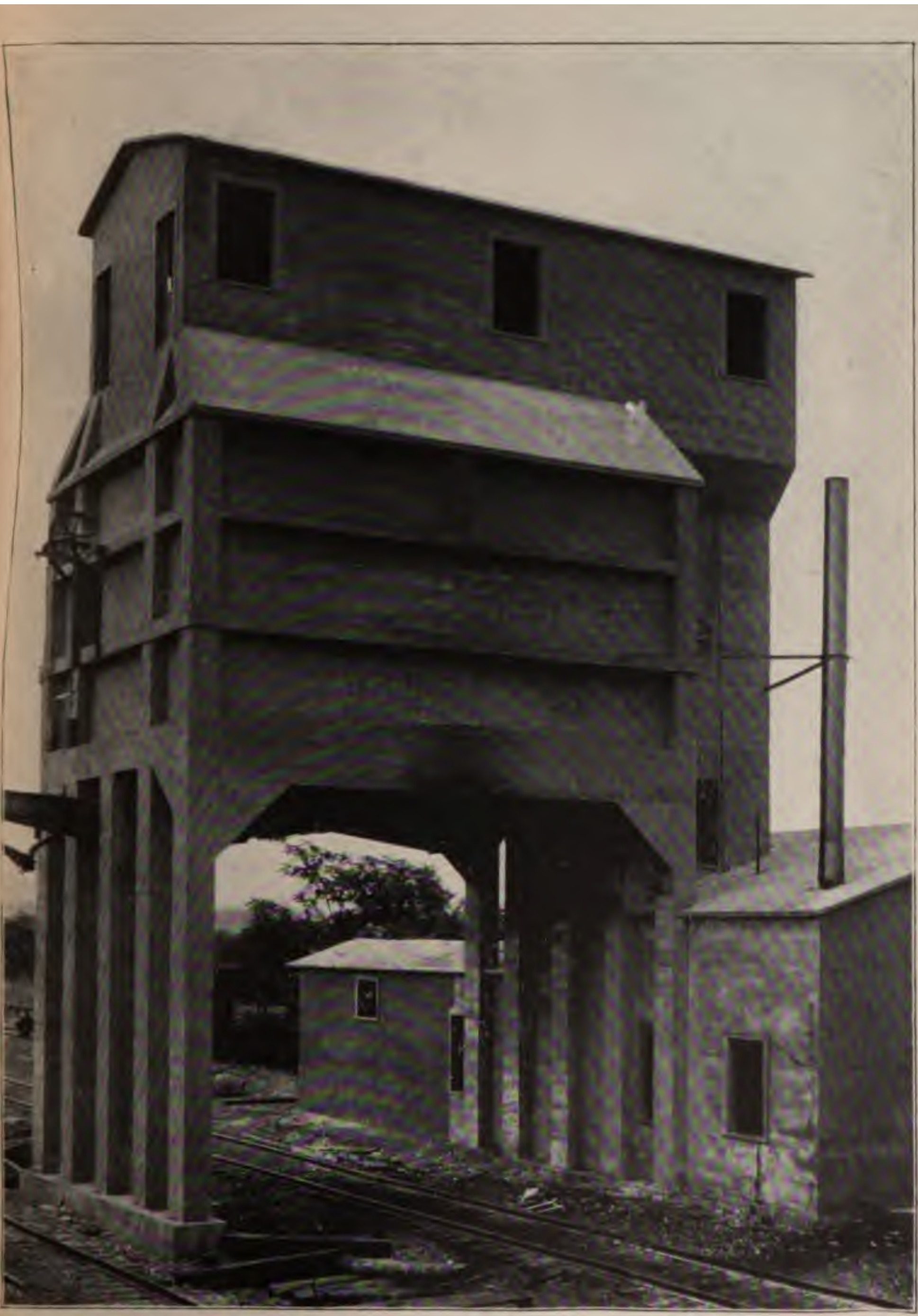
CONCRETE

Concrete is composed of a mixture of cement, sand, and crushed stone or gravel, which, after being mixed with water, soon sets and obtains a hardness and strength equal to that of a good building stone. These properties, together with its adaptability to monolithic construction, combined with its cheapness, render concrete very useful as a building material.

94. **General Principles of Proportioning Concrete.** Theoretically the proportioning of the sand and cementing material should be done by weight. It is always done in this way in laboratory testing. The volume of a given weight of cement is quite variable according as it is packed or loosely thrown in a pile. The same statement is true of sand. Since a barrel of Portland cement will increase in volume from 10 to 40 per cent by being merely dumped loosely in a pile and then shoveled into a measuring box, a contractor will frequently attempt to take advantage of this expansion by measuring the cement loose rather than by using the proportions as indicated by the original volume in the packed barrels. To a less extent the same uncertainty exists regarding the condition of the sand. Loose, dry sand occupies a considerably larger volume than wet sand, and this is still more the case when the sand is very fine.

The general principle to be adopted is that the amount of water should be just sufficient to supply that needed for crystallization of the cement paste; that the amount of paste should be just sufficient to fill the voids between the particles of sand; that the mortar thus produced should be just sufficient to fill the voids between the broken stones. If this ideal could be realized, the total volume of the mixed concrete would be no greater than that of the broken stone. But no matter how thoroughly and carefully the ingredients are mixed and rammed, the particles of cement will get between the grains of sand and thus cause the volume of the mortar to be greater than that of the sand; the grains of sand will get between the smaller stones and separate them; and the smaller stones will get between the larger stones and separate them. Experiments by Prof. I. O. Baker have shown that, even when the volume of the mortar was only 70 per cent of the volume of the voids in the broken stone, the volume of the rammed concrete was 5 per cent more than that of the broken stone. When the theoretical amount of mortar was added, the volume was 7.5 per cent in excess, which shows that it is practically impossible to ram such concrete and wholly prevent voids. When mortar amounting to 140 per cent of the voids was used, all voids were apparently filled, but the volume of the concrete was 114 per cent of that of the broken stone. Therefore, on account of the impracticability of securing perfect mixing, the amount of water used is always somewhat in excess (which will do no harm); the cement paste is generally made somewhat in excess of that required to fill the particles in the sand (except in those cases where, for economy, the mortar is purposely made very *lean*); and the amount of mortar is usually considerably in excess of that required to fill the voids in the stone. Even when we allow some excess in the above particulars, there is so much variation in the percentage of voids in the sand and broken stone, that the best work not only requires an experimental determination of the voids in the sand and stone which are being used; but, on account of the liability to variation in those percentages, even in materials from the same source of supply, the best work requires a constant testing and revision of the proportions as the work proceeds. For less careful work, the proportions ordinarily adopted in practice are considered sufficiently accurate.

On the general principle that the voids in ordinary broken stone are somewhat less than half of the volume, it is a very common practice to use one-half as much sand as the volume of the broken stone. The proportion of cement is then varied according to the strength required in the structure, and according to the desire to economize. On this principle we have the familiar ratios 1:2:4, 1:2½:5, 1:3:6, and 1:4:8. It should be noted that in each of these cases, in which the numbers give the relative proportions of the cement, sand, and stone respectively, the ratio of the sand to the broken stone is a constant, and the ratio of the cement is alone variable, for it would be just as correct to express the ratios as follows: 1:2:4; 0.8:2:4; 0.67:2:4; 0.5:2:4.



REINFORCED-CONCRETE COAL POCKET AT CONCORD, VIRGINIA
Designed by Webb & Gibson, Philadelphia, Pa.

95. **Compressive Strength.** The compressive strength of concrete is very important, as it is used more often in compression than in any other way. It is rather difficult to give average values of the compressive strength of concrete, as it is dependent on so many factors. The available aggregates are so varied, and the methods of mixing and manipulation so different, that tests must be studied before any conclusions can be drawn. For extensive work, tests should be made with the materials available to determine the strength of concrete, under conditions as nearly as possible like those in the actual structure.

A series of experiments made at the Watertown Arsenal for Mr. George A. Kimball, Chief Engineer of the Boston Elevated Railway Company, in 1899, was one of the best sets of tests that have been published, and the results are given in Table III. Portland cement, coarse, sharp sand, and stone up to 2½ inches were used; and when thoroughly rammed, the water barely flushed to the surface.

TABLE III

Compressive Strength of Concrete*

Tests Made at Watertown Arsenal, 1899

MIXTURE	BRAND OF CEMENT	STRENGTH (Pounds per Square Inch)			
		7 Days	1 Month	3 Months	6 Months
1:2:4	Saylor	1,724	2,238	2,702	3,510
	Atlas	1,387	2,428	2,966	3,953
	Alpha	904	2,420	3,123	4,411
	Germania	2,219	2,642	3,082	3,643
	Alsen	1,592	2,269	2,608	3,612
	Average	1,565	2,399	2,896	3,826
1:3:6	Saylor	1,625	2,568	2,882	3,567
	Atlas	1,050	1,816	1,538	3,170
	Alpha	892	2,150	2,355	2,750
	Germania	1,550	2,174	2,486	2,930
	Alsen	1,438	2,114	2,349	3,026
	Average	1,311	2,164	2,522	3,088

*From "Tests of Metals," 1899.

The values obtained in these tests are exceedingly high, and cannot be safely counted on in practice.

Tests made by Prof. A. N. Talbot (University of Illinois, Bulletin No. 14) on 6-inch cubes of concrete, show the average values given in Table IV. The cubes were about 60 days old when tested.

TABLE IV

Compressive Tests of Concrete

University of Illinois

No. of Tests	Mixture	Strength (Pounds per Square Inch)
3	1:2:4	2,350
6	1:3:5½	1,920
7	1:3:6	1,300

With fair conditions as to the character of the materials and workmanship, a mixture of 1:2:4 concrete should show a compressive strength of 2,000 to 2,300 pounds per square inch in 40 to 60 days; a mixture of 1:2½:5 concrete, a strength of 1,800 to 2,000 pounds per square inch; and a mixture of 1:3:6 concrete, a strength of 1,500 to 1,800 pounds per square inch. The rate of hardening depends upon the consistency and the temperature.

96. **Tensile Strength.** The tensile strength of concrete is usually considered about one-tenth of the compressive strength; that is, concrete which has a compressive value of 2,000 pounds per square inch should have a tensile strength of about 200 pounds per square inch. Although there is no fixed relation between the two values, the general law of increase in strength due to increasing the percentage of cement and the density, seems to hold in both cases.

97. **Shearing Strength.** The shearing strength of concrete is important on account of its intimate relation to the compressive strength and the shearing stresses to which it is subjected in structures reinforced with steel. But few tests have been made, as they are rather difficult to make; but the tests made show that the shearing strength of concrete is nearly one-half the crushing strength. By shearing is meant the strength of the material against a sliding failure when tested as a rivet would be tested for shear.

98. **Modulus of Elasticity.** The principal use of the modulus of elasticity in designing reinforced concrete is in determining the relative stresses carried by the concrete and the steel. The minimum value used in designing reinforced concrete is usually taken as 2,000,-

000, and the maximum value as 3,000,000, depending on the richness of the mixture used. A value of 2,500,000 is generally taken for ordinary concrete.

99. **Weight of Concrete.** The weight of stone or gravel concrete will vary from 145 pounds per cubic foot to 155 pounds per cubic foot, depending upon the specific gravity of the materials and the degree of compactness. The weight of a cubic foot of concrete is usually considered as 150 pounds.

100. **Cinder Concrete.** Cinder concrete has been used to some extent on account of its light weight. The strength of cinder concrete is from one-half to two-thirds the strength of stone concrete. It will weigh about 110 pounds per cubic foot.

101. **Rubble Concrete.** Rubble concrete is a concrete in which large stones are placed, and will be discussed in Part II.

102. **Cost of Concrete.** The cost of concrete depends upon the character of the work to be done, and the conditions under which it is necessary to do this work. The cost of the material, of course, will always have to be considered, but this is not so important as the character of the work. The cost of concrete in place will range from $4.50 per cubic yard to $20, or even $25, per cubic yard. When it is laid in large masses, so that the cost of forms is relatively small, the cost will range from $4.50 per cubic yard to $6 or $7 per cubic yard, depending on the local conditions and cost of materials. Foundations and heavy walls are good examples of this class of work. For sewers and arches, the cost will vary from $7 to $13. In building construction—floors, roofs, and thin walls—the cost will range from $14 to $20 per cubic yard.

103. **Cost of Cement.** The cost of Portland Cement varies with the demand. Being heavy, the freight is often a big item. The price varies from $1 to $2 per barrel. To this must be added the cost of handling.

104. **Cost of Sand.** The cost of sand, including handling and freight, ranges from $0.75 to $1.50 per cubic yard. A common price for sand delivered in the cities is $1.00 per cubic yard.

105. **Cost of Broken Stone or Gravel.** The cost of broken stone delivered in the cities varies from $1.25 to $1.75 per cubic yard. The cost of gravel is usually a little less than stone.

106. **Cost of Mixing.** Under ordinary conditions and where

the concrete will have to be wheeled only a very short distance, the cost of hand-mixing and placing will generally range from \$0.90 to \$1.30 per cubic yard, if done by men skilled in this work. If a mixer is used, the cost will range from \$0.50 to \$0.90 per cubic yard.

107. Cost of Forms. The cost of forms for heavy walls and foundations, varies from \$0.70 to \$1.20 per cubic yard of concrete laid. The cost of forms and mixing concrete will be further discussed in Part IV.

108. Practical Methods of Proportioning. A rich mixture, proportions 1:2:4—that is, 1 barrel (4 bags) packed Portland cement (as it comes from the manufacturer), 2 barrels (7.6 cubic feet) loose sand, and 4 barrels (15.2 cubic feet) loose stone—is used in arches, reinforced-concrete floors, beams, and columns for heavy loads; engine and machine foundations subject to vibration; tanks; and for water-tight work.

A medium mixture, proportions 1 : 2½ : 5—that is, 1 barrel (4 bags) packed Portland cement, 2½ barrels (9.5 cubic feet) loose sand, and 5 barrels (19 cubic feet) loose gravel or stone—may be used in arches, thin walls, floors, beams, sewers, sidewalks, foundations, and machine foundations.

An ordinary mixture, proportions 1 : 3 : 6—that is, 1 barrel (4 bags) packed Portland cement, 3 barrels (11.4 cubic feet) loose sand, and 6 barrels (22.8 cubic feet) loose gravel or broken stone—may be used for retaining walls, abutments, piers, floor slabs, and beams.

A lean mixture, proportions 1 : 4 : 8—that is, 1 barrel (4 bags) packed Portland cement, 4 barrels (15.2 cubic feet) loose sand, and 8 barrels (30.4 cubic feet) loose gravel or broken stone—may be used in large foundations supporting stationary loads, backing for stone masonry, or where it is subject to a plain compressive load.

These proportions must not be taken as always being the most economical to use, but they represent average practice. Cement is the most expensive ingredient; therefore a reduction of the quantity of cement, by adjusting the proportions of the aggregate so as to produce a concrete with the same density, strength, and impermeability, is of great importance. By careful proportioning and workmanship, water-tight concrete has been made of a 1 : 3 : 6

mixture. In floor construction where the span is very short and it is specified that the slab must be at least 4 inches thick, while with a high-grade concrete a 3-inch slab would carry the load, it is certainly more economical to use a leaner concrete.

An accurate and simple method to determine the proportions of concrete is by trial batches. The apparatus consists of a scale and a cylinder which may be a piece of wrought iron pipe 10 inches to 12 inches in diameter capped at one end. Measure and weigh the cement, sand, stone, and water and mix on a piece of sheet steel, the mixture having a consistency the same as to be used in the work. The mixture is placed in the cylinder, carefully tamped, and the height to which the pipe is filled is noted. The pipe should be weighed before and after being filled so as to check the weight of the material. The cylinder is then emptied and cleaned. Mix up another batch using the same amount of cement and water, slightly varying the ratio of the sand and stone but having the same total weight as before. Note the height in the cylinder, which will be a guide to other batches to be tried. Several trials are made until a mixture is found that gives the least height in the cylinder, and at the same time works well while mixing, all the stones being covered with mortar, and which makes a good appearance. This method gives very good results, but it does not indicate the changes in the physical sizes of the sand and stone so as to secure the most economical composition as would be shown in a thorough mechanical analysis.

There has been much concrete work done where the proportions were selected without any reference to voids, which has given much better results in practice than might be expected. The proportion of cement to the aggregate depends upon the nature of the construction and the required degree of strength, or water-tightness, as well as upon the character of the inert materials. Both strength and imperviousness increase with the proportion of cement to the aggregate. Richer mixtures are necessary for loaded columns, beams in building construction and arches, for thin walls subject to water pressure, and for foundations laid under water. The actual measurements of materials as actually mixed and used usually show leaner mixtures than the nominal proportions specified. This is largely due to the heaping of the measuring boxes.

TABLE V

Proportions of Cement, Sand, and Stone in Actual Structures

STRUCTURE	PROPORTIONS	REFERENCE
C. B. & Q. R. R.		
Reinforced Concrete Culverts	1:3:6	Engr. Cont., Oct. 3, '06
Phila. Rapid Transit Co.		
Floor Elevated Roadway....	1:3:6	" " Sept. 26, '06
Subway { Walls...........	1:2.5:5	
Subway { Floors...........	1:3:6	
C. P. R. R.		
Arch Rings.................	1:3:5	
Piers and Abutments........	1:4:7	Cement Era, Aug. '06
Hudson River Tunnel Caisson	1:2:4	Eng. Record, Sept. 29, '06
Stand Pipe at Attleboro, Mass. Height, 106 feet.	1:2:4	" " " 29, '06
C.C.& St.L.R.R., Danville Arch		
Footings....................	1:4:8 or 1:9.5	" " March 3, '06
Arch Rings..................	1:2:4	
Abutments, Piers............	1:3:6 or 1:6.5	
N. Y. C. & H. R. R. R.		
Ossining Tunnel { Footing..........	1:4:7.5	" " " 3, '06
Ossining Tunnel { Walls...........	1:3:6	
Ossining Tunnel { Coping..........	1:2:4	
American Oak Leather Co.		
Factory at Cincinnati, Ohio.	1:2:4	" " " 8, '03
Harvard University Stadium..	1:3:6	
New York Subway		
Roofs and Sidewalks........	1:2:4	
Tunnel Arches..............	1:2.5:5	
Wet Foundation 2' th. or less	1:2:4	
" " exceeding 2'	1:2.5:5	
Boston Subway...............	1:2.5:4	
P. & R. R. R.		
Arches......................	1:2:4	" " Oct. 13, '06
Piers and Abutments........	1:3:6	
Brooklyn Navy Yd. Laboratory		
Columns....................	1:2:3 Trap rock	Eng. News, March 23, '05
Beams and Slabs............	1:3:5 " "	
Roof Slab..................	1:3:5 Cinder	
Southern Railway		
Arches......................	1:2:4	
Piers and Abutments........	1:2.5:5	

109. **Methods of Mixing Concrete.** The method of mixing concrete is immaterial, if a homogeneous mass is secured of a uniform

consistency, containing the cement, sand, and stone in the correct proportions. The value of the concrete depends greatly upon the thoroughness of the mixing. The color of the mass must be uniform, every grain of sand and piece of the stone should have cement adhering to every point of its surface.

TABLE VI

Barrels of Portland Cement Per Cubic Yard of Mortar

(Voids in Sand Being 35 per cent and 1 Bbl. Cement Yielding 3.65 Cubic Feet of Cement Paste.)

PROPORTION OF CEMENT TO SAND	1:1	1:1.5	1:2	1:2.5	1:3	1:4
	Bbls.	Bbls.	Bbls.	Bbls.	Bbls.	Bbls.
Bbl. specified to be 3.5 cu. ft........	4.22	3.49	2.97	2.57	2.28	1.76
" " " 3.8 "	4.09	3.33	2.81	2.45	2.16	1.62
" " " 4.0 "	4.00	3.24	2.73	2.36	2.08	1.54
" " " 4.4 "	3.81	3.07	2.57	2.27	2.00	1 40
Cu. yds. sand per cu. yd. mortar...	0.6	0.7	0.8	0.9	1.0	1.0

TABLE VII

Barrels of Portland Cement Per Cubic Yard of Mortar

(Voids in Sand Being 45 per cent and 1 Bbl. Cement Yielding 3.4 Cubic Feet of Cement Paste.)

PROPORTION OF CEMENT TO SAND	1:1	1:1.5	1:2	1:2.5	1:3	1:4
	Bbls.	Bbls.	Bbls.	Bbls.	Bbls.	Bbls.
Bbl. specified to be 3.5 cu. ft........	4.62	3.80	3.25	2.84	2.35	1.76
" " " 3.8 "	4.32	3.61	3.10	2.72	2.16	1.62
" " " 4 0 "	4.19	3.46	3.00	2.64	2.05	1.54
" " " 4.4 "	3.94	3.34	2.90	2.57	1.86	1.40
Cu. yds. sand per cu. yds. mortar..	0.6	0.8	0.9	1.0	1.0	1.0

TABLE VIII

Ingredients in 1 Cubic Yard of Concrete

(Sand Voids, 40 per cent; Stone Voids, 45 per cent; Portland Cement Barrel Yielding 3.65 cu. ft. Paste. Barrel specified to be 3.8 cu. ft.)

PROPORTIONS BY VOLUME	1:2:4	1:2:5	1:2:6	1:2.5:5	1:2.5:6	1:3:4
Bbls. cement per. cu. yd. concrete..	1.46	1.30	1.18	1.13	1.00	1.25
Cu. yds. sand " " ..	0.41	0.36	0.33	0.40	0.35	0.53
" stone " " ..	0.82	0.90	1.00	0.80	0.84	0.71
Proportions by volum	1:3:5	1:3:6	1:3:7	1:4:7	1:4:8	1:4:9
Bbls. cement per cu. yd. concrete..	1.13	1.05	0.96	0.82	0.77	0.73
Cu. yds. sand " " ..	0.48	0.44	0.40	0.46	0.43	0.41
" stone " " ..	0.80	0.88	0.93	0.80	0.86	0.92

. **This table is to be used when cement is measured packed in the barrel, for the ordinary barrel holds 3.8 cu. ft.**

TABLE IX

Ingredients in 1 Cubic Yard of Concrete

(Sand Voids, 40 per cent; Stone Voids, 45 per cent; Portland Cement Barrel Yielding 3.65 cu. ft. of Paste. Barrel specified to be 4.4 cu. ft.)

PROPORTIONS BY VOLUME	1 : 2 : 4	1 : 2 : 5	1 : 2 : 6	1 : 2.5 : 5	1 : 2.5 : 6	1 : 3 : 4
Bbls. cement per cu. yd. concrete...	1.30	1.16	1.00	1.07	0.96	1.08
Cu. yds. sand " " ...	0.42	0.38	0.33	0.44	0.40	0.53
" stone " " ...	0.84	0.95	1.00	0.88	0.95	0.71
Proportions by volume...........	1 :3 :5	1 :3 :6	1 :3 :7	1 :4 :7	1 :4 :8	1 :4 :9
Bbls. cement per cu. yd. concrete...	0.96	0.90	0.82	0.75	0.68	0.64
Cu. yds. sand " " ...	0.47	0.44	0.40	0.49	0.44	0.42
" stone " " ...	0.78	0.88	0.93	0.86	0.88	0.95

This table is to be used when the cement is measured loose, after dumping it into a box, for under such conditions a barrel of cement yields 4.4 cu. ft. of loose cement.

[Tables V to IX have been taken from Gillette's "Handbook of Cost Data."]

110. **Wetness of Concrete.** In regard to plasticity, or facility of working and moulding, concrete may be divided into three classes: *dry*, *medium*, and *very wet*.

Dry concrete is used in foundations which may be subjected to severe compression a few weeks after being placed. It should not be placed in layers of more than 8 inches, and should be thoroughly rammed. In a dry mixture the water will just flush to the surface only when it is thoroughly tamped. A dry mixture sets and will support a load much sooner than if a wetter mixture is used, and generally is used only where the load is to be applied soon after the concrete is placed. This mixture requires the exercise of more than ordinary care in ramming, as pockets are apt to be formed in the concrete; and one argument against it is the difficulty of getting a uniform product.

Medium concrete will quake when rammed, and has the consistency of liver or jelly. It is adapted for construction work suited to the employment of mass concrete, such as retaining walls, piers, foundations, arches, abutments; and is sometimes also employed for reinforced concrete.

A *very wet* mixture of concrete will run off a shovel unless it is handled very quickly. An ordinary rammer will sink into it of its own weight. It is suitable for reinforced concrete, such as thin walls, floors, columns, tanks, and conduits.

Within the last few years there has been a marked change in the amount of water used in mixing concrete. The dry mixture has been superseded by a medium or very wet mixture, often so wet as to require no ramming whatever. Experiments have shown that *dry mixtures* give better results in *short time tests* and *wet mixtures* in *long time tests.* In some experiments made on dry, medium, and wet mixtures it was found that the medium mixture was the most dense, wet next, and dry least. This experimenter concluded that the medium mixture is the most desirable, since it will not quake in handling, but will quake under heavy ramming. He found medium 1 per cent denser than wet and 9 per cent denser than dry concrete; he considers thorough ramming important.

Concrete is often used so wet that it will not only quake but flow freely, and after setting it appears to be very dense and hard, but some engineers think that the tendency is to use far too much rather than too little water, but that thorough ramming is desirable. In thin walls very wet concrete can be more easily pushed from the surface so that the mortar can get against the forms and give a smooth surface. It has also been found essential that the concrete should be wet enough so as to flow under and around the steel reinforcement so as to secure a good bond between the steel and concrete.

Following are the specifications (1903) of the American Railway Engineering and Maintenance of Way Association:

"The concrete shall be of such consistency that when dumped in place it will not require tamping; it shall be spaded down and tamped sufficiently to level off and will then quake freely like jelly, and be wet enough on top to require the use of rubber boots by workman."

111. **Transporting and Depositing Concrete.** Concrete is usually deposited in layers of 6 inches to 12 inches in thickness. In handling and transporting concrete, care must be taken to prevent the separation of the stone from the mortar. The usual method of transporting concrete is by wheel-barrows, although it is often handled by cars and carts, and on small jobs it is sometimes carried in buckets. A very common practice is to dump it from a height of several feet into a trench. Many engineers object to this process as they claim

that the heavy and light portions separate while falling and the concrete is therefore not uniform through its mass, and they insist that it must be gently slid into place. A wet mixture is much easier to handle than a dry mixture, as the stone will not so readily separate from the mass. A very wet mixture has been deposited from the top of forms 43 feet high and the structure was found to be waterproof. On the other hand, the stones in a dry mixture will separate from the mortar on the slightest provocation. Where it is necessary to drop a dry mixture several feet, it should be done by means of a chute or pipe.

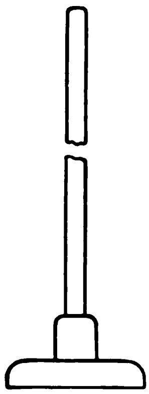


Fig. 8. Rammer for Dry Concrete. (Shoe 6 inches square.)

112. **Ramming Concrete.** Immediately after concrete is placed, it should be rammed or puddled, care being taken to force out the air-bubbles. The amount of ramming necessary depends upon how much water is used in mixing the concrete. If a very wet mixture is used, there is danger of too much ramming, which results in wedging the stones together and forcing the cement and sand to the surface. The chief object in ramming a very wet mixture is simply to expel the bubbles of air.

The style of rammer ordinarily used depends on whether a dry, medium, or very wet mixture is used. A rammer for dry concrete is shown in Fig. 8; and one for wet concrete, in Fig. 9. In very thin walls, where a wet mixture is used, often the tamping or puddling is done with a part of a reinforcing bar. A common spade is often employed for the face of work, being used to push back stones that may have separated from the mass, and also to work the finer portions of the mass to the face, the method being to work the spade up and down the face until it is thoroughly filled. Care must be taken not to pry with the spade, as this will spring the forms unless they are very strong.

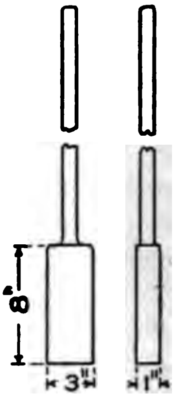


Fig. 9. Rammer for Wet Concrete.

113. **Bonding Old and New Concrete.** To secure a water-tight joint between old and new concrete, requires a great deal of care. Where

the strain is chiefly compressive, as in foundations, the surface of the concrete laid on the previous day should be washed with clean water, no other precautions being necessary. In walls and floors, or where a tensile stress is apt to be applied, the joint should be thoroughly washed and soaked, and then painted with neat cement or a mixture of one part cement and one part sand, made into a very thin mortar.

In the construction of tanks or any other work that is to be water-tight, in which the concrete is not placed in one continuous operation, one or more square or V-shaped joints are necessary. These joints are formed by a piece of timber, say 4 inches by 6 inches, being imbedded in the surface of the last concrete laid each day. On the following morning, when the timber is removed, the joint is washed and coated with neat cement or 1:1 mortar. The joints may be either horizontal or vertical. The bond between old and new concrete may be aided by roughening the surface after ramming or before placing the new concrete.

114. **Effects of Freezing of Concrete.** Many experiments have been made to determine the effect of freezing of concrete before it has a chance to set. From these and from practical experience, it is now generally accepted that the ultimate effect of freezing of Portland cement concrete is to produce only a surface injury. The setting and hardening of Portland cement concrete is retarded, and the strength at short periods is lowered, by freezing; but the ultimate strength appears to be only slightly, if at all, affected. A thin scale about $\frac{1}{16}$ inch in depth is apt to scale off from granolithic or concrete pavements which have been frozen, leaving a rough instead of a troweled wearing surface; and the effect upon concrete walls is often similar; but there appears to be no other injury. Concrete should not be laid in freezing weather, if it can be avoided, as this involves additional expense and requires greater precautions to be taken; but with proper care, Portland cement concrete can be laid at almost any temperature.

There are three methods which may be used to prevent injury to concrete when laid in freezing weather:

First: Heat the sand and stone, or use hot water in mixing the concrete.

Second: Add salt, calcium chloride, or other chemicals to lower the freezing point of the water.

Third: Protect the green concrete by enclosing it and keeping the temperature of the enclosure above the freezing point.

The first method is perhaps more generally used than either of the others. In heating the aggregate, the frost is driven from it; hot water alone is insufficient to get the frost out of the frozen lumps of sand. If the heated aggregate is mixed with water which is hot but not boiling, experience has shown that a comparatively high temperature can be maintained for several hours, which will usually carry it through the initial set safely. The heating of the materials also hastens the setting of the cement. If the fresh concrete is covered with canvas or other material, it will assist in maintaining a higher temperature. The canvas, however, must not be laid directly on the concrete, but an air space of several inches must be left between the concrete and the canvas.

The aggregate is heated by means of steam pipes laid in the bottom of the bins, or by having pipes of strong sheet iron, about 18 inches in diameter, laid through the bottom of the bins, and fires built in the pipes. The water may be heated by steam jets or other means. It is also well to keep the mixer warm in severe weather, by the use of a steam coil on the outside, and jets of steam on the inside.

The second method—lowering the freezing point by adding salt—has been commonly used to lower the freezing point of water. Salt will increase the time of setting and lower the strength of the concrete for short periods. There is a wide difference of opinion as to the amount of salt that may be used without lowering the ultimate strength of the concrete. Specifications for the New York Subway work required nine pounds of salt to each 100 pounds (12 gallons) of water in freezing weather. A common rule calls for 10 per cent of salt to the weight of water, which is equivalent to about 13 pounds of salt to a barrel of cement.

The third method is the most expensive, and is used only in building construction. It consists in constructing a light wooden frame over the site of the work, and covering the frame with canvas or other material. The temperature of the enclosure is maintained above the freezing point by means of stoves.

115. **Water-tightness of Concrete.** Water-tight concrete, or concrete made water-tight by some kind of waterproof coating, is frequently required, either for inclosing a space which must be kept dry, or for storing water or other liquids. Concrete, even when most carefully prepared from materials of the highest grade, is never of itself completely waterproof.

It is generally considered that in monolithic construction, a wet mixture, a rich concrete, and an aggregate proportioned for great density, are essential for water-tightness. With the wet mixtures of concrete now generally used in engineering work, concrete possesses far greater density, and is correspondingly less porous, than with the older, dryer mixtures. At the same time, in the large masses of actual work, it is difficult to produce concrete of such close texture as to prevent undesirable seepage at all points. Many efforts have been made to secure water-tightness of concrete in a practical manner—some with success, but others with unsatisfactory results. There are now a great many special preparations being advertised for making concrete water-tight.

It has frequently been observed that when concrete was green, there was a considerable seepage through it, and that in a short time absolutely all seepage stopped. Some experiments have been made to render porous concrete impermeable, by forcing water through a rich concrete under pressure. In these experiments, a mixture of 1 part Portland cement to 4 parts crushed gravel was used. The concrete tested was 6 inches thick. The flow through the concrete on the first day of the experiment, under a pressure of 36 pounds per square inch, was taken as 100 per cent. On the forty-sixth day, under a pressure of 48 pounds per square inch, the flow amounted to only 0.7 per cent.

While the pressure was constant, the rate of seepage of the water decreased with the lapse of time, showing a marked tendency of the seepage passages to become closed. The experimenter is of the opinion that the water, under pressure, *dissolves* some of the material and then deposits it in stalactitic form near the exterior surface of the concrete, where the water escapes under much reduced pressure. Others, however, think it quite possible that fine material carried *in suspension* by the water aids in producing the result.

For cistern work, two coats of Portland cement grout—1 part cement, 1 part sand—applied on the inside, have been found sufficient. About one inch of rich mortar has usually been found effective under high pressure. A coating of asphalt, or of asphalt with tarred or asbestos felt, laid in alternate layers between layers of concrete, has been used successfully. Coal-tar pitch and tarred felt, laid in alternate layers, have been used extensively and successfully in New York City for waterproofing.

Mortar may be made practically non-absorbent by the addition of alum and potash soap. One per cent by weight of powdered alum is added to the dry cement and sand, and thoroughly mixed; and about one per cent of any potash soap (ordinary soft soap) is dissolved in the water used in the mortar. A solution consisting of 1 pound of concentrated lye, 5 pounds of alum, and 2 gallons of water, applied while the concrete is green and until it lathers freely, has been successfully used. Coating the surface with boiled linseed oil until the oil ceases to be absorbed, is another method that has been used with success.

A reinforced concrete water-tank, 10 feet inside diameter and 43 feet high, designed and constructed by W. B. Fuller at Little Falls, N. J., has some remarkable features. It is 15 inches thick at the bottom and 10 inches thick at the top. The tank was built in eight hours, and is a perfect monolith, all concrete being dropped from the top, or 43 feet at the beginning of the work. The concrete was mixed very wet, the mixture being 1 part cement, 3 parts sand, and 7 parts broken stone. No plastering or waterproofing of any kind was used, but the tank was found to be absolutely water-tight although the mixture used has not generally been found or considered water-tight.

At Attleboro, Mass., a large reinforced concrete standpipe, 50 feet in diameter, 106 feet high from the inside of the bottom to the top of the cornice, and with a capacity of 1,500,000 gallons, has been constructed, and is in the service of the water works of that city. The walls of the standpipe are 18 inches thick at the bottom, and 8 inches thick at the top. A mixture of 1 part cement, 2 parts sand, and 4 parts broken stone, the stone varying from ¼ inch to 1½ inches, was used. The forms were constructed, and the concrete placed, in sections of 7 feet. When the walls of the tank had been completed, there was some leakage at the bottom with a head of water of 100 feet. The inside walls were then thoroughly cleaned and picked, and four coats of plaster applied. The first coat contained 2 per cent of lime to 1 part of cement and 1 part of sand; the remaining three coats were composed of 1 part sand to 1 part cement. Each coat was floated until a hard, dense surface was produced; then it was scratched to receive the succeeding coat.

On filling the standpipe after the four coats of plaster had been applied, the standpipe was found to be not absolutely water-tight. The water was drawn out; and four coats of a solution of castile soap, and one of alum, were applied alternately; and, under a 100-foot head, only a few leaks then appeared. Practically no leakage occurred at the joints; but in several instances a mixture somewhat wetter than usual was used, with the result that the spading and ramming served to drive the stone to the bottom of the batch being placed, and, as a consequence, in these places porous spots occurred. The joints were obtained by inserting beveled tonguing pieces, and by thoroughly washing the joint and covering it with a layer of thin grout before placing additional concrete.

In the construction of the filter plant at Lancaster, Pa., in 1905, a pure-water basin and several circular tanks were constructed of reinforced concrete. The pure-water basin is 100 feet wide by 200 feet long and 14 feet deep, with buttresses spaced 12 feet 6 inches center to center. The walls at the bottom are 15 inches thick, and 12 inches thick at the top. Four circular tanks are 50 feet in diameter and 10 feet high, and eight tanks are 10 feet in diameter and 10 feet high. The walls are 10 inches thick at the bottom, and 6 inches at the top. A wet mixture of 1 part cement, 3 parts sand, and 5 parts stone, was used. No waterproofing material was used, in the construction of the tanks; and when tested, two of them were found to be water-tight, and the other two had a few leaks where wires which had been used to hold the forms together had pulled out when the forms were taken down. These holes were stopped up and no furthur trouble was experienced. In constructing the floor of the pure-water basin, a thin layer of asphalt was used, as shown in Fig. 10; but no waterproofing material was used in the walls, and both were found to be water-tight.

116. **Sylvester Process.** The alternate application of washes of castile soap and alum, each being dissolved in water, is known as

the *Sylvester process* of waterproofing. Castile soap is dissolved in water, ¾ of a pound of soap in a gallon of water, and applied boiling hot to the concrete surface with a flat brush, care being taken not to form a froth. The alum dissolved in water—1 pound pure alum in 8 gallons of water— is applied 24 hours later, the soap having had time to become dry and hard. The second wash is applied in the same manner as the first, at a temperature of 60° to 70° F. The alternate coats of soap and alum are repeated every 24 hours. Usually four coats will make an impervious coating. The soap and alum combine and form an insoluble compound, filling the pores of the concrete and preventing the seepage of water. The walls should be clean and dry, and the temperature of the air not lower than 50° F., when the composition is applied. The composition should be applied while the concrete is still green. This method of waterproofing has been used extensively for years, and has generally given satisfactory results for moderate pressures.

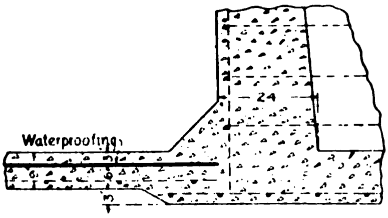
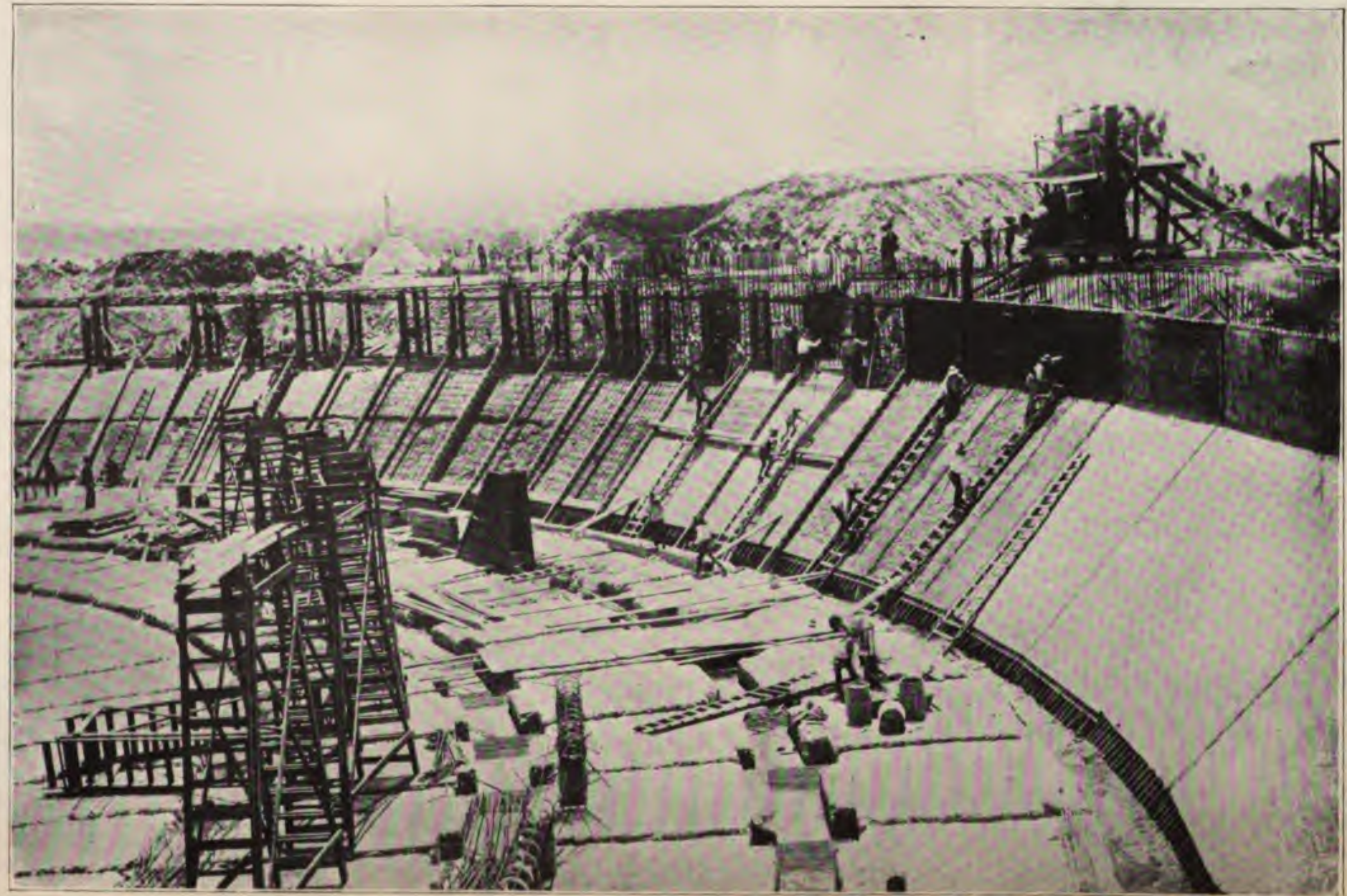


Fig. 10. Floor of Pure-Water Basin.

117. **Asphalt Waterproofing.** If asphalt is to be applied to a concrete surface, the concrete should be dry; and it will be found generally more satisfactory to coat the dry surface first with asphalt *cut* with naphtha. Unless the concrete is heated, it is generally very hard to make the asphalt adhere to the concrete. Hot asphalt applied to ordinary concrete surfaces will generally roll up like a blanket when it cools. The concrete should be heated by hot sand, or the asphalt should be cut with naphtha. When the coat containing the naphtha has been applied—like a coat of paint—and is dry, then the asphalt mastic is applied. The asphalt mastic is composed of 1 part asphalt to 4 parts of sand. This is smoothed off with hot irons, and thoroughly tamped into place. If stone or earth is to be placed next to the asphaltic surface, it is best to cover the surface with roofing gravel to protect the asphalt.

Asphalt paint has been used for a protective coat for all kinds of masonry where earth is to be placed against it.

A coat of asphalt ½ inch thick applied with mops to a grout



CONSTRUCTION OF A REINFORCED-CONCRETE CIRCULAR RESERVOIR FOR THE CITY OF MEXICO

View showing excavations for column bases and floor-girders: also construction work on side walls, surface, has been used satisfactorily for coating the interiors of tanks, for heads greater than 19 feet, by Mr. J. W. Schaub ("Transactions" of the American Society of Civil Engineers, Vol. LI). Mr. Schaub states that he believes the ¼-inch coat, in addition to the grout, is sufficient for a water pressure of 60 feet.

118. **Felt Laid with Asphalt.** In waterproofing floors, roofs, subways, tunnels, etc., alternate layers of paper or felt are laid with asphalt, bitumen, or tar. These materials range from ordinary tar paper laid with coal-tar pitch, to asbestos or asphalt felt laid in asphalt. Coal-tar products deteriorate when exposed to moisture. Some asphalts are more suitable than others for waterproofing purposes; therefore the properties of any asphalt intended for waterproofing should be thoroughly investigated.

In using these materials for rendering concrete water-tight, usually a layer of concrete or brick is first laid. On this is mopped a layer of hot asphalt; felt or paper is then laid on the asphalt, the latter being lapped from 6 to 12 inches. After the first layer of felt is placed, it is mopped over with hot asphalt compound, and another layer of felt or paper is laid, the operation being repeated until the desired thickness is secured, which is usually from 2 to 10 layers—or, in other words, the waterproofing varies from 2-*ply* to 10-*ply*. A waterproofing course of this kind, or a course as described in the paragraph on asphalt waterproofing, forms a distinct joint, and the strength in bending of the concrete on the two sides of the layer must be considered independently.

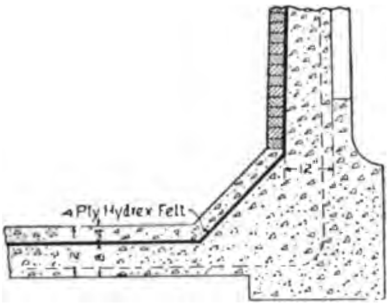


Fig. 11. Method of Waterproofing Reservoir.

When asphalt, or asphalt laid with felt paper, is used for waterproofing the interiors of the walls of tanks, a 4-inch course of brick is required to protect and hold in place the waterproofing materials. Fig. 11 shows a wall section of a reservoir (*Engineering Record*, Sept. 21, 1907) constructed for the New York, New Haven & Hartford Railroad, which illustrates the methods described above. The waterproofing materials for this reservoir consist of 4-ply "Hydrex" felt, and "Hydrex" compound was used to cement the layers together.

Fig. 12 is an illustration of the method used by the Barrett Manufacturing Company in applying their 5-ply coal-tar pitch and felt roofing material, and it shows in a general way the method of laying asphalt and felt for waterproofing purposes. The company's instructions for applying this roofing are as follows:

"First coat the concrete (*A*) with hot pitch (*B*) mopped on uniformly. Over the above coating of pitch, lay two thicknesses of tarred felt (*C*), lapping each sheet seventeen (17) inches over the preceding one, and mopping back with pitch (*D*) the full width of each lap.

"Over the felt thus laid, spread a uniform coating of pitch (*E*) mopped on. Then lay three (3) full thicknesses of tarred felt (*F*), lapping each sheet twenty-two (22) inches over the preceding one.

"When the felt is thus laid, mop back with pitch (*G*) the full width of twenty-two (22) inches under each lap. Then spread over the entire surface of the roof a uniform coating of pitch, into which, while hot, imbed slag or gravel (*H*)."

In applying asphalt and felt for general waterproofing purposes, the felt, as already stated, would be in a continuous roll, and not in sheets as shown for roofing purposes.

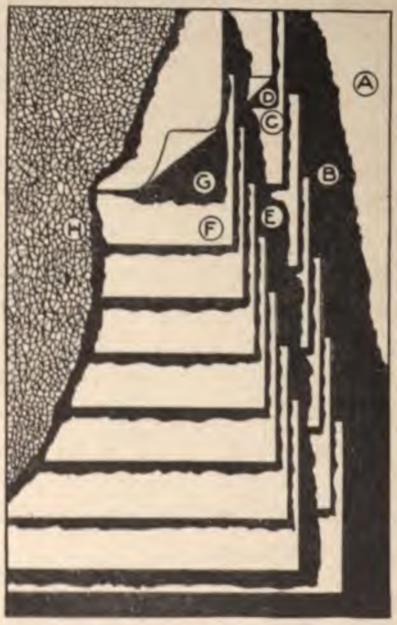


Fig. 12. Five-Ply Gravel Roofing.

119. Medusa Waterproof Compound. Among the many patented waterproofing materials on the market is the "Medusa." This compound is claimed to prevent efflorescence, as well as to make concrete waterproof. In using it, the following directions, given by the manufacturing company, are to be observed:

"To render cement work impervious to water, a small quantity of the compound is *thoroughly mixed* with the dry cement, before the addition of sand and water. For most purposes, from 1 to 2 per cent of the weight of cement used will be found sufficient. This is equivalent to from four to eight pounds of the compound to one barrel of cement. The precise amount to be used must be left to the experience of the user, and depends upon the proportion of sand, etc., employed, and on the kind of work to be done. Our own experience has been that the use of 1 per cent—4 pounds to the barrel, or 1 pound to the sack of cement—is enough to make hollow concrete building blocks water-tight. For cistern and reservoir linings and other work which must be absolutely impervious, a somewhat larger amount should be used. Thorough mixing is of the utmost importance."

In the operation of waterproofing, a very common mistake is made in applying the waterproofing materials on the wrong side of the wall to be made water-tight. That is, if water finds its way through a cellar wall, it is generally useless to apply a waterproofing coat on the inside surface of the wall, as the pressure of the water will push it off. If, however, there is no great pressure behind it, a waterproofing coat applied on the inside of the wall is usually successful in keeping moisture out of the cellar. To be successful in waterproofing a cellar wall, the waterproofing material should be applied on the *outside* surface of the wall; and if properly applied, the wall, as well as the cellar, will be entirely free of water.

In tank or reservoir construction, the conditions are different, in that it is desired to prevent the escape of water. In these cases, therefore, the waterproofing is applied on the inside surface, and is supported by the materials used in constructing the tank or reservoir. The structure should always be designed so that it can be properly waterproofed, and no asphaltic waterproofing should be laid at a temperature below 25° F.

The above-described methods of waterproofing are applicable to stone and brick masonry as well as to concrete.

BITUMEN

120. **Varieties.** One of the groups of mineral substances composed of different hydro-carbons, which are widely scattered throughout the world, is known as *bitumen*. There is a great variety of forms in which bitumen is found, ranging from volatile liquids to thick semi-fluids and solids. These are usually intermixed with different kinds of inorganic or organic matter, but are sometimes found in a free or pure state. Liquid varieties are known as *naphtha* and *petroleum*; the viscous or semi-fluid as *maltha* or *mineral tar*; and the solid as *asphalt* or *asphaltum*.

121. **Asphaltum.** The most noted deposit of asphaltum is found in the island of Trinidad and at Bermudez, Venezuela. De-

posits of nearly pure asphaltum are found in Utah, Mexico, Cuba, and different parts of the United States. Varieties of nearly pure asphalt are known as *wurtzilite*, *elaterite*, and *gilsonite*.

The main source of supply of asphaltum used in the United States for street paving, has been the Trinidad deposit. This is also the main source for asphaltic roofing materials.

122. **Asphalt.** The bituminous limestone deposits at Seyssel and Pyrimont, France; in the Val-de-Travers, Canton of Neuchatel, Switzerland; and at Ragusa, Sicily, are known as *rock asphalt.* It is more durable than asphaltum, and is extensively used in Europe for paving purposes.

There are two forms in which rock asphalt is prepared for shipment:

(*a*) *Compressed asphalt blocks*, which are used in about the manner of stone blocks.

(*b*) *Mastic asphalt*, which is made into blocks of different sizes, generally bearing the manufacturer's trade mark.

The mastic asphalt is used for waterproofing and damp-proofing purposes. For all work of this kind, the Val-de-Travers, or the Seyssel, or Sicilian rock asphalt should be used.

PRESERVATION OF STEEL IN CONCRETE

123. Tests have been made to find the value of Portland cement concrete as a protection of steel or iron from corrosion. Nearly all of these tests have been of short duration (from a few weeks to several months); but they have clearly shown, when the steel or iron is properly imbedded in concrete, that on being removed therefrom it is clean and bright. Steel removed from concrete containing cracks or voids usually shows rust at the points where the voids or cracks occur; but if the steel has been *completely covered* with concrete, there is no corrosion. Tests have shown that if corroded steel is imbedded in concrete, the concrete will remove the rust. To secure the best results, the concrete should be mixed quite wet, and care should be taken to have the steel thoroughly imbedded in the concrete.

124. **Cinder vs. Stone Concrete.** A compact cinder concrete has proven about as effective a protection for steel as stone concrete. The corrosion found in cinder concrete is mainly due to iron oxide

or rust in the cinders, and not to the sulphur. The amount of sulphur in cinders is extremely small, and there seems to be little danger from that source. A steel-frame building erected in New York in 1898 had all its framework, except the columns, imbedded in cinder concrete; when the building was demolished in 1903, the frame showed practically no rust which could be considered as having developed after the material was imbedded.

125. **Practical Illustrations.** Cement washes, paints, and plasters have been used for a long time, in both the United States and Europe, for the purpose of protecting iron and steel from rust. The engineers of the Boston Subway, after making careful tests and investigations, adopted Portland cement paint for the protection of the steel work in that structure. The railroad companies of France use cement paint extensively to protect their metal bridges from corrosion. Two coats of the cement paint and sand are applied with leather brushes.

A concrete-steel water main on the Monier system, 12 inches in diameter, $1\frac{6}{10}$ inches thick, containing a steel framework of $\frac{1}{4}$-inch and $\frac{1}{16}$-inch steel rods, was taken up after 15 years' use in wet ground, at Grenoble, France. The adhesion was found perfect, and the metal absolutely free from rust.

William Sooy Smith, M. Am. Soc. C. E., states that in removing a bed of concrete at a lighthouse in the Straits of Mackinac, twenty years after it was laid, and ten feet below water surface, imbedded iron drift-bolts were found free from rust.

A very good example of the preservation of steel imbedded in concrete is given by Mr. H. C. Turner (*Engineering News*, Jan. 16, 1908). Mr. Turner's company has recently torn down a one-story reinforced-concrete building erected by his company in 1902, at New Brighton, Staten Island. The building had a pile foundation, the piles being cut off at mean tide level. The footings, side walls, columns, and roof were all constructed of reinforced concrete. The portion removed was 30 by 60 feet, and was razed to make room for a five-story building. In concluding his account, Mr. Turner says:

> "All steel reinforcement was found in perfect preservation, excepting in a few cases where the hoops were allowed to come closer than $\frac{3}{4}$ inch to the surface. Some evidence of corrosion was found in such cases, thus demonstrating the necessity of keeping the steel reinforcement at least $\frac{3}{4}$ inch from the surface. The footings were covered by the tide twice daily. The concrete

was extremely hard, and showed no weakness whatever from the action of the salt water. The steel bars in the footings were perfectly preserved, even in cases where the concrete protection was only ¾ inch thick."

126. **Tests by Professor Norton.** Prof. Chas. L. Norton made several experiments with concrete bricks, 3 by 3 by 8-inch, in which steel rods, sheet metal, and expanded metal were imbedded. The specimens were enclosed in tin boxes with unprotected steel, and were exposed for three weeks. One portion was exposed to steam, air, and carbon dioxide; another to air and steam; another to air and carbon dioxide; and another was left in the testing room. In these tests, Portland cement was used. The bricks were made of neat cement of 1 part cement and 3 parts sand; of 1 part cement and 5 parts stone; and of 1 part cement and 7 parts cinders. After the steel had been imbedded in these blocks three weeks, they were opened and the steel examined and compared with specimens which had been unprotected in corresponding boxes in the open air. The unprotected specimens consisted of rather more rust than steel; the specimens imbedded in neat cement were found to be perfectly protected; the rest of the specimens showed more or less corrosion. Professor Norton's conclusions were as follows:

1. Neat Portland cement is a very effective preventive against rusting.
2. Concrete, to be effective in preventing rust, should be dense and without voids or cracks. It should be mixed wet when applied to steel.
3. The corrosion found in cinder concrete is mainly due to iron oxide in the cinders, and not to sulphur.
4. Cinder concrete, if free from voids and well rammed when wet, is about as effective as stone concrete.
5. It is very important that the steel be clean when imbedded in concrete.

FIRE PROTECTION

127. The various tests which have been conducted—including the involuntary tests made as the result of fires—have shown that the fire-resisting qualities of concrete, and even resistance to a combination of fire and water, are greater than those of any other known type of building construction. Fires and experiments which test buildings of reinforced concrete have proved that where the temperature ranges from 1,400° to 1,900° F., the surface of the concrete may be injured to a depth of ½ to ¾ inch or even of one inch; but the body of the concrete is not affected, and the only repairs required, if any, consist of a coat of plaster.

128. Theory. The theory given by Mr. Spencer B. Newberry is that the fireproofing qualities of Portland cement concrete are due to the capacity of the concrete to resist fire and prevent its transference to steel by its *combined water and porosity*. In hardening, concrete takes up 12 to 18 per cent of the water contained in the cement. This water is chemically combined, and not given off at the boiling point. On heating, a part of the water is given off at 500° F., but dehydration does not take place until 900° F. is reached. The mass is kept for a long time at comparatively low temperature by the vaporization of water absorbing heat. A steel beam imbedded in concrete is thus cooled by the volatilization of water in the surrounding concrete.

Resistance to the passage of heat is offered by the porosity of concrete. Air is a poor conductor, and an air space is an efficient protection against conduction. The outside of the concrete may reach a high temperature; but the heat only slowly and imperfectly penetrates the mass, and reaches the steel so gradually that it is carried off by the metal as fast as it is supplied.

129. Cinder vs. Stone Concrete. Mr. Newberry says: "Porous substances, such as asbestos, mineral wool, etc., are always used as heat-insulating material. For this same reason, cinder concrete, being highly porous, is a much better non-conductor than a dense concrete made of sand and gravel or stone, and has the added advantage of being light."

Professor Norton, in comparing the actions of cinder and stone concrete in the great Baltimore fire of February, 1904, states that there is but little difference in the two concretes. The burning of bits of coal in poor cinder concrete is often balanced by the splitting of stones in the stone concrete. "However, owing to its density, the stone concrete takes longer to heat through."

130. Thickness of Concrete Required for Fireproofing. Actual fires and tests have shown that 2 inches of concrete will protect an I-beam with good assurance of safety. Small rods in girders are more effectively coated, and 1½ inches of concrete is usually considered sufficient protection, although some city building laws specify 2 inches of concrete. Beams usually have the same thickness of concrete for fireproofing purposes as the main girders, although perhaps 1 to 1½ inches would be sufficient. For ordinary slabs, ¾ inch is ample protection; but for long-span slabs the fireproofing thickness should be from ¾ inch to 1½ inches. Columns should have at least 2 inches of concrete outside of the steel; often 3 inches is specified.

131. **The Baltimore Fire.** Engineers and architects, who made reports on the Baltimore fire of February, 1904, generally state that reinforced concrete construction stood very well—much better than terra-cotta. Professor Norton, in his report to the Insurance Engineering Experiment Station, says:

> "Where concrete floor-arches and concrete-steel construction received the full force of the fire, it appears to have stood well, distinctly better than the terra-cotta. The reasons, I believe, are these: *First*, because the concrete and steel expand at sensibly the same rate, and hence, when heated, do not subject each other to stress; but terra-cotta usually expands about twice as fast with increase in temperature as steel, and hence the partitions and floor-arches soon become too large to be contained by the steel members which under ordinary temperature properly enclose them."

132. **Fire and Water Tests.** Under the direction of Prof. Francis C. Van Dyck, a test was made on December 26, 1905, on stone and cinder reinforced concrete, according to the standard fire and water tests of the New York Building Department. A building was constructed 16 feet by 25 feet, with a wall through the middle. The roof consisted of the two floors to be tested. One floor was a reinforced cinder concrete slab and steel I-beam construction; and the other was a stone concrete slab and beam construction. The floors were designed for a safe load of 150 pounds per square foot, with a factor of safety of four.

The object of the test was to ascertain the result of applying to these floors, *first*, a temperature of about 1,700° F. during four hours, a load of 150 lbs. per square foot being upon them; and *second*, a stream of water forced upon them while still at about the temperature above stated. A column was placed in the chamber roofed by the rock concrete, and it was tested the same way.

The fuel used was seasoned pine wood, and the stoking was looked after by a man experienced in a pottery; hence a very even fire was maintained, except at first, on the cinder concrete side, where the blaze began in one corner and spread rather slowly for some time.

The water was supplied from a pump at which 90 lbs. pressure was maintained, and was delivered through 200 feet of new cotton hose and a $1\frac{1}{8}$-inch nozzle. Each side was drenched with water while at full temperature, apparently; and the water was thrown as uniformly as possible over the surface to be tested, for the required time. The floors were then flooded on top, and again treated underneath.

Inasmuch as the floors and the column were the only parts submitted for tests, the slight cracking and pitting of the walls and partition need not be detailed.

The column was practically intact, except that a few small pieces of the concrete were washed out where struck by the stream at close range. The metal, however, remained completely covered. On the rock concrete side, the beams showed naked metal up to within about 7 inches of the ends on one beam, and about 2 feet from the ends on the other beam. The reinforcing bars were denuded over an area of about 30 square feet near the center; but no cracks developed, and the water poured on top seemed to come down only through the pipe set in for the pyrometer.

On the cinder concrete side, the beams lost only a little of the edges of the covering, not showing the metal at all. There were no cracks on this side either, and the water came down through the pyrometer tube as on the other side. The metal in the slab was bared over an area of about 24 square feet near the center.

During the firing, both chambers were occasionally examined, and no cracking or flaking-off of the concrete could be detected. Hence the water did all the damage that was apparent at the end.

During the test the floors supported the load they were designed to carry; and on the following day the loads were increased to 600 pounds per square foot.

The following is taken from Professor Van Dyck's report:

"The maximum deflection of the stone concrete *before* the application of water, was $2\frac{15}{16}$ inches; *after* application of water, $3\frac{3}{16}$ inches; with normal temperature and original load, $3\frac{1}{16}$ inches; deflection after load of 600 pounds was added, $3\frac{13}{16}$ inches.

"The maximum deflection of the cinder concrete before the application of water, was $6\frac{1}{16}$ inches; after application of water, $6\frac{1}{2}$ inches; with normal temperature and original load, $5\frac{11}{16}$ inches; deflection after a load of 600 pounds was added, 6 inches. These measurements were taken at the center of the roof of each chamber."

METHODS OF MIXING

133. The two methods used in mixing concrete are by hand and by machinery. Good concrete may be made by either method Concrete mixed by either method should be carefully watched by a good foreman. If a large quantity of concrete is required, it is cheaper to mix it by machinery. On small jobs where the cost of erecting the plant, together with the interest and depreciation, divided by the number of cubic yards to be made, constitute a large item, or if frequent moving is required, it is very often cheaper to mix the concrete by hand. The relative cost of the two methods usually depends upon circumstances, and must be worked out in each individual case.

134. **Hand Mixing.** The placing and handling of materials and arranging the plant are varied by different engineers and contractors. In general the mixing of concrete is a simple operation, but should be carefully watched by an inspector. He should see

(1) That the exact amount of stone and sand are measured out;
(2) That the cement and sand are thoroughly mixed;
(3) That the mass is thoroughly mixed;
(4) That the proper amount of water is used;
(5) That care is taken in dumping the concrete in place;
(6) That it is thoroughly rammed.

The mixing platform, which is usually 10 to 20 feet square, is made of 1-inch or 2-inch plank planed on one side and well nailed to stringers, and should be placed as near the work as possible, but so situated that the stone can be dumped on one side of it and the sand on the opposite side. A very convenient way to measure the stone and sand is by the means of bottomless boxes. These boxes are of such a size that they hold the proper proportions of stone or sand to mix a batch of a certain amount. Cement is usually measured by the package, that is by the barrel or bag, as they contain a definite amount of cement.

The method used for mixing the concrete has little effect upon the strength of the concrete, if the mass has been turned a sufficient number of times to thoroughly mix them. One of the following methods is generally used. (Taylor and Thompson's *Concrete.*)

(a) Cement and sand mixed dry and shoveled on the stone or gravel, leveled off, and wet as the mass is turned.

(b) Cement and sand mixed dry, the stone measured and dumped on top of it, leveled off, and wet, as turned with shovels.

(c) Cement and sand mixed into a mortar, the stone placed on top of it and the mass turned.

(d) Cement and sand mixed with water into a mortar which is shoveled on the gravel or stone and the mass turned with shovels.

(e) Stone or gravel, sand, and cement spread in successive layers, mixed slightly and shoveled into a mound, water poured into the center, and the mass turned with shovels.

The quantity of water is regulated by the appearance of the concrete. The best method of wetting the concrete is by measuring the water in pails. This insures a more uniform mixture than by spraying the mass with a hose.

135. **Mixing by Machinery.** On large contracts the concrete is generally mixed by machinery. The economy is not only in the mixing itself but in the appliances introduced in handling the raw materials and the mixed concrete. If all materials are delivered to the mixer in wheel-barrows, and if the concrete is conveyed away in wheel-barrows, the cost of making concrete is high, even if machine mixers are used. If the materials are fed from bins by gravity into the mixer, and if the concrete is dumped from the mixer into cars and hauled away, the cost of making the concrete should be very low. On small jobs the cost of maintaining and operating the mixer will usually exceed the saving in hand labor and will render the expense with the machine greater than without it.

136. **Machine vs. Hand Mixing.** It has already been stated that good concrete may be produced by either machine or hand mixing, if it is thoroughly mixed.

Tests made by the U. S. Government engineers at Duluth, Minn., to determine the relative strength of concrete mixed by hand and mixed by machine (a cube mixer), showed that at 7 days, hand-mixed concrete possessed only 53 per cent of the strength of the machine-mixed concrete; at 28 days, 77 per cent; at 6 months, 84 per cent; and at one year, 88 per cent. Details of these tests are given in Table X.

It should be noted in this connection, that the variations in strength from highest to lowest were greatest in the hand-mixed samples, and that the strength was more uniform in the machine-mixed.

TABLE X

Tensile Tests of Concrete

(From "Concrete and Reinforced Concrete Construction," by H. A. Reid)

Age, and Method of Mixing	High	Low	Average
Age 7 Days			
Machine-Mixed Sample	260	243	253
Hand-Mixed Sample	159	113	134
Age 28 Days			
Machine-Mixed Sample	294	249	274
Hand-Mixed Sample	231	197	211
Age 6 Months			
Machine-Mixed Sample	441	345	388
Hand-Mixed Sample	355	298	324
Age One Year			
Machine-Mixed Sample	435	367	391
Hand-Mixed Sample	369	312	343

The mixture tested was composed of 1 part cement and 10.18 parts aggregate.

STEEL FOR REINFORCING CONCRETE

137. Steel for reinforcing concrete is not usually subjected to so severe treatment as ordinary structural steel, as the impact effect is likely to be less; but the quality of the steel should be carefully specified. To reduce the cost of reinforced-concrete structures, there has been a great tendency to use cheap steel. It has been generally recognized in the design of reinforced concrete, that the *yield point* or *elastic limit* of the steel shall be considered as the *failing point*. It has been shown by beam tests, that when the yield point of the steel is reached, the beam sags because of the stretching or slipping of the steel, and the top of the beam is likely to crush.

138. **Quality of Reinforcing Steel.** The grades of steel used in reinforced concrete range from soft to fairly hard. These grades of steel may be classified under three heads: *soft*, *medium*, and *hard*.

Soft steel should have an ultimate strength of 50,000 to 60,000 pounds per square inch, and an elastic limit of 28,000 to 35,000 pounds per square inch. The elongation should be 25 per cent in 8 inches; and the specimen should bend cold 180 degrees flat on itself, without fracture on the outside.

Medium steel, ordinary market steel, has an ultimate strength of 60,000 to 70,000 pounds per square inch; and the elastic limit ranges from 35,000 to 40,000 pounds per square inch. The elongation should be 22 per cent in 8 inches, and the specimen should bend cold around a diameter equal to the thickness of the piece tested. This steel is manufactured and sold under standard conditions, and usually it can safely be used without being tested.

Hard steel, better known as *high-carbon steel*, should have an ultimate strength of 85,000 to 105,000 pounds per square inch; and the elastic limit should be from 50,000 to 65,000 pounds per square inch. The elongation should not be less than 10 per cent in 8 inches for a test piece ⅜ to ¾ inch in diameter. A test piece ½ inch in thickness should bend 100 degrees without fracture, around a diameter equal to its own. The high steel has a larger percentage of carbon than the medium steel, and therefore the yield point is higher. This steel is to be preferred for reinforced-concrete work; but it should be thoroughly tested, as many engineers object to it on account of its brittleness and the poor quality of the material from which it is sometimes rolled. On account of its higher elastic limit, a smaller percentage of steel is required; and when rolled under proper specifications and inspection, high steel is more economical for use than low-carbon steel.

In high-carbon steel, the chemical properties should conform to the following limits:

Phosphorus not to exceed	0.06 per cent.
Sulphur not to exceed	0.06 per cent.
Manganese not to exceed	0.80 nor less than 0.40 per cent.
Carbon not to exceed	0.65 nor less than 0.45 per cent.

In comparing the two processes of making steel, the products of Bessemer steel found in the general market are apt to be extremely irregular in their composition, although they may be rolled into like forms and sold for the same purpose. Open-hearth products purchased in the open market and designed to serve the same purpose, are more uniform in quality. Test specimens cut from different parts of the same Bessemer steel plate, often show a wide difference in their mechanical properties. In the open-hearth steel, this wide difference is not found, this grade of steel being more homogeneous than the Bessemer plates.

139. **Types of Reinforcing Steel.** The reinforcing steel usually consists of small bars of such shape and size that they may easily be bent and placed in the concrete so as to form a monolithic structure. To distribute the stress in the concrete, and secure the necessary bond between the steel and concrete, the steel required must be supplied in comparatively small sections. All types of the regularly rolled small bars of square, round, and rectangular section, as well as some of the smaller sections of structural steel, such as angles, T-bars, and channels, and also many special rolled bars, have been used for reinforcing concrete. These bars vary in size from ¼ inch for light construction, up to 1½ inches for heavy beams, and up to 2 inches for large columns. In Europe, plain round bars have been extensively used for many years; and in the United States also, they have been extensively used, but not to the same extent as in Europe; that is, in America a very much larger percentage of work has been done with *deformed* bars.

140. **Plain Bars.** With plain bars, the transmission of stresses is dependent upon the adhesion between the concrete and the steel. Square and round bars show about the same adhesive strength, but the adhesive strength of flat bars is far below that of the round and square bars. The round bars are more convenient to handle and easier obtained, and have, therefore, generally been used when plain bars were desirable.

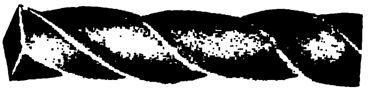


Fig. 13. Ransome Twisted Steel Bar.

141. **Structural Steel.** Small angles, T-bars, and channels have been used to a greater extent in Europe than in this country. They are principally used where riveted skeleton work is prepared for the steel reinforcement; and in this case, usually, it is desirable to have the steel work self-supporting.

142. **Deformed Bars.** There are many forms of reinforcing materials on the market, differing from one another in the manner of forming the irregular projections on their surface. The object of all these special forms of bars is to furnish a bond with the concrete, independent of adhesion. This bond formed between the deformed bar and the concrete, is usually called a *mechanical bond.* Some of the most common types of bars used are the *Ransome, Thacher, Johnson, Diamond, Kahn,* and *Twisted Lug.*

The *Ransome* or twisted bar, shown in Fig. 13, was one of the first steel bars shaped to give a mechanical bond with concrete. This type of bar is a commerical square bar twisted while cold. There are two objects in twisting the bar—*first*, to give the metal a mechanical bond with the concrete; *second*, to increase the elastic limit and ultimate strength of the bar. In twisting the bars, usually one complete turn is given the bar in eight or nine diameters of the bar, with the result that the elastic limit of the bar is increased from 40 to 50 per cent, and the ultimate strength is increased from 25 to 35 per cent. These bars can readily be bought already twisted; or, if it is desired, square bars may be bought and twisted on the site of the work.

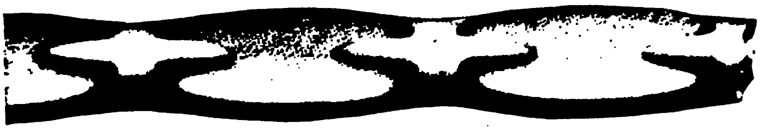


Fig. 14. Thacher Patent Bar.

The *Thacher* bar (Fig. 14) was patented by Mr. Edwin Thacher, M. Am. Soc. C. E. These bars are rolled from medium steel, and range in size from ¼ inch to 2 inches. The cross-sectional area is practically uniform throughout, and all changes in shape of section are made by gradual curves.

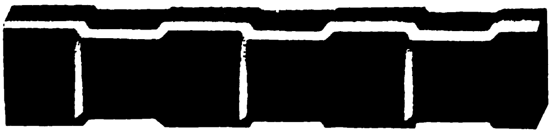


Fig. 15. Johnson Bar.

The *Johnson* or *Corrugated* bar (Fig. 15), with corrugations on all four sides, was invented by Mr. A. L. Johnson, M. Am. Soc. C. E. The corrugations are so placed that the cross-sectional area is the same at all points. The angles of the sides of these corrugations or square shoulders, vary from the axis of the bars not exceeding the angle of friction between the bar and concrete. These bars are usually rolled from high-carbon steel having an elastic limit of 55,000 to 65,000 pounds per square inch and an ultimate strength of about 100,000 pounds per square inch. They are also rolled from any desired quality of steel. In size they range from $\frac{1}{4}$ inch to $1\frac{1}{4}$ inches, their sectional area being the same as that of commercial square bars of the same size.

The *Diamond* bar (Fig. 16) was devised by Mr. William Mueser. This bar has a uniform cross-section throughout its length, exerts a uniform bonding strength at every section, and every portion is available for tensile strength. In design, this bar consists of a round bar with interlacing longitudinal semicircular ribs, and without any sharp angles. The Diamond bar is one of the newer types of bars.

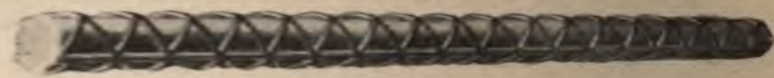


Fig. 16. Diamond Bar.

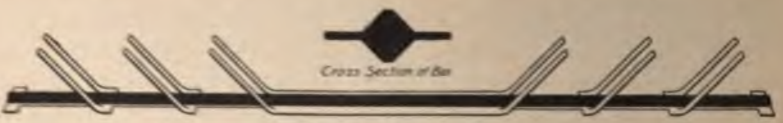


Fig. 17. Kahn Trussed Bar.

The *Kahn* bar (Fig. 17) was invented by Mr. Julius Kahn, Assoc. M. Am. Soc. C. E. This bar is designed with the assumption that the shear members should be rigidly connected to the horizontal members. The bar is rolled with a cross-section as shown in the figure. The thin edges are cut and turned up, and form the shear members. These bars are manufactured in several sizes.

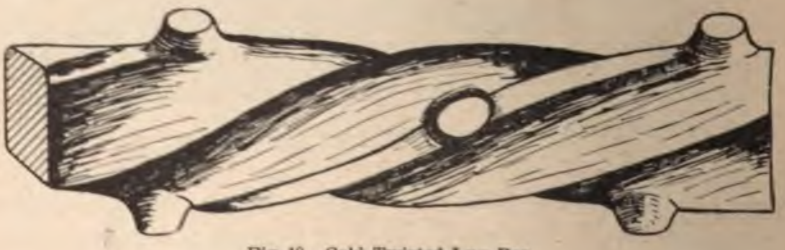
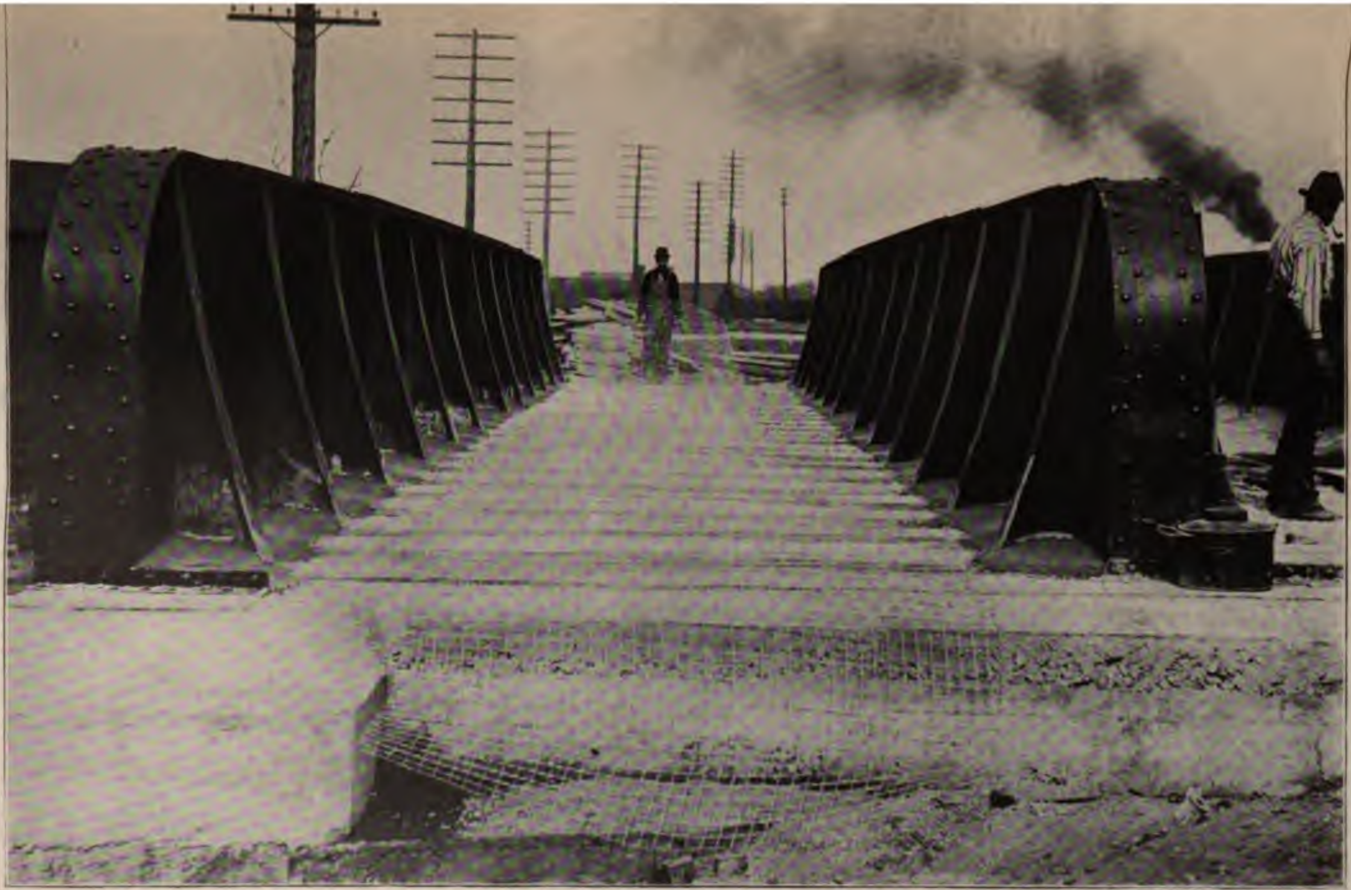


Fig. 18. Cold Twisted Lug Bar.

The *Twisted Lug* bar (Fig. 18) is similar in form to the Ransome cold-twisted bar, with the addition of lugs or truncated cones placed at regular intervals along the spirals. These bars are rolled with the lugs, and the twisting is done either while the bars are hot or at any



REINFORCING CONCRETE FLOOR OF PLATE-GIRDER BRIDGE AT STREET CROSSING

Work of track elevation, City of Chicago.

time after they are cold. If the bars are twisted while hot, their elastic limit and ultimate strength are not raised; that is, their physical properties are not changed.

Expanded metal (Fig. 19) is made from plain sheets of steel, slit in regular lines and opened into meshes of any desired size or section of strand. It is commercially designated by giving the gauge of the steel and the amount of displacement between the junctions of the meshes. The most common manufactured sizes are as follows:

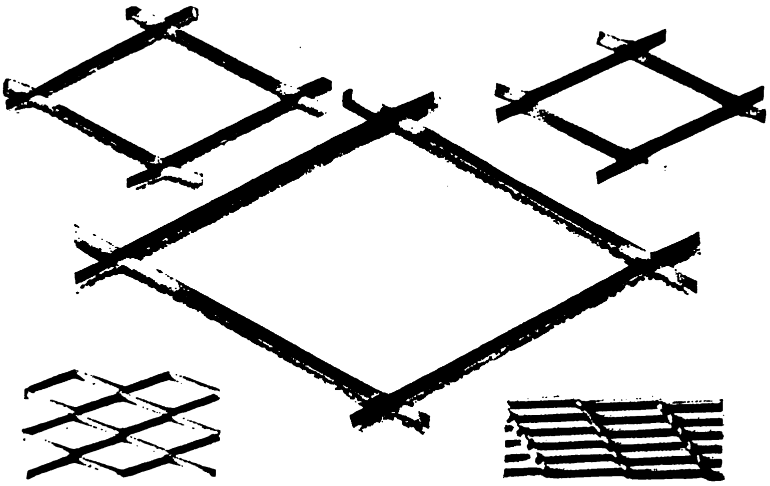


Fig. 19. Styles of Expanded Metal.

Standard Sizes of Expanded Metal

Mesh	Gauge	Weight per Sq. Ft.	Sectional Area 1 foot wide
3-inch	No. 16	.30 lbs.	.082 sq. in.
3-inch	No. 10	.625 lbs.	.177 sq. in.
6-inch	No. 4	.86 lbs.	.243 sq. in.

Steel *wire fabric* reinforcement consists of a netting of heavy and light wires, usually with rectangular meshes. The heavy wires carry the load, and the light ones are used to space the heavier ones. There are many forms of wire fabric on the market.

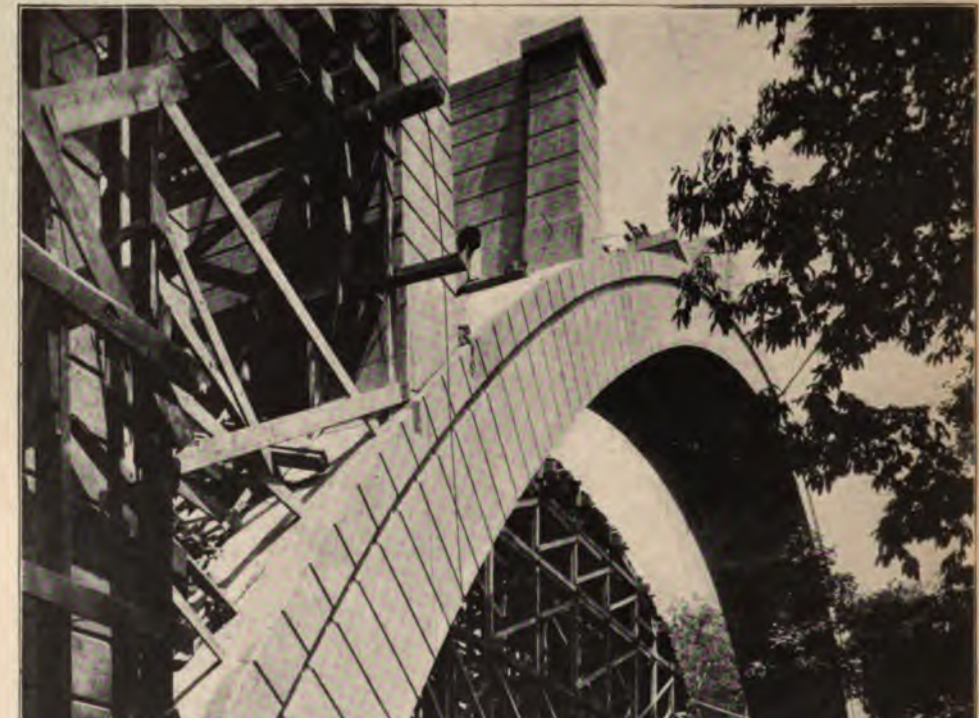
Table XI is condensed from the handbook of the Cambria Steel Company, and gives the standard weights and areas of plain round and square bars as commonly used in reinforced-concrete construction:

TABLE XI

Weights and Areas of Square and Round Bars

(One cubic foot of steel weighs 489.6 pounds)

THICKNESS OR DIAMETER (Inches)	WEIGHT OF SQUARE BAR. 1 FOOT LONG (Pounds)	WEIGHT OF ROUND BAR. 1 FOOT LONG (Pounds)	AREA OF SQUARE BAR (Sq In)	AREA OF ROUND BAR (Sq In)	CIRCUM OF ROUND BAR (Inches)
1/4	.213	.167	.0525	.0491	.7854
5/16	.332	.261	.0977	.0767	.9817
3/8	.478	.376	.1406	.1104	1.1781
7/16	.651	.511	.1914	.1503	1.3744
1/2	.850	.668	.2500	.1963	1.5708
5/8	1.328	1.043	.3906	.3068	1.9635
3/4	1.913	1.502	.5625	.4418	2.3562
1	3.400	2.670	1.0000	.7854	3.1416
1 1/8	4.303	3.379	1.2656	.9940	3.5343
1 1/4	5.312	4.173	1.5625	1.2272	3.9270
1 1/2	7.650	6.008	2.2500	1.7671	4.7124
1 3/4	10.41	8.178	3.0625	2.4053	5.4078
2	13.60	10.68	4.0000	3.1416	6.1823



REINFORCED-CONCRETE ARCH RIB, WALNUT LANE BRIDGE, PHILADELPHIA, PA.

Courtesy of Geo. S. Webster, Chief Engineer, Bureau of Surveys, Dept. of Public Works.

MASONRY AND REINFORCED CONCRETE

PART II

STONE MASONRY

143. Definitions. In the following paragraphs, the meanings of various technical terms frequently used in stone masonry, are clearly explained:

Arris—The external edge formed by two surfaces, whether plane or curved, meeting each other.

Ashlar—A stone wall built of stones having rectangular faces and with joints dressed so closely that "the distance between the general planes of the surfaces of the adjoining stones is one-half inch or less."

Ax* or *Pean Hammer—This tool (Fig. 20) is similar to a double-bladed wood-ax. It is used after the stone is rough-pointed, to make drafts along the edges of the stone. For rubble work, and even for squared-stone work, no finer tool need be used.

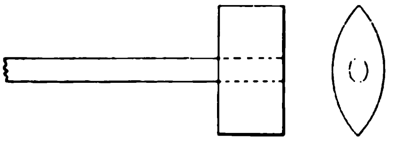


Fig. 20. Ax or Pean Hammer.

Backing—The masonry on the back side of a wall; usually of rougher quality than that on the face.

Batter—The variation from the perpendicular, of a wall surface. It is usually expressed as the ratio of the horizontal distance to the vertical height. For example, a batter of 1:12 means that the wall has a slope of one inch horizontally to each twelve inches of height.

Bearing Block—A block of stone set in a wall with the special purpose of forming a bearing for a concentrated load (such as the load of a beam).

Bed-Joint—A horizontal joint, or one which is nearly perpendicular to the resultant line of pressure (see *Joint*).

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Belt-Course—A horizontal course of stone extending around one or more faces of a building; it is usually composed of larger stones which sometimes project slightly, and is usually employed only for architectural effect.

Bonding—A system of arranging the stones so that they are mutually tied together by the overlapping of joints.

Bush-Hammering—A method of finishing by which the surface of the stone, after being roughly dressed to a surface which is nearly plane, is smoothed still more with a *bush-hammer* (see Fig. 21). The face of the bush-hammer has a large number of small pyramidal points, which, in skilful hands, speedily reduce the surface to a uniformly granular condition.

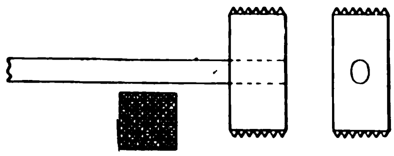
Fig. 21. Bush-Hammer.

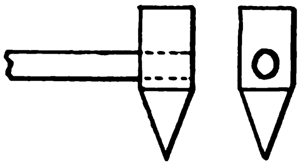
Fig. 23. Cavil.

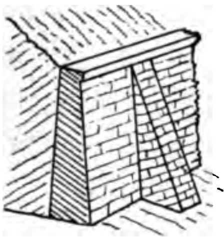
Fig. 22. Buttress.

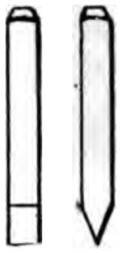
Fig. 24. Chisel.

Buttress—A very short wall (Fig. 22) built perpendicular to a main wall which may be subjected to lateral thrust, in order to resist by *compression* the tendency to tip over. (See *Counterfort.*)

Cavil—A tool which has one blunt face, and a pyramidal point at the other end (Fig. 23). It is used for roughly breaking up stone.

Chisel—A tool made of a steel bar which has one end forged and ground to a chisel edge (Fig. 24). It is used for cutting drafts for the edges of stones.

Coping—A course of stone which caps the top of a wall.

Corbel—A stone projecting from the face of a wall for the purpose of supporting a beam or an arch which extends out from the wall.

Counterfort—A short wall built *behind* a retaining wall, to relieve by *tension* the overturning thrust against the wall. (See *Buttress*.)

Course—A row of stones of equal height laid horizontally along a wall.

Coursed Masonry—Masonry having courses of equal height throughout.

Coursed Rubble—Rubble masonry (see *Rubble*) which is leveled off at certain definite heights so as to make continuous horizontal joints. The expediency of this is doubtful. It certainly adds something to the cost; it probably makes the wall somewhat weaker, and is no advantage either mechanically or in appearance.

Cramp—A bar of iron, the ends of which are bent at right angles, which is inserted in holes and grooves specially cut for it in adjacent stones in order to bind the stones together. When they are carefully packed with cement mortar, they are effectively prevented from rusting.

Crandall—A tool made by fitting a series of steel points into a handle, using a wedge (see Fig. 25), by means of which a series of fine picks at the stone are made with each stroke, and the surface is more quickly reduced to a true plane.

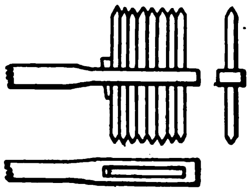
Fig. 25. Crandall.

Crandalling—A system of dressing stone by which the surface, after having been rough-pointed to a fairly plane surface, is hammered with a crandall such as is illustrated in Fig. 25.

Dimension Stone—Cut stone whose precise dimensions in a building are specified in the plans. The term refers to the highest grade of ashlar work.

Dowel—A straight bar of iron, copper, or even stone, which is inserted in two corresponding holes in adjacent stones. They may be vertical across horizontal joints, or horizontal across vertical joints. In the latter case, they are frequently used to tie the stones of a coping or cornice. The extra space between the dowels and the stones should be filled with melted lead, sulphur, or cement grout.

Draft—A line on the surface of a stone which is cut to the breadth of the draft chisel.

Dry Stone Masonry—Masonry which is put in place without mortar.

Extrados—The upper surface of an arch.

Face—The exposed surface of a wall.

Face-Hammer—A tool having a hammer face and an ax face. It is used for roughly squaring up stones, either for rubble work or in preparation for finer stone dressing. See Fig. 26.

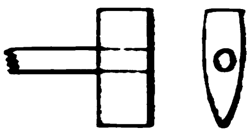

Fig. 26. Face-Hammer.

Feathers—See *Plugs*.

Footing—The foundation masonry for a wall or pier, usually composed (in stone masonry) of large stones having a sufficient area so that the pressure upon the subsoil shall not exceed a safe limit, and having sufficient transverse strength to distribute the pressure uniformly over the subsoil.

Grout—A thin mixture of cement, sand, and water, which is sometimes forced by pressure into the cracks in defective masonry or to fill cavities which have formed behind masonry walls. Sometimes grout has been used to solidify quicksand. Its use must always be considered as a makeshift with which to improve a bad condition of affairs. It is frequently used in the endeavor to hide defective work.

Header—A stone laid with its greatest dimension perpendicular to the face of a wall. Its purpose is to bond together the facing and the backing.

Intrados—The inner (or under) surface of an arch.

Jamb—The vertical sides of an opening left in a wall for a door or window.

Joint—The horizontal and vertical spaces between the stones, which are filled with mortar, are called the *joints*. When they are horizontal, they are called *bed-joints*. Their width or thickness depends on the accuracy with which the stones are dressed. The joint should always have such a width that any irregularity on the surface of a stone shall not penetrate completely through the mortar joint and cause the stones to bear directly on each other, thus producing concentrated pressures and transverse stresses which might rupture the stones. The criterion used by a committee of the American Society of Civil Engineers in classifying different grades of masonry, is to make the classification depend on the required thickness of the joint. These thicknesses have been given when defining various grades of stone masonry.

Lintel—The stone, iron, wood, or concrete beam covering the opening left in a wall for a door or window.

Natural Bed—The surfaces of a stone parallel to its stratification.

One-Man Stone—A term used to designate roughly the size and weight of stone used in a wall. It represents, approximately, the size of stone which can be readily and continuously handled by one man.

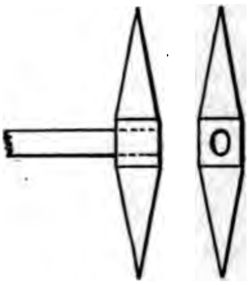

Fig. 27. Pick.

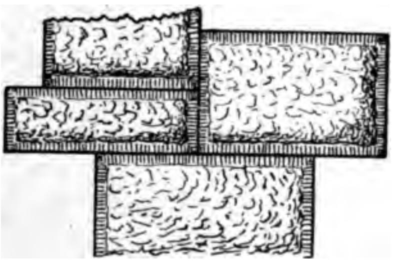

Fig. 28. Pitched-Faced Masonry.

Pick—A tool which roughly resembles an earth pick, but which has two sharp points. It is used like a cavil for roughly breaking up and forming the stones as desired. (See Fig. 27.)

Pitched-Faced Masonry—That in which the edges of the stone are dressed to form a rectangle which lies in a true plane, although the portion of the face between the edges is not plane. (See Fig. 28.)

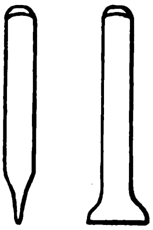

Fig. 29. Pitching Chisel.

Pitching Chisel—A tool which is used with a mallet to prepare pitched-face masonry. The usual dimensions are as illustrated in Fig. 29.

Plinth—Another term for *Water-Table*, which see.

Plug—A plug is a truncated wedge (see Fig. 30). Corresponding with them are wedge-shaped pieces made of half-round malleable iron. A plug is used in connection with a pair of *feathers* to split a section of stone uniformly. A row of holes is drilled in a straight line along the surface of the stone, and a plug and pair of feathers are inserted in each hole. The plugs in succession are tapped lightly with a hammer so that the pressure produced by all the plugs is increased as uniformly as possible. When the pressure is uniform, the stone usually splits along the line of the holes without injury to the portions split apart.

Point—A tool made of a bar of steel whose end is ground to a point. It is used in the intermediate stage of dressing an irregular surface which has already been roughly trued up with a face-hammer or an ax. For rough masonry, this maybe the finishing tool. For higher-grade masonry, such work will be followed by bush-hammering, crandalling, etc.

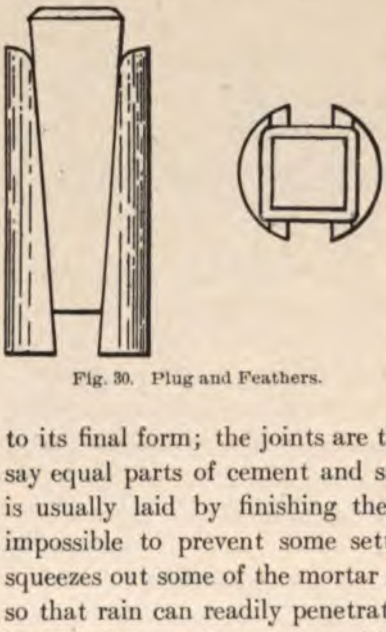

Fig. 30. Plug and Feathers.

Pointing—A term applied to the process of scraping out the mortar for a depth of an inch or more on the face of a wall after the wall is complete and is supposed to have become compressed to its final form; the joints are then filled with a very rich mortar—say equal parts of cement and sand. Although ordinary brickwork is usually laid by finishing the joints as the work proceeds, it is impossible to prevent some settling of the masonry, which usually squeezes out some of the mortar and leaves it in a cracked condition so that rain can readily penetrate through the cracks into the wall. By scraping out the mortar, which may be done with a hook before it has become thoroughly hard, the joint may be filled with a high grade of mortar which will render it practically impervious to rain-water. The pointing may be done with a masons' trowel, although, for architectural effect, such work is frequently finished off with specially formed tools which will mould the outer face of the mortar into some desired form.

Quarry-Faced Stone—Stone laid in the wall as it comes from the quarry. The term usually applies to stones which have such regular cleavage planes that even the quarry faces are sufficiently regular for use without dressing.

Quoin—A stone placed in the corner of a wall so that it forms a header for one face and a stretcher for the other.

Random—The converse of *Coursed Masonry*; masonry which is *not* laid in courses.

Range—Masonry in which each course has the same thickness throughout, but the different courses vary in thickness.

Rip-Rap—Consists of rough stone just as it comes from the quarry, which is placed on the surface of an earth embankment.

Rough-Pointing—Dressing the face of a stone by means of a *pick*, or perhaps a *point*, until the surface is approximately plane. This may be the first stage preliminary to finer dressing of the stones.

Rubble—Masonry composed of stones as they come from the quarry without any dressing other than knocking off any objectionable protruding points. The thickness may be quite variable, and therefore the joints are usually very thick in places.

Slope-Wall Masonry—A wall, usually of dry rubble, which is built on a sloping bank of earth and supported by it, the object of the wall being chiefly to protect the embankment against scour.

Spalls—Small stones and chips, selected according to their approximate fitness, which are placed between the larger, irregular stones in rubble masonry in order to avoid in places an excessive thickness of the mortar joint. Specifications sometimes definitely forbid their use.

Squared-Stone Masonry—Masonry in which the stones are roughly dressed so that at the joints "the distance between the general planes of the surface of adjoining stones is one-half inch or more."

Stretcher—A stone which is placed in the wall so that its greatest dimension is parallel with the wall.

String-Course—A course of stone or brick running horizontally around a building, whose sole purpose is architectural effect (see *Belt-Course*).

Template—A wooden form used as a guide in dressing stones to some definite shape (see Figs. 33 and 34).

Two-Men Stone—A rather indefinite term applied to a size and weight of stone which cannot be readily handled except by two men. The term has a significance in planning the masonry work.

Water-Table—A course of stone which projects slightly from the face of the wall and which is usually laid at the top of the foundation wall. Its function is chiefly architectural, although, as its name implies, it is supposed to divert the water which might drain down the wall of a building, and to prevent it from following the face of the foundation wall.

Wooden Brick—A block of wood placed in a wall in a situation where it will later be convenient to drive nails or screws. Such a block is considered preferable to the plan of subsequently drilling a hole and inserting a plug of wood into which the screws or nails may be driven, since such a plug may act as a powerful wedge and crack the masonry.

144. **Classification of Dressed Stones.** Stone masonry is classified according to the shape of the stones, and also according to the quality and accuracy of the dressing of the joints so that the joints may be close. The definitions of these various kinds of stonework have already been given in the previous section, and therefore will not be repeated here; but the classification will be repeated in the order of the quality and usual relative cost of the work. The term *ashlar* refers to the rectangular shape of the stone and the accuracy of dressing the joints, and may be applied to *coursed ashlar*, *range*, and even *random*. The next grade in quality is *squared-stone masonry*, which likewise refers only to the accuracy in dressing the joints. The variations in the coursing of the stones may be the same as for ashlar. The term *rubble* is usually applied to stone masonry on which but little work has been done in dressing the stones, although the cleavage planes may be such that very regular stones may be produced with very little work. Rubble masonry usually has joints which are very irregular in thickness. In order to reduce the amount of clear mortar which otherwise might be necessary in places between the stones, small pieces of stone called *spalls* are placed between the larger stones. Such masonry is evidently largely dependent upon the shearing and tensile strength of the mortar and is therefore comparatively weak. *Random rubble* (Fig. 31), which has joints that are not in general horizontal or vertical, or even approximately so, must be considered as a weak type of masonry. In fact the real strength of such walls, which are frequently built for architectural effect, depends on the backing to which the facing stones are sometimes secured by cramps.

145. Stone Cutting and Dressing. Many of the requirements and methods of stone dressing have already been stated in the definitions given above. Frequently a rock is so stratified that it can be split up into blocks whose faces are so nearly parallel and perpendicular that they may be used with little or no dressing in building a substantial wall with comparatively close joints. On the other hand, an igneous rock such as granite must be dressed to a regular form.

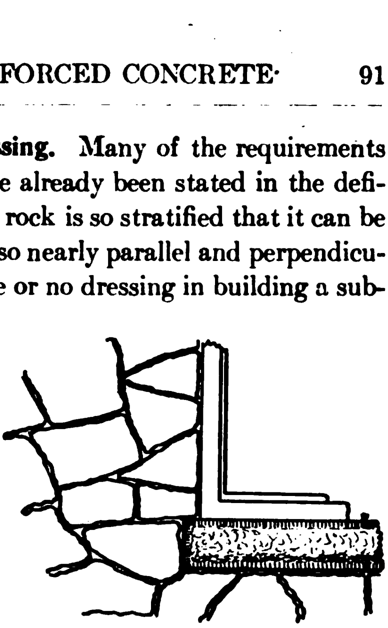

Fig. 31. Random Rubble.

The first step in making rectangular blocks from any stone is to decide from its stratification, if any, or its cleavage planes, how the stone may be dressed with the least labor in cutting. The stone is then marked in straight lines with some form of marking chalk, and drafts are cut with a drafting chisel so as to give a rectangle whose four lines lie all in one plane. The other faces are then dressed off with as great accuracy as is desired, so that they are perpendicular (or parallel) to this plane. For squared-stone masonry, and especially for ashlar masonry, the drafts should be cut for the bed-joints, and the surface between the drafts on any face should be worked down to a true plane, or nearly so. The bed-joints should be made slightly concave rather than convex, but the concavity should be very slight. If the surface is very convex, there is danger that the stones will come in contact with each other and produce a concentration of pressure, unless the joints are made undesirably thick. If they are very concave, there is a danger of developing transverse stresses in the stones, which might cause a rupture. The engineer or contractor must be careful to see that the bed-joints are made truly perpendicular to the face. A frequent trick of masons is to make the stones like truncated wedges, as illustrated in Fig. 32. Such masonry, when finished, may look almost

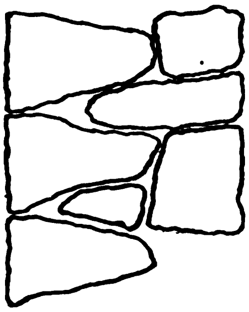

Fig. 32. Defective Work.

like ashlar; but such a wall is evidently very weak, even dangerously so.

To produce a cylindrical surface on a stone, a draft must be cut along the stone, which shall be parallel with the axis of the cylinder. See Fig. 33. A template made with a curve of the desired radius, and with a guide which runs along the draft, may be used in cutting down the stone to the required cylindrical form. A circular template swung around a point which may be considered as a pole, may be used for making spherical surfaces, although such work is now usually done in a lathe instead of by hand.

To make a warped surface or helicoidal surface, a template must be made, as in Fig. 34, by first cutting two drafts which shall fit a template made as shown in the figure. After these two drafts are cut, the surface between them is dressed down to fit a straight edge, which is moved along the two drafts and perpendicular to them. Such stonework is very unusual, and almost its only application is in the making of oblique or helicoidal arches.

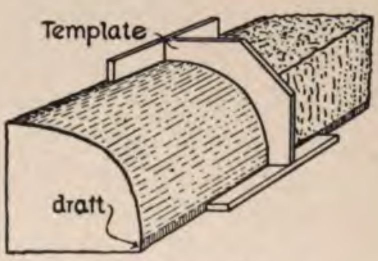

Fig. 33. Cylindrical Surface.

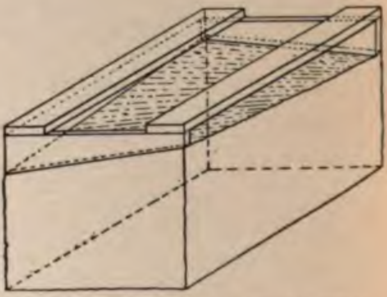

Fig. 34. Template for Warped-Surface Cutting.

The size of the blocks has a very great influence on the cost of dressing the stones *per cubic yard* of masonry. For example, to quote a very simple case, a stone 3 feet long, 2 feet wide, and 18 inches high has 12 square feet of bed-joints, 6 square feet of end joints, and 4.5 square feet of facing, and contains 9 cubic feet of masonry. If the stones are 18 inches long, 1 foot wide, and 9 inches high (just one-half of each dimension), the area of each kind of dressed joint is one-fourth that in the case of the larger stones, but the *volume* of the masonry is only *one-eighth*. In other words, for stones of *similar* shape, increasing the size increases the area of dressing in proportion to the *square* of the dimensions, but it also increases the volume in proportion to the *cube* of the dimensions. Therefore large stones are far more economical than small stones, so far as the cost of dressing is a factor.

The size of stones, the thickness of courses, and the type of masonry should depend largely on the product of the quarry to be utilized. An unstratified stone like granite must have *all* faces of the stone plug-and-feathered; and therefore the larger the stone, the less will be the area to be dressed per cubic foot or yard of masonry. On the other hand, the size of blocks which can be broken out from a quarry of stratified rock, such as sandstone or limestone, is usually fixed somewhat definitely by the character of the quarry itself. The stratification reduces very greatly the work required, especially on the bed-joints. But since the stratification varies, even in any one quarry, it is generally most economical to use a stratified stone for *random* masonry, while granite can be cut for *coursed* masonry at practically the same expense as for stones of variable thickness.

146. **Cost of Dressing Stone.** Although, as explained above, the cost of dressing stone should properly be estimated by the *square foot* of surface dressed, most figures which are obtainable give the cost per *cubic yard* of masonry, which practically means that the figures are applicable only to stones of the average size used in that work. A few figures are here quoted from Gillette's "Handbook of Cost Data:"

(*a*) HAND DRESSING—Wages, 50 cents per hour. Soft, 25 to 30 cents; medium, 40 to 45 cents; hard, 75 to 80 cents, per square foot of surface dressed.

(*b*) HAND DRESSING—Wages, $3 per day. Limestone, bush-hammered, 25 cents per square foot.

(*c*) HAND DRESSING LIMESTONE—36 square feet of beds and joints per 9-hour day (or 4 square feet per hour); wages, 40 cents per hour, or 10 cents per square foot.

(*d*) HAND DRESSING GRANITE—For ½-inch joints, 26 cents per square foot.

(*e*) SAWING SLABS BY MACHINERY—Costs approximately 17 cents per square foot.

147. **Constructive Features—Bonding.** It is a fundamental principle of masonry construction, that vertical joints (either longitudinal or lateral) should not be continuous for any great distance.

Masonry walls (except those of concrete blocks) are seldom or never constructed entirely of single blocks which extend clear through the wall. The wall is essentially a double wall which is frequently connected by *headers*. These break up the continuity of the longitudinal vertical joints. The continuity of the lateral vertical joints is broken up by placing the stones of an upper course over the joints in the course below. Since the headers are made of the same quality of stone (or brick) as the face masonry, while the backing is of comparatively inferior quality, it costs more to put in numerous headers, although strength is sacrificed by neglect to do so. For the best work, stretchers and headers should alternate. This would usually mean that about one-third of the face area would consist of headers. One-fourth or one-fifth is a more usual ratio. Cramps and dowels are merely devices to obtain a more efficient bonding. An inspector must guard against the use of *blind headers*, which are *short* blocks of stone (or brick), which have the same external appearance on the finished wall, but which furnish no bond. After an upper course has been laid, it is almost impossible to detect them.

Amount of Mortar. For the same reasons given when discussing the relation of size of stones to amount of dressing required, more mortar per cubic yard of masonry is needed for small stones than for large. The larger and rougher joints, of course, require more mortar per cubic yard of masonry. In the tabular form at top of page 95, are given figures which, for the above reasons, are necessarily approximate; the larger amounts of mortar represent the requirements for the smaller sizes of stone, and *vice versâ*:

The stones should be very thoroughly wetted before laying in the wall, so that they will not absorb the water in the mortar and ruin it before it can set. It is very important that the bed-joints should be thoroughly flushed with mortar. All vertical joints should likewise be tightly filled with mortar.

148. **Allowable Unit-Pressures.** In estimating such quantities, the following considerations must be kept in mind:

(*a*) The accuracy of the dressing of the stone, **particularly the bed-joints**, has a very great influence.

(*b*) The strength is largely dependent on that of the mortar.

(*c*) The strength is so little dependent on that of the stone itself that the strength of the stone cannot be considered a guide to the strength of the masonry. For example, masonry has been known to fail under a load not

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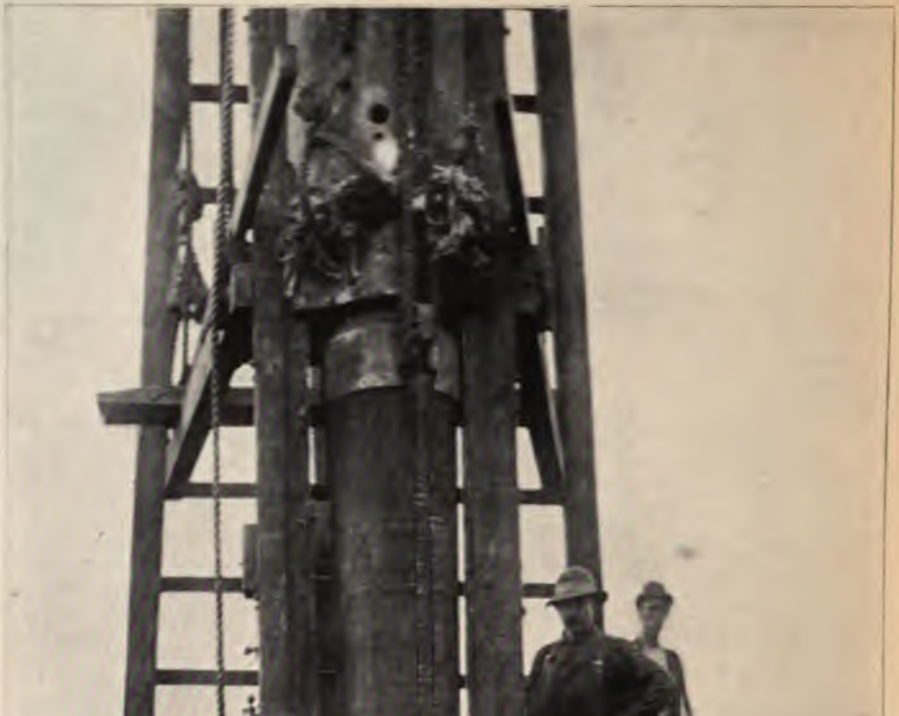

DRIVING A "RAYMOND" CONCRETE PILE

The pile is here being driven into the previously driven steel shell.

Mortar per Cubic Yard of Masonry

GRADE OF MASONRY	VOLUME OF MORTAR PER CUBIC YARD OF MASONRY
Ashlar	1 to 2 cubic feet
Squared-Stone	4.5 to 7 " "
Rubble	5.5 to 9 " "

more than five per cent of the ultimate crushing strength of the stone itself.

(*d*) The strength of a miniature or small-scale prism of masonry is evidently no guide to the strength of large prisms. The ultimate strength of these is beyond the capacity of testing machines.

(*e*) So much depends on the workmanship, that in any structure where the unit-stresses are so great as to raise any question concerning the strength, the best workmanship must be required.

Judging from the computed pressures now carried by noted structures, and also from the pressures sustained by piers, etc., which have shown distress and have been removed, it is evident that, assuming good workmanship, we may depend on masonry as follows:

Allowable Pressures on Masonry

Granite Ashlar	up to 400 pounds per sq. inch
Limestone or Sandstone Ashlar	" " 300 " " " "
Squared Stone	" " 250 " " " "
Rubble	" " 100 " " " "

It is interesting to note that, although concrete has been considered inferior even to rubble, unit-stresses of 400 pounds per square inch are now being freely employed for concrete.

149. Cost of Stone Masonry. The total cost is a combination of several very variable items as follows:

1. Value of quarry privilege;
2. Cost of stripping superincumbent earth or disintegrated rock;
3. Cost of quarrying;
4. Cost of dressing;
5. Cost of transportation (teaming, railroad, etc.) from quarry to site of work;
6. Cost of mortar;
7. Cost of centering, scaffolding, derricks, etc.;
8. Cost of laying;
9. Interest and depreciation on plant;
10. Superintendence.

Some of the above items may be practically nothing, in cases. The cost of some of the items has already been discussed. The cost of many items is so dependent on local conditions and prices that the quotation of the cost of definite jobs would have but little value and might even be deceptive. The following very general values may be useful to give a broad idea of the cost:

Cost of Stone Masonry

Rubble Masonry in Mortar	$3.00 to $ 5.00	per cubic yard.
Squared-Stone Masonry	6.00 to 10.00	" " "
Dimension Stone, Granite Ashlar	up to 60.00	" " "

BRICK MASONRY

Many of the terms employed in stone masonry, and of the directions for properly doing the work, are equally applicable to brick masonry, and therefore will not be here repeated. The following sections will be devoted to those terms and specifications which are applicable only to brick masonry.

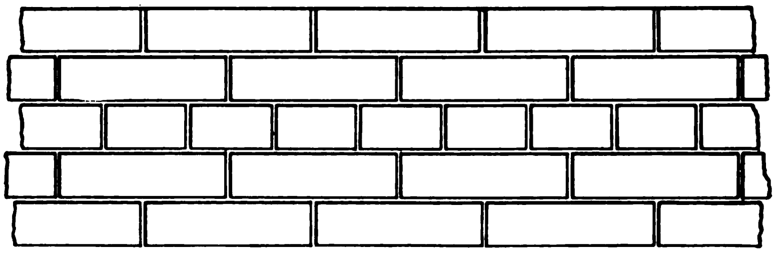

Fig. 35. Common Bond.

150. **Bonding Used in Brick Masonry.** Some of the principles involved in the effect of bonding on the strength of a wall, have already been discussed under "Stone Masonry." The other consideration is that of architectural appearance. The common method of bonding (Fig. 35) is to lay five or six courses of brick entirely as stretchers, then a course of brick will be laid entirely as headers. There is probably some economy in the work required of a bricklayer in following this policy. The so-called *English Bond* (Fig. 36) consists of alternate courses of headers and stretchers. If the face bricks are of better quality than those used in the backing of the wall, this system means that one-half the face area of the wall consists of headers; which is certainly not an economical way of using the facing brick. The *Flemish Bond* (Fig. 37) employs alternate headers and

stretchers in each course, and also disposes of the vertical joints so that there is a definite pattern in the joints, which has a pleasing architectural effect.

151. **Constructive Features.** On account of the comparatively high absorptive power of brick, it is especially necessary that they shall be thoroughly soaked with water before being laid in the wall. An excess of water can do no harm, and will further insure the bricks being clean from dust, which would affect the adhesion of the mortar. It is also important that the brick shall be laid with what is called a *shove joint.* This term is even put in specifications, and has a definite meaning to masons. It means that after laying the mortar for the bed-joints, a brick is placed with its edge projecting somewhat over that of the lower brick, and is then pressed down into the mortar, and, while still being pressed down, is *shoved* into its proper position. In this way is obtained a proper adhesion between the mortar and the brick.

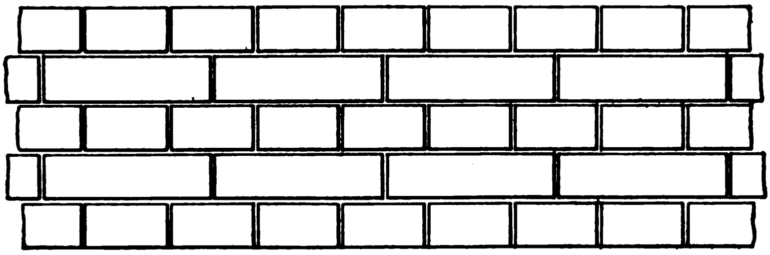

Fig. 36. English Bond.

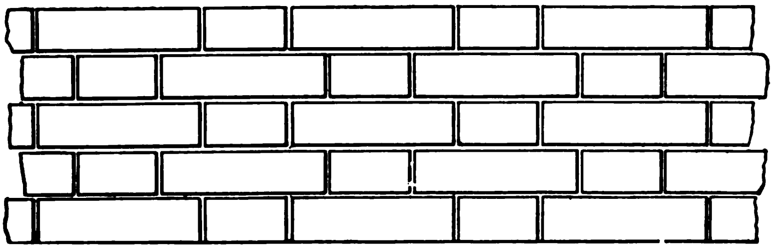

Fig. 37. Flemish Bond.

The thickness of the mortar joint should not be over one-half inch; one-fourth inch, or even less, is far better, since it gives stronger masonry. It requires more care to make thin joints than thick joints, and therefore it is very difficult to obtain thin joints when masons are paid by piecework. Pressed brick fronts are laid with joints of one-eighth inch or even less, but this is considered high-grade work and is paid for accordingly.

152. **Strength of Brickwork.** As previously stated with respect to stone masonry, the strength of brick masonry is largely dependent upon the strength of the mortar; but, unlike stone masonry, the strength of brick masonry is, in a much larger proportion, dependent on the strength of the brick composing it. The ultimate strength of brick masonry has been determined by a series of tests, to vary from 1,000 to 2,000 pounds per square inch, using lime mortar; and from 1,500 to 3,000 pounds per square inch, using cement mortar—the variation in each group (for the same kind of mortar) depending on the quality of the brick. A large factor of safety, perhaps 10, should be used with such figures.

153. **Methods of Measuring Brickwork.** There is unfortunately a considerable variation in the methods of measuring brickwork, the variation depending on local trade customs. Brickwork is often paid for by the *perch*. The volume of a perch was originally taken from a similar volume of stone masonry, the unit being a section of the wall one rod (16½ feet) long and one foot high. Since the usual custom made such a wall 18 inches thick, the volume 24¾ cubic feet came to be considered as one perch of masonry; then this number was modified to the round number 25 cubic feet, for convenience of computation. The construction of walls one foot thick and with the same face unit of measurement, gave rise to a unit volume of 16½ cubic feet, which was also called a perch. Such units have undoubtedly arisen from the fact that it requires more work per cubic yard to build a thin wall than a thick wall, and the brick mason desires a unit of measurement more nearly in accordance with the labor involved.

Brick is generally paid for by the cubic yard or by the thousand, and the bidder must make his own allowance, if necessary, for any extra work due to thin walls. The number of brick per cubic yard depends on the thickness of the joints and on the size of the bricks. A very slight variation in the thickness of the joint will change very materially the number of brick per cubic yard, and also the amount of mortar. The exact values (according to the size of the brick and the thickness of the mortar joint) are as given below; but the values are not closely to be depended on, because of these variations:

Quantities of Brick and Mortar

Kind of Brick	Size (Inches)	Thickness of Joints	No. of Brick per Cubic Yard	Mortar	
				Per Cubic Yard of Masonry	Per 1,000 Brick
Common brick	8¼ × 4 × 2¼	½ in.	430	.34 cu. yd.	.80 cu. yd.
" "	8¼ × 4 × 2¼	¼ in.	516	.21 " "	.40 " "
Pressed "	8⅜ × 4⅛ × 2¼	⅛ in.	544	.11 " "	.21 " "

It is very common and convenient to estimate that 1,000 brick will make two cubic yards of masonry. The number of brick per cubic yard given above is the equivalent of 16, 19, and 20 brick per cubic foot. Bricklayers (backed up by their unions) sometimes demand pay per 1,000 brick laid, but compute the number on the basis of 7½ bricks per superficial foot of a wall 4 inches thick, 15 bricks for a "9-inch wall," and 22½ bricks for a "13-inch wall." The number actually used in a 13-inch wall varies from 17 to 20.

154. **Cost of Brickwork.** A laborer should handle 2,000 brick per hour in loading them from a car to a wagon. If they are not unloaded by dumping, it will require as much time again to unload them. A mason should lay from 1,200 to 1,500 brick per 9-hour day on ordinary wall work. For large, massive foundation work with thick walls, the number should rise to 3,000 per day. On the other hand, the number may drop to 200 or 300 on the best grade of pressed-brick work. About one helper is required for each mason. Masons' wages vary from 40 to 60 cents per hour; helpers' wages are about one-half as much.

155. **Impermeability.** As previously stated, brick is very porous; ordinary cement mortar is not water-tight; and therefore, when it is desirable to make brick masonry impervious to water, some special method must be adopted as described in Part I, under the head of "Waterproofing."

156. **Efflorescence.** This name is applied to the white deposit which frequently forms on brickwork and concrete, and has already been described in Part I. The Sylvester wash has frequently been used as a preventive, and with fairly good results. Diluted acid

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has been used successfully to remove the efflorescence. These methods have already been described in Part I.

157. **Brick Piers.** A brick pier, as a general rule, is the only form of brickwork that is subjected to its full resistance. Sections of walls under bearing plates also receive a comparatively large load; but only a few courses receive the full load, and therefore a greater unit-stress may be allowed than for piers.

Kidder gives the following formulæ for the safe strength of brick piers exceeding 6 diameters in height:

$$\left.\begin{array}{l}
\text{PIERS LAID WITH RICH LIME MORTAR,} \\
\qquad \text{Safe load in pounds per square inch} = 110 - 5\frac{H}{D} \ldots (1) \\
\text{PIERS LAID WITH 1 TO 2 NATURAL CEMENT MORTAR,} \\
\qquad \text{Safe load in pounds per square inch} = 140 - 5\tfrac{1}{2}\frac{H}{D} \ldots (2) \\
\text{PIERS LAID WITH 1 TO 3 PORTLAND CEMENT MORTAR,} \\
\qquad \text{Safe load in pounds per square inch} = 200 - 6\frac{H}{D} \ldots (3)
\end{array}\right\} \ldots (1)$$

In the above formulæ, H is the height of the column in feet, and D is the diameter of the column in feet.

For example, a column 16 feet in height and 1¾ feet square, laid with rich lime mortar, may be subjected to a load of 65 pounds per square inch, or 9,360 pounds per square foot; for a 1 to 2 natural cement mortar, 90 pounds per square inch, or 12,960 pounds per square foot; and for a 1 to 3 Portland cement mortar, 146 pounds per square inch, or 20,914 pounds per square foot.

The building laws of some cities require a bonding stone spaced every 3 to 4 feet, when brick piers are used. This stone is 5 to 8 inches thick, and is the full size of the pier. Engineers and architects are divided in their opinion as to the results obtained by using the bonding stone.

CONCRETE

158. Concrete is extensively used for constructing the many different types of foundations, retaining walls, dams, culverts, etc. The ingredients of which concrete is made, the proportioning and the methods of mixing these materials, etc., have been discussed in Part I. Methods of mixing and handling concrete by machinery will be discussed in Part IV. Various details of the use of concrete in the construction of foundations, etc., will be discussed during the treatment of the several kinds of work.

RUBBLE CONCRETE

159. Rubble concrete includes any class of concrete in which large stones are placed. The chief use of this concrete is in constructing dams, lock walls, breakwaters, retaining walls, and bridge piers.

The cost of rubble concrete in large masses should be less than that of ordinary concrete, as the expense of crushing the stone used as rubble is saved, and each large stone replaces a portion of cement and aggregate; therefore this portion of cement is saved, as well as the labor of mixing it. The weight of a cubic foot of stone is greater than that of an equal amount of ordinary concrete, because of the pores in the concrete; the rubble concrete is therefore heavier, which increases its value for certain classes of work. In comparing rubble concrete with rubble masonry, the former is usually found cheaper because it requires very little skilled labor. For walls 3 or 3½ feet thick, the rubble masonry will usually be cheaper, owing to the saving in forms.

160. **Proportion and Size of Stone.** Usually the proportion of rubble stone is expressed in percentage of the finished work. This percentage varies from 20 to 65 per cent. The percentage depends largely on the size of the stone used, as there must be nearly as much space left between small stones as between large ones. The percentage therefore increases with the size of the stones. When "one-man" or "two-men" rubble stone is used, about 20 per cent to 25 per cent of the finished work is composed of these stones. When the stones are large enough to be handled with a derrick, the proportion is increased to about 33 per cent; and to 55 per cent, or even 65 per cent, when the rubble stones average from 1 to 2½ cubic yards each.

The distance between the stones may vary from 3 inches to 15 or 18 inches. With a very wet mixture of concrete, which is generally used, the stones can be placed much closer than if a dry mixture is used. With the latter mixture, the space must be sufficient to allow of the concrete being thoroughly rammed into all of the crevices. Specifications often state that no rubble stone shall be placed nearer the surface of the concrete than 6 to 12 inches.

161. **Rubble Masonry Faces.** The faces of dams are very often

built of rubble, ashlar, or cut stone, and the filling between the faces made of rubble concrete. For this style of construction, no forms are required. For rubble concrete, when the faces are not constructed of stone, wooden forms are constructed as for ordinary concrete.

162. **Comparison of Quantities of Materials.** The mixture of concrete used for this class of work is often 1 part Portland cement, 3 parts sand, and 6 parts stone. The quantities of materials required for one yard of concrete, according to Table VI, are 1.05 bbls. cement, 0.44 cu. yd. sand, and 0.88 cu. yd. stone. If rubble concrete is used, and if the rubble stone laid averages 0.40 cubic yard for each yard of concrete, then 40 per cent of the cubic contents is rubble, and each of the other materials may be reduced 40 per cent. Reducing these quantities gives 1.05 × 0.60 = 0.63 bbl. of cement; 0.44 × 0.60 = 0.26 cu. yd. sand; and 0.88 × 0.60 = 0.53 cu. yd. of stone, per cubic yard of rubble concrete.

The construction of a dam on the Quinebaug river is a good example of rubble concrete. The height of the dam varies from 30 to 45 feet above bed-rock. The materials composing the concrete consist of bank sand and gravel excavated from the bars in the bed of the river. The rock and boulders were taken from the site of the dam, and were of varying sizes. Stones containing 2 to 2½ cubic yards were used in the bottom of the dam, but in the upper part of the dam smaller stones were used. The total amount of concrete used in the dam was about 12,000 cubic yards. There was 1½ cubic yards of concrete for each barrel of cement used. The concrete was mixed wet, and the large stones were so placed that no voids or hollows would exist in the finished work.

DEPOSITING CONCRETE UNDER WATER

163. **Methods.** In depositing concrete under water, some means must be taken to prevent the separation of the materials while passing through the water. The three principal methods are as follows:

(1) By means of closed buckets;
(2) By means of cloth or paper bags;
(3) By means of tubes.

164. **Buckets.** For depositing concrete by the first method, special buckets are made with a closed top and hinged bottom.

Concrete deposited under water must be disturbed as little as possible, and in tipping a bucket the material is apt to be disturbed. Several different types of buckets with hinged bottoms have been devised to open automatically when the place for depositing the concrete has been reached. In one type, the latches which fasten the trap doors are released by the slackening of the rope when the bucket reaches the bottom, and the doors are open as soon as the bucket begins to ascend. In another type, in which the handle extends down the sides of the bucket to the bottom, the doors are opened by the handles sliding down when the bucket reaches the bottom. The doors are hinged to the sides of the bucket, and when opened permit the concrete to be deposited in one mass. In depositing concrete by this means, it is found rather difficult to place the layers uniformly and to prevent the formation of mounds.

165. **Bags.** This method of depositing concrete under water is by means of open, woven bags or paper bags, two-thirds to three-quarters filled. The bags are sunk in the water and placed in courses, if possible, header and stretcher system, arranging each course as laid. The bagging is close enough to keep the cement from washing out, and at the same time, open enough to allow the whole to unite into a compact mass. The fact that the bags are crushed into irregular shapes which fit into each other, tends to lock them together in a way which makes even an imperfect joint very effective. When the concrete is deposited in paper bags, the water quickly soaks the paper; but the paper retains its strength long enough so that the concrete can be deposited properly.

166. **Tubes.** The third method of depositing concrete under water is by means of long tubes, 4 to 14 inches in diameter. The tubes extend from the surface of the water to the place where the concrete is to be deposited. If the tube is small, 4 to 6 inches in diameter, a cap is placed over the bottom, the tube filled with concrete, and lowered to the bottom. The cap is then withdrawn; and as fast as the concrete drops out of the bottom, more concrete is put in at the top of the tube, and there is thus a continuous stream of concrete deposited.

When a large tube is used to deposit concrete in this manner, it will be too heavy to handle conveniently if filled before being lowered. The foot of the tube is lowered to the bottom, and the water rises into the chute to the same level as that outside; and into this water the concrete must be dumped until the water is wholly replaced or absorbed by the concrete. This has a tendency to separate the cement from the sand and gravel, and will take a yard or more concrete to displace the water in the chute. There is a danger that this amount of badly washed concrete will be deposited whenever it is necessary to charge the chute. This danger occurs not only when the charge is accidentally lost, but whenever the work is begun in the morning or at any other time. Whenever the work is stopped, the charge must be allowed to run out, or it would set in the tube. The tubes are usually charged by means of wheelbarrows, and a continuous flow of concrete must be maintained. When the chute has been filled, it is raised slowly from the bottom, allowing a part of the concrete to run out in a conical heap at the foot.

This method has also been used for grouting stone. In this case, a 2-inch pipe, perforated at the bottom, was used. The grout, on account of its great specific gravity, is sufficient to replace the water in the interstices between the stones, and firmly cement them into a mass of concrete. A mixture of one part cement and one part sand is the leanest mixture than can be used for this purpose, as there is a great tendency for the cement and sand to separate.

CLAY PUDDLE

Clay puddle consists of clay and sand made into a plastic mass with water. It is used principally to fill cofferdams, and for making embankments and reservoirs water-tight.

167. **Quality of Clay.** Opaque clays with a dull earthy fracture, of an argillaceous nature, which are greasy to the touch, and which readily form a plastic paste when mixed with water, are the best clays for making puddle. Large stones should be removed from the clay, and it should also be free from vegetable matter. Sufficient sand and water should be added to make a homogeneous mass. If too much sand is used, the puddle will be permeable; and if too little is used, the puddle will crack by shrinkage in drying. It is very important that clay for making puddle should show great cohesive power and also the property of retaining water.

A simple test to find the cohesive property, can easily be made. A small quantity of the clay is mixed with water and made into a roll about 1 inch in diameter and 8 to 10 inches long; and if, on being suspended by one end while wet, it does not break, the cohesive strength is ample. The test to find its water-retaining properties is made by mixing up 1 or 2 cubic yards of the clay with water, making it into a homogeneous plastic mass. A round hole is made in the top of the mass, large enough to hold 4 or 5 gallons of water. The hole is filled with water, and the top covered and left 24 hours; when the cover is removed, the properties of the clay will be indicated by the presence or absence of water.

168. **Puddling.** The clay should be spread in layers about 3 inches thick and well chopped with spades, aided by the addition of sufficient water to reduce it to a pasty condition. Water should be given a chance to pass through freely as the clay is being mixed. The different layers, as they are mixed, should be bonded together by the spade passing through the upper layer into the under layer. The test for thorough puddling is that the spade will pass through the layer with ease, which it will not do if there are any hard lumps.

When a large amount of puddle is required, harrows are sometimes used instead of spades. Each layer of clay is thoroughly harrowed, aided by being sprinkled freely with water, and is then rolled with a grooved roller to compact it.

Puddle, when finished, should not be exposed to the drying action of the air, but covered with dry clay or sand.

FOUNDATIONS

169. It would be impossible to over-emphasize the importance of foundations, because the very fact that the foundations are underground and out of sight detracts from the consideration that many will give to the subject. It is probably true that a yielding of the subsoil is responsible for a very large proportion of the structural failures which have occurred. It is also true that many failures of masonry, especially those of arches, are considered as failures of the superstructure, because of breaks occurring in the masonry of the superstructure, which have really been due, however, to a settlement of the foundations, resulting in unexpected stresses in the superstructure. It is also true that the design of foundations is one which calls for the exercise of experience and broad judgment, to be able to interpret correctly such indications as are obtainable as to the real

character of the subsoil and its probable resistance to concentrated pressure.

170. **Classification of Subsoil.** The character of soil on which it may be desired to place a structure, varies all the way from the most solid rock to that of semi-fluid soils whose density is but little greater than that of water. The gradation between these extremes is so uniform that it is practically impossible to draw a definite line between any two grades. It is convenient, however, to group subsoils into three classes, the classification being based on the method used in making the foundation. These three classes of subsoils are: (*a*) *Firm*; (*b*) *Compressible*; and (*c*) *Semi-fluid*.

(*a*) *Firm Subsoils*. These comprise all soils which are so firm, at least at some reasonably convenient depth, that no treatment of the subsoil or any other special method needs to be adopted to obtain a sufficiently firm foundation. This, of course, practically means that the soil is so firm that it can safely withstand the desired unit-pressure. It also means that a soil which might be classed as firm soil for a light building should be classed as compressible soil for a much heavier building. It frequently happens that the top layers must be removed from rock because the surface rock has become disintegrated by exposure to the atmosphere. Nothing further needs to be done to a subsoil of this kind.

(*b*) *Compressible Subsoils*. These include soils which might be considered as firm soils for light buildings such as dwelling-houses, but which could not withstand the concentrated pressure that would be produced, for example, by the piers or abutments of a bridge. Such soils may be made sufficiently firm by methods described later.

(*c*) *Semi-Fluid Subsoils*. These are soils such as are frequently found on the banks or in the beds of rivers, which are so soft that they cannot sustain without settlement even the load of a house, to say nothing of a heavier structure. Nor can they be materially improved by any reasonable method of compression. The only possible method of placing a heavy structure in such a locality, consists in sinking some sort of a foundation through such soft soil until it reaches and is supported by a firm soil or by rock, which may be 50 or even 100 feet below the surface. The general methods of accomplishing these results will be detailed in the following sections.

171. **Testing the Bearing Power.** The first step is to excavate the surface soil to the depth at which it would be convenient to place the foundation and at which the soil appears, from mere inspection, to be sufficiently firm for the purpose. An examination of the trenches or foundation pits with a post-auger or steel bar will generally be sufficient to determine the nature of the soil for any ordinary building. The depth to which such an examination can be made with a post-auger or steel bar will depend on the nature of the soil. In ordinary soils there will not be much difficulty in extending such an examination 3 to 6 feet below the bottom of the foundation pits. In common soils or clay, borings 40 feet deep (or even deeper) can readily be made with a common wood-auger, turned by men. From the samples brought up by the auger, the nature of the soil can be determined; but nothing of the compactness of the soil can be determined in this manner.

In order to test a soil to find its compressive value, the bottom of the pit should be leveled for a considerable area, and stakes should be driven at short intervals in each direction. The elevations of the tops of all the stakes should be very accurately taken with a spirit level. For convenience, all stakes should be driven to the same level. A mast whose base has an area one foot square can support a platform which may be loaded with several tons of building material, such as stone, brick, steel, etc. This load can be balanced with sufficient closeness so that some very light guys will maintain the unstable equilibrium of the platform. As the load on the platform is greatly increased, at some stage it will be noted that the mast and platform have begun to sink slightly, and also that the soil in a circle around the base of the mast has begun to rise. This is indicated by the rising of the tops of the stakes. Even a very ordinary soil may require a load of five or six tons on a square foot before any yielding will be observable. One advantage of this method lies in the fact that the larger the area of the foundation, the *greater* will be the load *per square foot* which may be safely carried, and that the uncertainty of the result is on the *safe* side. A soil which might yield under a load concentrated on a mast one foot square, would probably be safe under that same unit-load on a continuous footing which was perhaps three feet wide; and if, in addition, a factor of safety of three or four was used, there would probably be no question as to the safety. Such a test need be applied only to an earthy soil. It would be practically impossible to produce a yielding by such a method on any kind of rock or even on a compacted gravel.

172. **Bearing Power of Ordinary Soils.** A distinction must be maintained between the crushing strength of a cube of rock or soil, and the bearing power of that soil when it lies as a mass of indefinite extent under some structure. A soil can fail only by being actually displaced by the load above it, or because it has been undermined, perhaps by a stream of water. A sample of rock which might crush with comparative ease when tested as a six-inch cube in a testing machine, will probably withstand as great a concentration of load as it is practicable to put upon it by any engineering structure. Even a gravel which would have absolutely no strength if it were attempted to place a cube of it in a testing machine, will be practically immovable when lying in a pit where it is confined laterally in all directions.

173. *Rock.* The ultimate crushing strength of stone varies greatly. The crushing strength is usually determined by making tests on small cubes. Tests made on prisms of a less height than width show a much greater strength than tests made on cubes of the same material, which shows that the bearing strength of rock on which foundations are built is much greater than the cubes of this stone. In Table I, Part I, the lowest value given for the crushing strength of a cube is 2,894 pounds per square inch, which is equal to 416,736 pounds per square foot. This shows that any ordinary stone which is well imbedded will carry any load of masonry placed on it.

174. *Sand and Gravel.* Sand and gravel are often found together. Gravel alone, when of sufficient thickness, makes one of the firmest and best foundations. Dry sand or wet sand, when prevented from spreading laterally, forms one of the best beds for foundations; but it must be well protected from running water, as it is easily moved by scouring. Clean, dry sand will safely support a load of 3,000 to 8,000 pounds per square foot; and when compact and well cemented, from 8,000 to 10,000 pounds per square foot. Ordinary gravel well bedded will safely bear a load of 6,000 to 8,000 pounds per square foot; and when well cemented, from 12,000 to 16,000 pounds per square foot.

175. *Clay.* There is great variation in clay soils, ranging from a very soft mass which will squeeze out in all directions when a very small pressure is applied, to shale or slate which will support a very heavy load. As the bearing capacity of ordinary clay is largely dependent upon its dryness, it is therefore very important that a clay soil should be well drained, and that a foundation laid on such a soil should be at a sufficient depth to be unaffected by the weather. If the clay cannot be easily drained, means should be taken to prevent the penetration of water. When the strata are not horizontal, great care must be taken to prevent the flow of the soil under pressure. When gravel or coarse sand is mixed with the clay, the bearing capacity of the soil is greatly increased.

The bearing capacity of a soft clay is from 2,000 to 4,000 pounds per square foot; of a thick bed of medium dry clay, 4,000 to 8,000 pounds per square foot, and for a thick bed of dry clay, 8,000 to 10,000 pounds per square foot.

176. *Soft or Semi-Liquid Soils.* The soils of this class include mud, silt, quicksand, etc., and it is necessary to remove them entirely or to reach a more solid stratum under the softer soil; or sometimes the soil can be consolidated by adding sand, stone, etc. The manner of improving such a soil will be discussed later. In the same way that water will bear up a boat, a semi-liquid soil will support, by the upward pressure, a heavy structure. For a soil of this kind, it is very difficult to give a safe bearing value; perhaps 500 to 1,500 pounds per square foot is as much as can be supported without too great a settlement.

177. **Improving a Compressible Soil.** The general method of doing this consists in making the soil more dense. This may be done by driving a large number of piles into the soil, especially if the piles will be always under the water line in that ground. Driving the piles compresses the soil; and if the piles are always under water, they will be free from decay. If the soil is sufficiently firm so that the pile can be withdrawn and the hole will retain its form even temporarily, it is better to draw the pile and then immediately fill the hole with sand, which is rammed into the hole as compactly as possible. This gives us a type of piling known as *sand piles.*

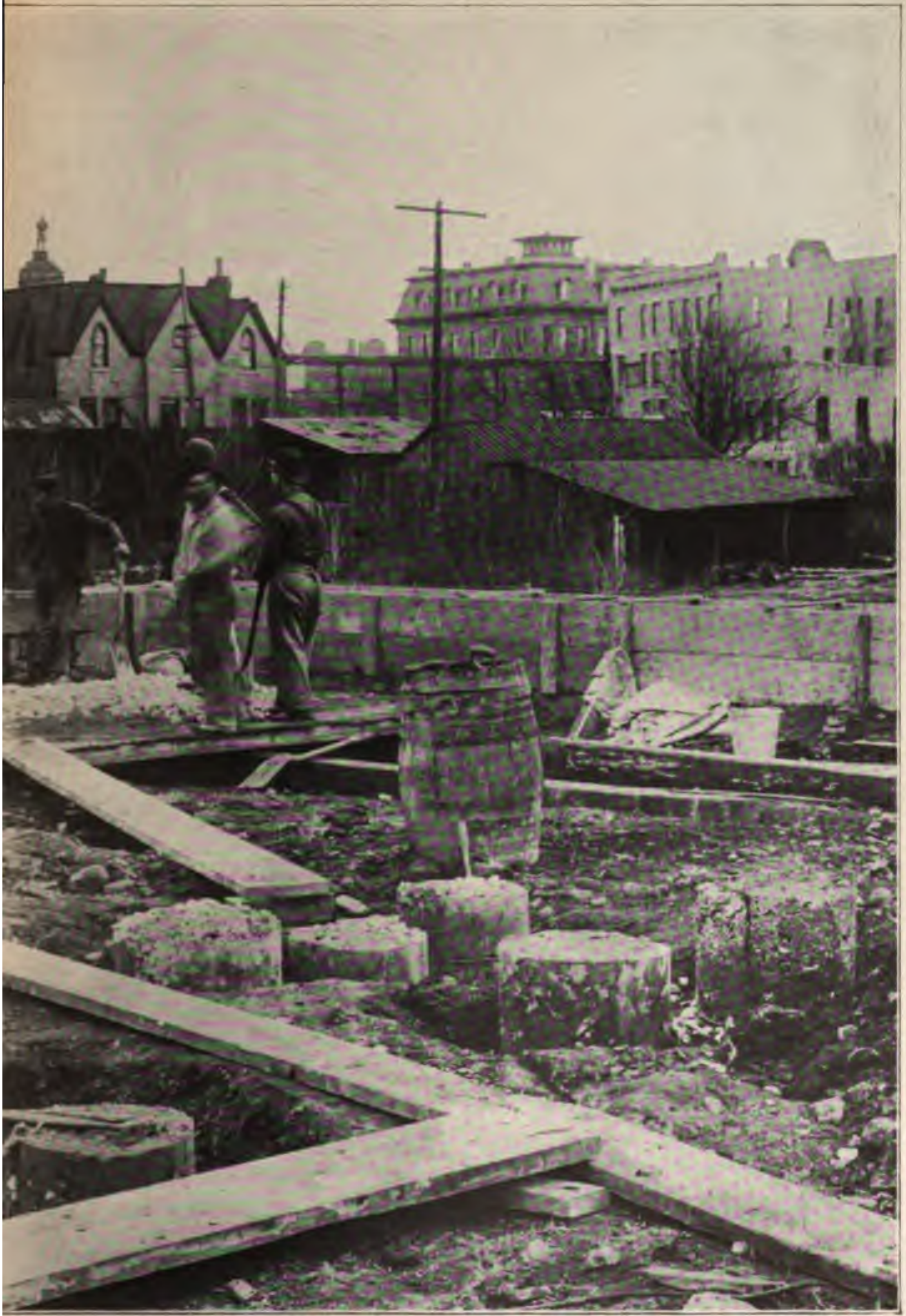
A soft, clayey subsoil may frequently be improved by covering it with gravel, which is rammed and pressed into the clay. Such a device is not very effective, but it may sometimes be sufficiently effective for its purpose.

A subsoil is often very soft because it is saturated with water which cannot readily escape. Frequently a system of deep drainage which will reduce the natural level of the ground-water considerably below the desired depth of the bottom of the foundation, will transform the subsoil into a dry, firm soil which is amply strong for its purpose. Even when the subsoil is very soft, it will sustain a heavy load, provided that it can be confined. While excavating for the foundations of the tower of Trinity Church in New York City, a large pocket of quicksand was discovered directly under the proposed tower. Owing to the volume of the quicksand, it was found to be impracticable to drain it all out; but it was also discovered that the quicksand was confined within a pocket of firm soil. A thick layer of concrete was then laid across the top, which effectively sealed up the pocket of quicksand, and the result has been perfectly satisfactory.

178. **Preparing the Bed on Rock.** The preparation of a rock bed on which a foundation is to be placed, is a simple matter compared with that required for some soils on which foundations are placed. The bed-rock is prepared by cutting away the loose and decayed portions of the rock and making the plane on which the foundation is placed perpendicular to the direction of the pressure. If the rock bed is an inclined plane, a series of steps can be made for the support of the foundation. Any fissures in the rock should be filled with concrete.

Whenever it is necessary to start the foundation of a structure at different levels, great care is required to prevent a break in the joints at the stepping places. The precautions to be taken are that the mortar-joints must be kept as thin as possible; the lower part of the foundations should be laid in cement mortar; and the work should proceed slowly. By following these precautions, the settlement in the lower part will be reduced to a minimum. These precautions apply to foundations of all classes.

179. **Preparing the Bed on Firm Earth.** Under this heading is included hard clay, gravel, and clean, dry sand. The bed is prepared by digging a trench deep enough so that the bottom of the foundation is below the frost line, which is usually 3 to 6 feet below the surface.



COMPLETED CONCRETE PILES

The five piles in the center are to serve as the foundation for a chimney. A comparison with the barrel shown in the picture will give an accurate idea of the size of the piles.

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TILDEN FOUNDATIONS

Some provision, similar to that shown in Fig. 38, should be made for drainage.

Care should be taken to proportion the load per unit of area so that the settlement of the foundation will be uniform.

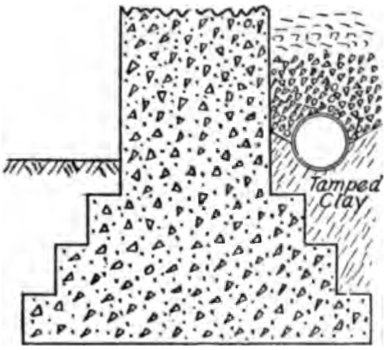

Fig. 38. Drainage of Foundation Wall.

180. Preparing the Bed on Wet Ground. The chief trouble in making an excavation in wet ground, is in disposing of the water and preventing the wet soil from flowing into the excavation. In moderately wet soils, the area to be excavated is enclosed with sheet piling (see Fig. 39). This piling usually consists of ordinary plank 2 inches thick and 6 to 10 inches wide, and is often driven by hand; or it may be driven by methods that will be described later. The piling is driven in close contact, and in very wet soil it is necessary to drive a double row of the sheeting. To prevent the sheeting from being forced inwards, cross-braces are used between the longitudinal timbers. When one length of sheeting is not long enough, an additional length can be placed inside. A more extended discussion of pile-driving will be given in the treatment of the subject of "Piles."

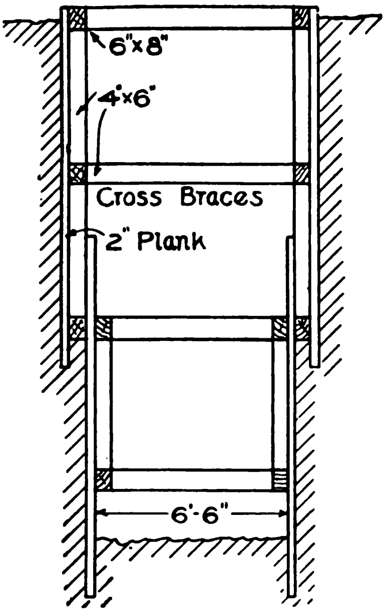

Fig. 39. Sheet Piling in Foundation Trenches.

The water can sometimes be bailed out, but it is generally necessary to use a hand or steam pump to free the excavation of water. Quicksand and very soft mud are often pumped out along with the water by a centrifugal or mud pump.

Sometimes areas are excavated by draining the water into a hole the bottom of which is always kept lower than the general level of bottom of the excavation. A pump may be used to dispose of the water drained into the hole or holes.

When a very soft soil extends to a depth of several feet, piles are usually driven at uniform distances over the area, and a grillage is constructed on top of the piles. This method of constructing a foundation is discussed in the section on "Piles."

181. **Footings.** The three requirements of a footing course are:

(1) That the area shall be such that the total load divided by the area shall not be greater than the allowable unit-pressure on the subsoil.

(2) That the line of pressure of the wall (or pier) shall be directly over the center of gravity (and hence the center of upward pressure) of the base of the footings.

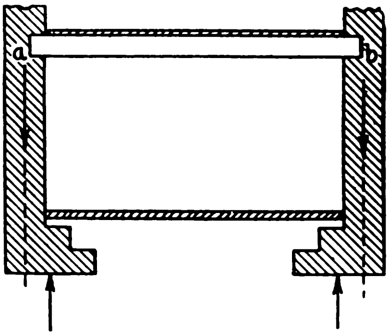

Fig. 40. Construction where Lines of Downward and Upward Pressure on Footings do not Coincide.

(3) That the footing shall have sufficient structural strength so that it can distribute the load uniformly over the subsoil.

When it has been determined with sufficient accuracy how much pressure per square foot may be allowed on the subsoil (see sections 172–176), and when the total load of the structure has been computed, it is a very simple matter to compute the *width* of continuous footings or the *area* of column footings.

The second requirement is very easily fulfilled when it is possible to spread the footings in all directions as desired, as shown in Fig. 43. A common exception occurs when putting up a building which entirely covers the width of the lot. The walls are on the building line; the footings can expand inward only. The lines of pressure do not coincide, as shown in Fig. 40. A construction as shown in the figure will almost inevitably result in cracks in the building, unless some special device is adopted to prevent them. One general method is to introduce a tie of sufficient strength from *a* to *b*. The other general method is to introduce cantilever beams under the basement, which either extend clear across the building or else carry the load of interior columns so that the center of gravity of the combined loads will coincide with the central pressure line of the upward pressure of the footings.

The third requirement practically means that the thickness of the footing (*bc*, Fig. 41) shall be great enough so that the footing can resist the transverse stresses caused by the pressure of the subsoil on the area between *c* and *d*. When the thickness must be made very great (such as *fh*, Fig. 42), on account of the wide offset *gh*, material may be saved by cutting out the rectangle *e k m l*. The thickness *m o* is computed for the offset *g o*, just as in the first case; while the thickness *k m* of the second layer may be computed from the offset *k f*. Where the footings are made of stone or of plain concrete, whose transverse strength is always low, the offsets are necessarily small; but when using timber, reinforced concrete, or steel I-beams, the offsets may be very wide in comparison with the depth of the footing.

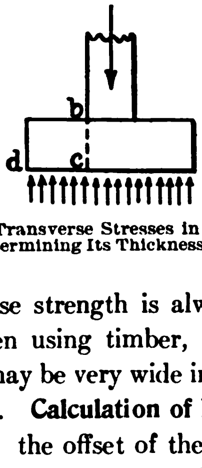

Fig. 41. Transverse Stresses in Footing Determining Its Thickness.

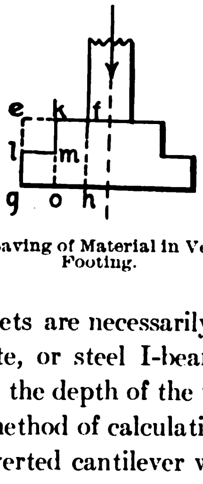

Fig. 42. Saving of Material in Very Thick Footing.

182. **Calculation of Footings.** The method of calculation is to consider the offset of the footing as an inverted cantilever which is loaded with the calculated *upward* pressure of the subsoil against the footing. If Fig. 41 is turned upside down, the resemblance to the ordinary loaded cantilever will be more readily apparent. Considering a unit-length (l) of the wall and the amount of the offset o ($= dc$ in Fig. 41), and calling P the unit-pressure from the subsoil, we have $P o l$ as the pressure on that area, and its lever-arm about the point c is $\frac{1}{2} o$. Therefore its moment $= \frac{1}{2} P o^2 l$. If t represents the thickness $b c$ of the footing, the moment of resistance of that section $= \frac{1}{6} R l t^2$, in which $R =$ the unit-compression (or unit-tension) in the section. We therefore have the equation:

$$\tfrac{1}{2} P o^2 l = \tfrac{1}{6} R l t^2.$$

By transposition,

$$\frac{o^2}{t^2} = \frac{R}{3P}\ ;\ \text{or,}\ \frac{o}{t} = \sqrt{\frac{R}{3P}} \ldots\ldots\ldots\ldots (2)$$

The fraction $\frac{o}{t}$ is the ratio of the offset to its thickness. The solution of the above equation, using what are considered to be conservatively safe values for R for various grades of stone and concrete, is given in Table XII.

TABLE XII

Ratio of Offset to Thickness for Footings of Various Kinds of Masonry

KIND OF MASONRY	MODULUS OF RUPTURE (Minimum and Maximum Values)	AVERAGE	ASSUMED SAFE VALUE (R)	PRESSURE ON BOTTOM OF FOOTING (Tons per Square Foot)						
				0.5	1.0	1.5	2.0	2.5	3.0	3.5
Granite	1,200–1,365	1,280	130	2.5	1.8	1.45	1.25	1.1	1.0	0.95
Limestone	450– 900	675	70	1.8	1.3	1.05	0.9	0.8	0.75	0.7
Sandstone	135–1,100	525	55	1.6	1.15	0.95	0.8	0.75	0.65	0.6
Concrete (plain)										
1:2:4	400– 480	440	75	1.9	1.35	1.1	0.95	0.85	0.75	0.7
1:3:6	213– 246	230	40	1.4	1.0	0.8	0.7	0.6	0.55	0.5

183. *Example* The load on a wall has been computed as 19,000 pounds per running foot of the wall, which has a thickness of 18 inches just above the footing. What must be the breadth and thickness of granite slabs which may be used as a footing on soil which is estimated to bear safely a load of 2.0 tons per square foot?

Solution. Dividing the computed load (19,000) by the allowable unit-pressure (2.0 tons = 4,000 pounds), we have 4.75 feet as the required width of the footing.

$\frac{1}{2}$ (4.75 − 1.5) = 1 625 feet, the breadth of the offset (*b*).

From the table we find that for a subsoil loading of 2.0 tons per square foot, the offset for granite may be 1.25 times its thickness. Therefore, $\frac{1.625}{1.25} = 1.30$ feet = 15.6 inches, is the required thickness of the footing.

The computation of the dimensions of such footings when they are made of reinforced concrete is taken up during the development of this specialized form of Masonry in Part III.

Although brick *can* be used as a footing course, the maximum possible offset (no matter how strong the brick may be) can only be a small part of the length of the brick—the brick being laid perpendicular to the wall. One requirement of a footing course is that the blocks shall be so large that they will equalize possible variations in the density and compressibility of the subsoil. This cannot be done by bricks or small stones. Their use should therefore be avoided in footing courses.

184. **Beam Footings.** Steel, and even wood, in the form of beams, are used to construct very wide offsets. This is possible on account of their greater transverse strength. The general method of calculation is identical with that given above, the only difference being that beams of definite transverse strength are so spaced that one beam can safely resist the moment developed in the footing in that length of wall. Wood can be used only when it will be always under water. Steel beams should always be surrounded by concrete for protection from corrosion.

If we call the spacing of the beams s, the length of the offset o, the unit-pressure from the subsoil P, the moment acting on one beam $= \frac{1}{2} P o^2 s$. Calling w the width of the beam, t its thickness or depth, and R the maximum permissible fibre stress, the maximum permissible moment $= \frac{1}{6} R w t^2$. Placing these quantities equal, we have the equation:

$$\tfrac{1}{2} P o^2 s = \tfrac{1}{6} R w t^2 \dots\dots\dots\dots\dots\dots (3)$$

Having decided on the size of the beam, the required spacing may be determined.

185. *Example.* An 18-inch brick wall carrying a load of 12,000 pounds per running foot, is to be placed on a soft, wet soil where the unit-pressure cannot be relied on for more than one-half a ton per square foot. What must be the spacing of 10 by 12-inch footing timbers of long-leaf yellow pine?

Solution. The width of the footing is evidently 12,000 ÷ 1,000 = 12 feet. The offset o equals $\frac{1}{2}$ (12 − 1.5) = 5.25 feet = 63 inches. Since the unit of measurement for computing the transverse strength is the inch, the same unit must be employed throughout. Therefore $P = \frac{1,000}{144}$; R = 1,200 pounds per square inch; w = 10 inches; and t = 12 inches. Equation (3) may be rewritten:

$$s = \frac{R w t^2}{3 P o^2}.$$

Substituting the above values, we have:

$$s = \frac{1{,}200 \times 10 \times 144}{3 \times \frac{1{,}000}{144} \times 3{,}969} = 20.9 \text{ inches.}$$

This shows that the beams must be spaced 20.9 inches apart, center to center, or with a clear space between them but little more than their width. Under the above conditions, the plan would probably be inadvisable, unless timber were abnormally cheap and no other method seemed practicable.

186. **Steel I-Beam Footings.** The method of calculation is the same as for wooden beams, except that, since the strength of I-beams is not readily computable except by reference to tables in the handbooks published by the manufacturers, such tables will be utilized. The tables always give the *safe* load which may be carried on an I-beam of given dimensions on any one of a series of spans varying by single feet. If we call W the total load (or upward pressure) to be resisted by a single cantilever beam, this will be *one-fourth* of the load which can safely be carried by a beam of the same size and on a span equal to the offset.

187. *Example.* Solve the previous example on the basis of using steel I-beams.

The offset is necessarily 5 feet 3 inches; at 1,000 pounds per square foot, the pressure to be carried by the beams is 5,250 pounds for each foot of length of the wall. By reference to the tables and interpolating, an 8-inch I-beam weighing 17.75 pounds per linear foot will carry about 28,880 pounds on a 5 foot 3 inch span. One-fourth of this (or 7,220 pounds) is the load carried by a cantilever of that length. Therefore, 7,220 ÷ 5,250 = 1.375 feet = 16.5 inches, is the required spacing of such beams. When comparing the cost of this method with the cost of others, the cost of the masonry concrete filling must not be overlooked.

188. **Design of Pier Footings.** The above designs for footings have been confined solely to the simplest case of the footing required for a continuous wall. A column or pier must be supported by a footing which is offset from the column in all four directions. It is usually made square. The area is very readily obtained by dividing the total load by the allowable pressure per square foot on the soil. The quotient is the required number of square feet in the area of the

footing. If a square footing is permissible (and it is usually preferable), the square root of that number gives the length of one side of the footing. Special circumstances frequently require a rectangular footing or even one of special shape. The problem of designing a footing so that the center of pressure of the load on a column shall be vertical over the center of pressure of the subsoil, is usually even more complicated than the problem referred to in section 189. A column placed at the corner of a building which is located at the extreme corner of the property and which cannot extend over the property line, must usually be supported by a cantilever (or by two of them at right angles), balanced at the other end by the load on another pier or column. While the general principle involved in such methods of construction is very simple, a correct solution often requires the exercise of considerable ingenuity.

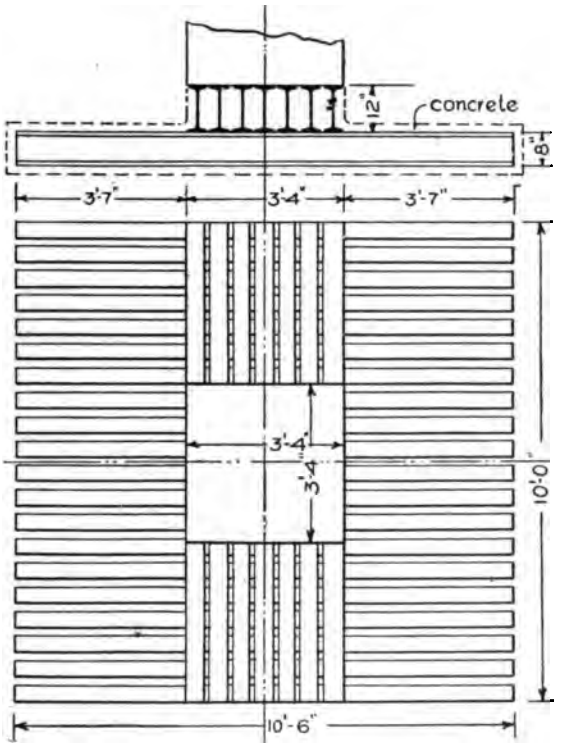

Fig. 43. Grillage of I-Beams.

The determination of the thickness of such a footing depends somewhat upon the method used. When the grillage is constructed of I-beams as illustrated in Fig. 43, the required strength of each series of beams is readily computed from the offset of each layer. If the footing consists of a single block of stone or a plate of concrete, either plain or reinforced, the thickness must be computed on the basis of the mechanics of a plate loaded on one side with a uniformly distributed load and on the other side with a load which is practically concentrated in the center. The theory of the stresses in such a plate is very complicated. It is usually considered safe to design the footing in *each* direction on the basis of one-half the actual loading.

189. *Example.* **A column 3 feet 4 inches square, carrying a total load of 630,000 pounds, is to be supported on a soil on which the permissible loading is estimated as three tons per square foot; an I-beam footing is to be used. Required, the design of the I-beams.**

Solution. The required area of the footing is evidently 630,000 ÷ 6,000 = 105 square feet. Using a footing similar to that illustrated in Fig. 43, we shall make the lower layer of the footing, say 10 feet 6 inches by 10 feet wide. The length of the beams being 126 inches, and the column being 40 inches square, the offset from the column is 43 inches (= 3.58 feet) on each side. Looking at a table of standard I-beams, we find that an 8-inch beam weighing 17.75 pounds per linear foot will carry 37,920 pounds on a span of four feet. For a span of 3.58 feet, the allowable load is $\frac{4.00}{3.58} \times 37{,}920$, or 42,368 pounds. Taking one-fourth of this, as in the example in section 187, we have 10,592 pounds which may be carried by each beam acting as a cantilever. The upward pressure on an offset 3.58 feet long and 1 foot wide, at the rate of 6,000 pounds per square foot, would be 21,500 pounds. Therefore, if two 8-inch beams were placed in each foot of width, they would carry the pressure. Therefore 20 such beams, each 10 feet 6 inches long, would be required in the lower layer. The upper layer must consist of beams 10 feet long on which the offset from the pier is 40 inches on each side. The group of beams under each of these upper offsets carries an upward pressure of 6,000 pounds per square foot on an area 10 feet 6 inches by 3 feet 4 inches; total pressure, 210,000 pounds. The total load on each foot of width of the upper layer is 63,000 pounds. Looking at the tables, a 12-inch I-beam weighing 40 pounds per foot can carry a load, on a 10-foot span, of 43,720 pounds. The permissible load on a cantilever of this length would be one-fourth of this, or 10,930 pounds. The permissible load on a cantilever 3 feet 4 inches long will be in the ratio of 10 feet to 3 feet 4 inches, or, in this case, exactly three times as much, which would be 32,790 pounds. If, therefore, such beams are placed 6 inches apart, their strength would be slightly in excess of that required. Or, as a numerical check, the total of 210,000 pounds, divided by 32,790, will show that although seven such beams will have a somewhat excessive strength, six would be hardly sufficient; therefore seven beams should be used. It should not be forgotten that surrounding all these beams in both layers with concrete adds very largely to the strength of the whole footing, but that no allowance is made for this additional strength in computing dimensions. It merely adds an indefinite amount to the factor of safety.

PILE FOUNDATIONS

190. **Piles.** The term *pile* is generally understood to be a stick of timber driven in the ground to support a structure. This stick of timber is generally thought of as the body of a small tree; but timber in many shapes is used for piling. Sheet piling, for example, is generally much wider than thick. Cast iron and wrought iron have also been used for all forms of piling. Structural steel has also been used for this purpose. Within the last few years, concrete and reinforced concrete piles have been used quite extensively in place of wood piles.

191. **Cast-Iron Piles.** Cast-iron piles have been used to some extent. The advantages claimed for these piles are that they are not subject to decay; they are more readily driven than wooden piles in stiff clays or stony ground; and they have a greater crushing strength than wooden piles. The latter quality will apply only when the pile acts as a column. The greatest objection to these piles is that they are deficient in transverse strength to resist sudden blows. This objection applies only in handling them before they are driven, and to those which, after being driven, are exposed to blows from ice and logs, etc. When driving cast-iron piles, a block of wood is placed on top of the pile to receive the blow; and, after being driven, a cap with a socket in its lower side is placed upon the pile to receive the load. Generally lugs or flanges are cast on the sides of the piles, to which bracing may be attached for fastening them in place.

192. **Steel Sections.** Structural steel sections, as well as many special sections, are being used for piling. This form of piling is generally used for dams, cofferdams, or locks, and seldom or never

used as bearing piles. Fig. 44 illustrates some of these sections of piling.

193. **Screw Piles.** This term refers to a type of metal pile whose use is limited, but which is apparently very effective where it has been used. It consists essentially of a steel shaft, 3 to 8 inches in diameter, strong enough to act as a column, and also to withstand the twisting to which it is subjected while the pile is being sunk (see Fig. 45). At the lower end of the shaft is a helicoidal surface having a diameter of perhaps five feet. Such piles can be used only in comparatively soft soil, and their use is practically confined to foundations in sandbanks on the shore of the ocean. To sink such piles, they are screwed into place by turning the vertical shaft with a long lever. Such a sinking is usually assisted by a water-jet, as will be described later.

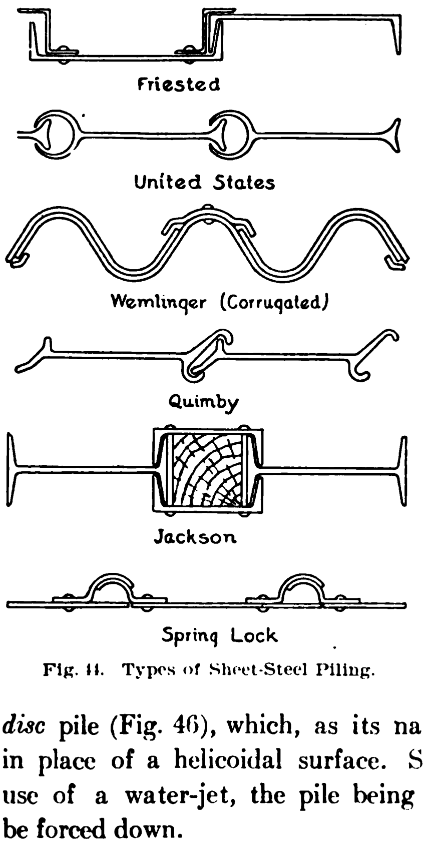

Fig. 44. Types of Sheet-Steel Piling.

194. **Disc Piles.** A variation of the screw pile is the *disc* pile (Fig. 46), which, as its name implies, has a circular disc in place of a helicoidal surface. Such a pile can be sunk only by use of a water-jet, the pile being heavily loaded so that it shall be forced down.

195. **Sheet Piles.** Ordinary planks, two or more inches thick, and wider than they are thick, are, when driven close together, known as *sheet piling*. The leakage between the piles may be very materially diminished by using piles which interlock with each other instead of making merely a butt joint. (See Fig. 47.) The simplest form is the ordinary tongue-and-groove joint similar to that of matched boarding. A development of this in timber sheet piling is a combination of three planks which are so bolted together as to make a large-

scale tongue and groove on each side. The increasing cost of timber, and the large percentage of deterioration and destruction during its use for a single cofferdam, have developed the manufacture of *steel sheet piling*, which can be redrawn and used many times. The forms of steel for sheet piling are nearly all patented. The cross-sections of a few of them are shown in Fig. 44. One feature of some of the designs is the possible flexibility secured in the outline of the dam without interfering with the water-tightness.

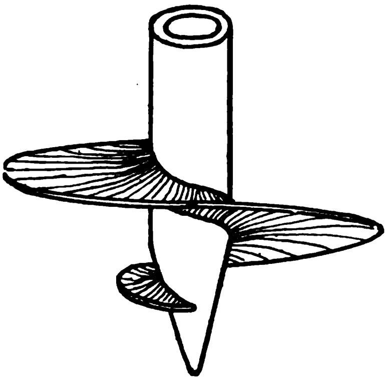

Fig. 45. Screw Pile.

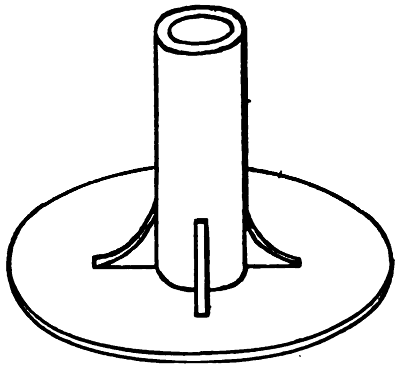

Fig. 46. Disc Pile.

Sheet piling is usually driven in close contact (as shown in Fig. 48), either to prevent leakage, or to confine puddle in cofferdams, to prevent the materials of a foundation from spreading, or to guard a foundation from being undermined by a stream of water. To make wooden piles drive with their parts close against each other, they are cut obliquely at the bottom, as shown in Fig. 49. They are kept in place while being driven, by means of two longitudinal stringers or

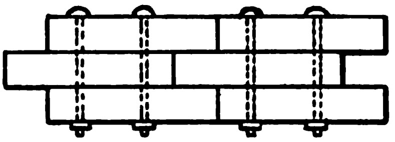

Fig. 47. Lapped Sheet-Piling.

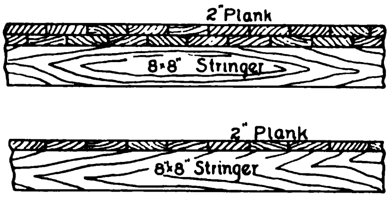

Fig. 48. Single and Sheet Piling Plan View.

wales. These wales are supported by gauge-piles previously driven, which are several feet apart. Sheet piling is seldom used as bearing piles.

196. **Wooden Bearing Piles.** Specifications for wooden piles generally require that they shall have a diameter of from 7 to 10 inches at the smaller end, and 12 to 15 inches at the larger end. Older specifications were quite rigid in insisting that the tree trunks should be straight, and that the piles should be free from various kinds of minor defects; but the growing scarcity of timber is modifying the rigidity of these specifications, provided the most essential qualifications of strength and durability are provided for. Timber piles should have the bark removed before being driven, unless the piles are to be always under water. They should be cut square at the driving end, and pointed at the lower end. When they are to be driven in hard, gravelly soil, it is often specified that they shall be shod with some form of iron shoe. This may be done by means of two straps of wrought iron, which are bent over the point so as to

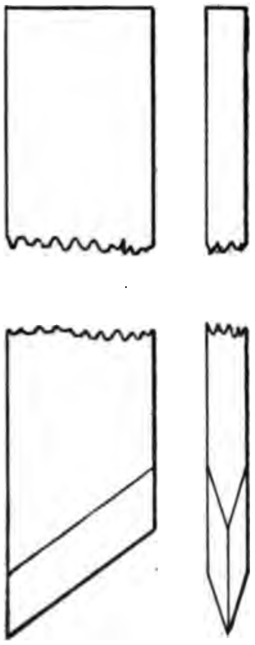

Fig. 49. Bevel Point for Sheet Pile.

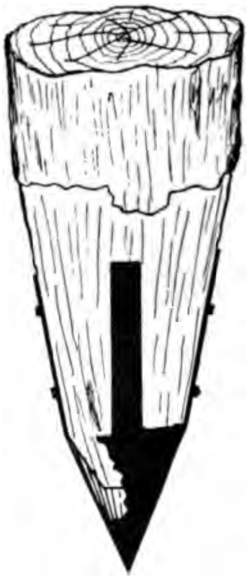

Fig. 50. Wrought-Iron Pile-Shoe.

form four bands radiating from the point of the pile (see Fig. 50). By means of holes through them, these bands are spiked to the piles. Another method, although it is considered less effective on account of its liability to be displaced during driving, is to use a cast-iron shoe. These shoes are illustrated in Fig. 51. It is sometimes specified that piles shall be driven with the butt end or larger end down, but there seems to be little if any justification for such a specification. The resistance to driving is considerably greater, while their ultimate bearing power is but little if any greater. If the driving of piles is considered from the standpoint of compacting the soil (as already discussed in section 177), then driving the piles with the small end down will compact the soil more effectively than driving them butt end down.

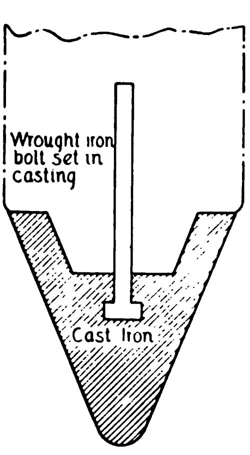

Fig. 51. Cast-Iron Pile-Shoe.

White pine, spruce, or even hemlock may be used in soft soils; yellow pine in firmer ones; and oak, elm, beech, etc., in the more compact soils. They are usually driven from $2\frac{1}{2}$ to 4 feet apart each way, center to center, depending on the character of the soil and the load to be supported. Timber piles, when partly above and partly under water, will decay very rapidly at the water line. This is owing to the alternation of dryness and wetness. In tidal waters, they are destroyed by the marine worm known as the *teredo*.

The American Railway Engineering & Maintenance of Way Association recommends the following specifications for piling:

> "Piles shall be cut from sound, live trees; shall be close-grained and solid; free from defects such as injurious ring shakes, large and unsound knots, decay, or other defects that will materially impair their strength. The taper from butt to top shall be uniform and free from short bends.
>
> "All piles except foundation piles shall be peeled."

197. Bearing Power of Piles. Pile foundations act in a variable combination of two methods of support. In one case the piles are driven into the soil to such a depth that the *frictional resistance* of the soil to further penetration of the pile is greater than any load which will be placed on the pile. As the soil becomes more and more soft, the frictional resistance furnished by the soil is less and less;

and it then becomes necessary that the pile shall penetrate to a strata of much greater density into which it will penetrate but little if any. Under such conditions, the structure rests on a series of columns (the piles) which are supported by the hard subsoil, and whose action as columns is very greatly assisted by the density of the very soft soil through which the piles have passed. It practically makes but little difference which of these methods of support exists in any particular case. The piles are driven until the resistance furnished by each pile is as much as is desired. The resistance against the sinking of a pile depends on such a very large variety of conditions, that attempts to develop a formula for that resistance based on a theoretical computation of all these various factors are practically useless. There are so many elements of the total resistance which are so large, and also so very uncertain, that they entirely overshadow the few elements which can be precisely calculated. Most formulæ for pile-driving are based on the general proposition that the resistance of the pile, multiplied by its motion during the last blow, equals the weight of the hammer multiplied by the distance through which it falls. To express this algebraically:

If R = Resistance of pile;
s = Penetration of pile during last blow;
w = Weight of hammer;
h = Height of fall of hammer;

then, according to the above principle, we have:

$$Rs = wh.$$

Practically, such a formula is considerably modified, owing to the fact that the resistance of a pile (or its penetration for any blow) depends considerably on the time which has elapsed since the previous blow. This practically means that it is far easier to drive the pile, provided the blows are delivered very rapidly; and also that when a pause is made in the driving for a few minutes or for an hour, the penetration is very much less (and the apparent resistance very much greater), on account of the earth settling around the pile during the interval. The most commonly used formula for pile-driving is known as the *Engineering News formula*, which, when used for ordinary hammer-driving, is as follows:

$$R = \frac{2wh}{s+1} \quad \dots\dots\dots\dots \quad (4)$$

This formula is fundamentally the same as the formula given above, except that,

R = *Safe* load, in *pounds;*
s = Penetration, in *inches,*
w = Weight of hammer, in *pounds;*
h = Height of fall of hammer, in *feet.*

In the above equation, s is considered a free-falling hammer (not retarded by hammer rope) striking a pile having a sound head. If a friction-clutch driver is used, so that the hammer is retarded by the rope attached to it, the penetration of the pile is commonly assumed to be just one-half what it would have been had no rope been attached (that is, had it been free-falling).

Also, the quantity s is arbitrarily increased by 1, to allow for the influence of the settling of the earth during ordinary hammer pile-driving, and a factor of safety of 6 for safe load has been used. In spite of the extreme simplicity of this formula compared with that of others which have attempted to allow for all possible modifying causes, this formula has been found to give very good results. When computing the bearing power of a pile, the penetration of the pile during the last blow is determined by averaging the total penetration during the last five blows.

The pile-driving specifications adopted by the American Railway Engineering & Maintenance of Way Association, require that,

> "All piles shall be driven to a firm bearing satisfactory to the Engineer, or until five blows of a hammer weighing 3,000 pounds, falling 15 feet (or a hammer and fall producing the same mechanical effect), are required to drive a pile one-half (½) inch per blow, except in soft bottom, when special instructions will be given."

This is equivalent to saying (applying the *Engineering News formula*) that the piles must have a bearing power of 60,000 pounds.

198. *Example* 1. The total penetration during the last five blows was 14 inches for a pile driven with a 3,000-pound hammer. During these blows the average drop of the hammer was 24 feet. How much is the safe load?

$$\frac{2\,w\,h}{s+1} = \frac{2 \times 3{,}000 \times 24}{(\frac{1}{5} \times 14) + 1} = \frac{144{,}000}{3.7} = 38{,}919 \text{ pounds.}$$

199. *Example* 2. It is required (if possible) to drive piles with a 3,000-pound hammer until the indicated resistance is 70,000 pounds. What should be the average penetration during the last five blows when the fall is 25 feet?

$$70,000 = \frac{2wh}{s+1} = \frac{2 \times 3,000 \times 25}{s+1} = \frac{150,000}{s+1}$$

$$s = \frac{150,000}{70,000} - 1 = 2.14 - 1 = 1.14 \text{ inches.}$$

200. The last problem suggests a possible impracticability, for it may readily happen that when the pile has been driven to its full length its indicated resistance is still far less than that desired. In some cases, such piles would merely be left as they are, and additional piles would be driven beside them, in the endeavor to obtain as much total resistance over the whole foundation as is desired.

The above formula applies only to the *drop-hammer* method of driving piles, in which a weight of 2,500 to 3,000 pounds is raised and dropped on the pile.

When the steam pile-driver is used, the blows are very rapid, about 55 to 65 per minute. On account of this rapidity the soil does not have time to settle between the successive blows, and the penetration of the pile is much more rapid, while of course the resistance after the driving is finished is just as great as is secured by any other method. On this account, the above formula is modified so that the arbitrary quantity added to s is changed from one to 0.1, and the formula becomes:

$$R = \frac{2wh}{s + 0.1} \quad \text{. . . . .} \quad (5)$$

201. **Methods of Driving Piles.** There are three general methods of driving piles, namely, by using (1) a falling weight; (2) the erosive action of a water-jet; or (3) the force of an explosive. The third method is not often employed, and will not be further discussed. In constructing foundations for small highway bridges, well augers have been used to bore holes, in which piles are set and the earth rammed around them.

202. *Drop-Hammer Pile-Driver.* This method of driving piles consists in raising a hammer made of cast iron, and weighing from 2,000 to 3,000 pounds, to a height of 5 to 30 feet, and then allowing it to fall freely on the head of the pile. The weight is hoisted by means of a hoisting engine or sometimes by horses. When an engine is used [illegible] sometimes merely [illegible] the weight [illegible] drags the rope and turns the hoisting drum as it falls. This reduces the effectiveness of the blow, and

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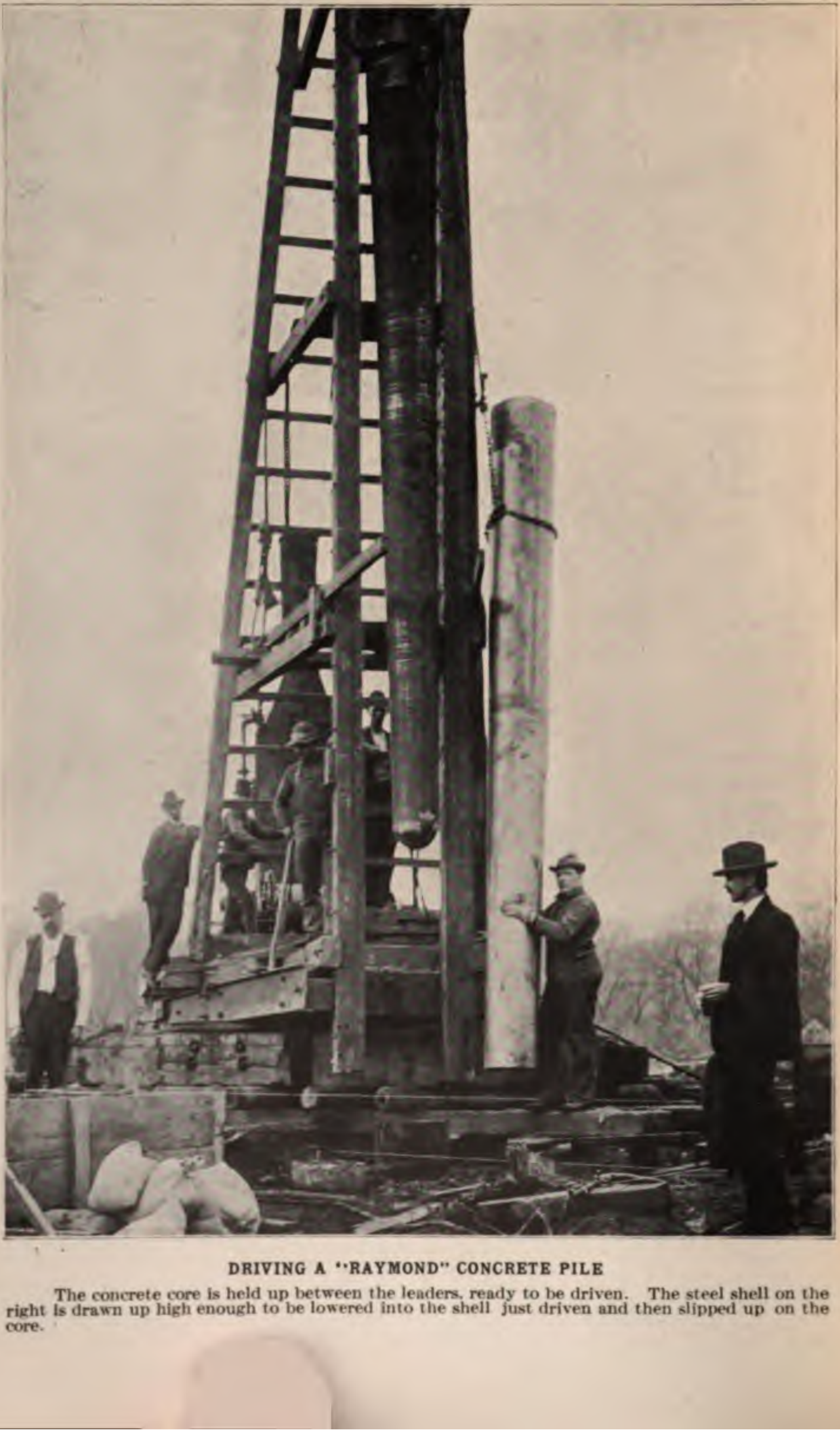


DRIVING A "RAYMOND" CONCRETE PILE

The concrete core is held up between the leaders, ready to be driven. The steel shell on the right is drawn up high enough to be lowered into the shell just driven and then slipped up on the core.

lowers the value of *s* in the formula given, as already mentioned. To guide the hammer in falling, a frame, consisting of two uprights called *leaders*, about 2 feet apart, is erected. The uprights are usually wooden beams, and are from 10 to 60 feet long. Such a simple method of pile-driving, however, has the disadvantage, not only that the blows are infrequent (not more than 20 or even 10 per minute), but also that the effectiveness of the blows is reduced on account of the settling of the earth around the piles between the successive blows. On this account, a form of pile-driver known as the *steam pile-driver* is much more effective and economical, even though the initial cost is considerably greater.

203. *Steam-Hammer Pile-Driver.* The steam pile-driver is essentially a hammer which is attached directly to a piston in a steam cylinder. The hammer weighs about 4,000 pounds, is raised by steam to the full height of the cylinder, which is about 40 inches, and is then allowed to fall freely. Although the height of fall is far less than that of the ordinary pile-driver, the weight of the hammer is about double, and the blows are very rapid (about 50 to 65 per minute). As before stated, on account of this rapidity, the soil does not have time to settle between blows, and the penetration of the pile is much more rapid, while, of course, the ultimate resistance after the driving is finished, is just as great as that secured by any other method.

204. *Driving Piles with Water-Jet.* When piles are driven in a situation where a sufficient supply of water is available, their resistance during driving may be very materially reduced by attaching a pipe to the side of the pile and forcing water through the pipe by means of a pump. The water returns to the surface along the sides of the pile and thus reduces its frictional resistance. The water also softens and scours out the soil immediately underneath the pile, and enables the pile to penetrate the soil much more easily. In very soft soils, piles may be thus driven by merely loading a comparatively small weight on top of the pile while the force pump is being operated; and yet the resistance shortly after stopping the pump will be found to be very great. Of course the only method of testing such resistance is by actually loading a considerable weight on the pile. This method of using a water-jet is chiefly applicable in structures which are on the banks of streams or large bodies of water. The water-jet

and the hammer are advantageously used together, especially in stiff clay.

205. **Splicing Piles.** On account of the comparatively slight resistance offered by piles in swampy places, it sometimes becomes necessary to *splice* two piles together. The splice is often made by cutting the ends of the piles perfectly square so as to make a good butt joint. A hole 2 inches in diameter and 12 inches deep is bored in each of the butting ends, and a dowel-pin 23 inches long is driven in the hole bored in the first pile; the second pile is then fitted on the first one. The sides of the piles are then flattened, and four 2 by 4-inch planks, 4 to 6 feet long, are securely spiked on the flattened sides of the piles. Such a joint is weak at its best, and the power of lateral resistance of a joint pile is less than would be expected from a single stick of equal length. Nevertheless, such an arrangement is in some cases the only solution.

206. **Pile Caps.** One practical trouble in driving piles, especially those made of soft wood, is that the end of the pile will become crushed or *broomed* by the action of the heavy hammer. Unless this crushed material is trimmed off the head of the pile, the effect of the hammer is largely lost in striking this cushioned head. This crushed portion of the top of a pile should always be cut off just before the test blows are made to determine the resistance of the pile, since the resistance of a pile indicated by blows upon it, if its end is broomed, will apparently be far greater than the actual resistance of the pile.

Another advantage of the steam pile-driver is that it does not produce such an amount of brooming as is caused by the ordinary pile-driver. Whenever the hammer bounces off the head of the pile, it shows either that the fall is too great or that the pile has already been driven to its limit. Whenever the pile refuses to penetrate appreciably for each blow, it is useless to drive it any further, since added blows can only have the effect of crushing the pile and rendering it useless. It has frequently been discovered that piles which have been hammered after they have been driven to their limit, have become broken and crushed, perhaps several feet underground. In such cases, their supporting power is very much reduced.

Usually about two inches of the head is chamfered off to prevent this bruising and splitting in driving the pile. A steel band 2 to 3 inches wide and ½ to 1 inch thick, is often hooped over the head of the pile to assist in keeping it from splitting. These devices have led to the use of a cast-iron cap for the protection of the head of the pile. The cap is made with two tapering recesses, one to fit on the chamfered head of the pile, and in the other is placed a piece of hardwood on which the hammer falls. The cap preserves the head of the pile.

207. **Sawing Off the Piles.** When the piles have been driven, they are sawed off to bring the top of them to the same elevation so that they will have an even bearing surface. When the tops of the piles are above water, this sawing is usually done by hand; and when under water, by machinery. The usual method of cutting piles off under water is by means of a circular saw on a vertical shaft which is supported on a special frame, the saw being operated by the engine used in driving the piles.

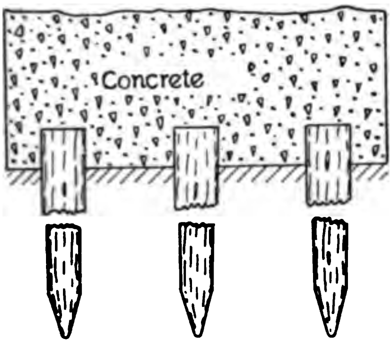


Fig. 52. Concrete Foundation on Wooden Piles.

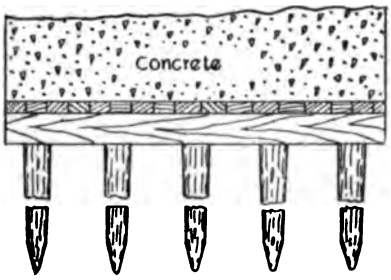


Fig. 53. Foundation on Wooden Piles.

208. **Finishing the Foundations.** When the heads of the piles are above water, a layer of concrete is usually placed over them, the concrete resting on the ground between the piles, as well as on the piles themselves. It is necessary to use a thick plate of concrete, so that a concentrated load will be distributed over a number of piles (see Fig. 52). Sometimes a platform of heavy timbers is constructed on top of the piles, to assist in distributing the load; and in this case the concrete is placed on the platform (see Fig. 53).

When the heads of the piles are under water, it is always necessary to construct a grillage of heavy timber and float it into place, unless a cofferdam is constructed and the water pumped out, in which case the foundation can be completed as already described. The timbers used to cap the piles in constructing a grillage are usually about 12 by 12 inches, and are fastened to the head of each pile by a *drift-bolt* (a plain bar of steel). A hole is bored in the cap and into the head of the pile, in which the drift-bolt is driven. The section of the drift-bolt is always larger than the hole into which it is to be driven; that is, if a 1-inch round drift-bolt is to be used, a $\frac{7}{8}$-inch auger would be used to bore the hole. The transverse timbers of the grillage are drift-bolted to the caps.

209. Concrete and Reinforced-Concrete Piles. A recent development of the use of concrete and reinforced concrete is to construct piles of this material. A reinforced-concrete pile foundation does not materially differ in construction from a timber pile foundation. The piles are driven and capped with concrete ready for the superstructure in the usual manner. In comparing this type of piles with timber piles, they have the advantage of being equally durable in a wet or dry soil, and the disadvantage of being more expensive in first cost. Sometimes their use will effect a saving in the total cost of the foundation by obviating the necessity of cutting the piles off below the water line. The depth of the excavation and the volume of masonry may be greatly reduced, as shown in Fig. 54. In this figure is shown a comparison of the relative amount of excavation which would be necessary, and also of the concrete which would be required for the piles, thus indicating the economy which is possible in the items of excavation and concrete. There is also shown a possible economy in the number of piles required, since concrete piles

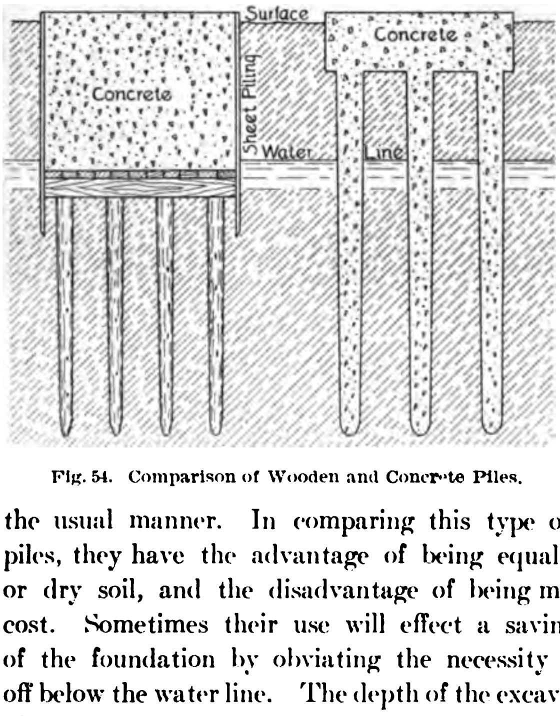


Fig. 54. Comparison of Wooden and Concrete Piles.

can readily be made of any desired diameter, while there is a practical limitation of the diameter of wooden piles. Therefore a less number of concrete piles will furnish the same resistance as a larger number of wooden piles. In Fig. 54 it is assumed that the three concrete piles not only take the place of the four wooden piles in the width of the foundation, but there will also be a corresponding reduction in the number of piles in a direction perpendicular to the section shown. The extent of these advantages depends very greatly on the level of the ground-water line. When this level is considerably below the surface of the ground, the excavation and the amount of concrete required in order that the timber grillage and the tops of the piles shall always be below the water line will be correspondingly great, and the possible economy of concrete piles will also be correspondingly great.

The pile and cap being of the same material, they readily bond together and form a monolithic structure. Reinforced-concrete piles can be driven in almost any soil that a timber pile can penetrate, and they are driven in the same manner as the timber piles. A combination of the hammer and water-jet has been found to be the most successful manner of driving them. The hammer should be heavy and drop a short distance with rapid blows, rather than using a light hammer dropping a greater distance. For protection while being driven, a hollow, cast-iron cap filled with sand is placed on the head of the pile.

Concrete and reinforced-concrete piles may be classified under two headings: (*a*) those where the piles are formed, hardened, and driven very much the same as any pile is driven; (*b*) those where a hole is made in the ground, into which concrete is rammed and left to harden.

Reinforced-concrete piles which have been formed on the ground are designed as columns with vertical reinforcement connected at intervals with horizontal bands. These piles are usually made square or triangular in section, and a steel or cast-iron point is used.

Fig. 55 shows the cross-section of a corrugated pile used in the foundations of the buildings for the Simmons Hardware Company, Sioux City, Iowa, and for John J. Latteman, Brooklyn, N. Y. The pile tapers from 16 inches at the large end to 11 inches at the small end. The reinforcement consists of Clinton electrically-welded fabric, the size being approximately $\frac{3}{8}$-inch wires longitudinally, and $\frac{1}{8}$-inch wires, 12 inches on centers, for the bands. The hole in the center is $3\frac{1}{2}$ inches at the top, and tapers to 2 inches at the bottom. The piles were driven by means of a water-jet and hammer. The jet extended through the opening in the pile, and protruded three inches below the bottom of the pile. The pressure of the water was sufficient to dig a hole and carry the loosened soil up the corrugations, and the weight of the hammer drove the pile down. When the pile was nearly in place, the jet was removed, and the hammer was used to force the pile until it was solid. The cap was made as shown in Fig. 56; and in driving the pile, a hammer weighing 2,500 pounds was dropped 25 feet, 20 to 30 times, without injury to the head.

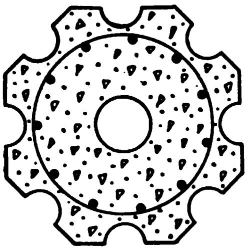


Fig. 55. Section of Corrugated Pile.

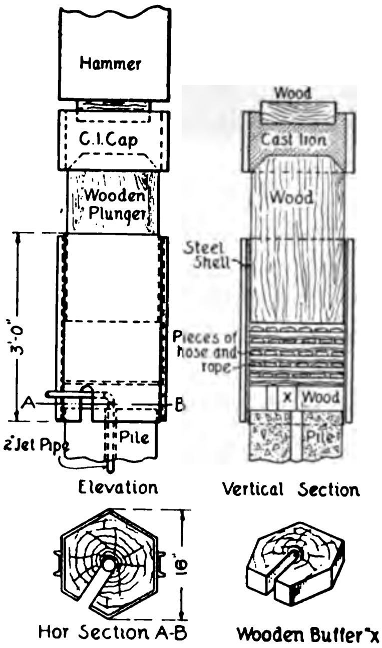


Fig. 56. Cushion Head for Driving Piles.

210. **Raymond Concrete Pile.** The *Raymond* concrete pile (Fig. 57) is constructed in place. A collapsible steel pile-core is encased in a thin, closely-fitting, sheet-steel shell. The core and shell are driven to the required depth by means of a pile-driver. The core is so constructed that when the driving is finished, it is collapsed and withdrawn, leaving the shell in the ground, which acts as a mould for the concrete. When the core is withdrawn, the shell is filled with concrete, which is tamped during the filling process. These piles are usually 18 inches to 20 inches in diameter at the top, and 6 inches to 8 inches at the point. When it is desirable, the pile can be made larger at the small end. The sheet steel used for these piles

is usually No. 20 gauge. When it is desirable to reinforce these piles, the bars are inserted in the shell after the core has been withdrawn and before the concrete is placed.

211. **Simplex Concrete Pile.** The different methods for producing the *Simplex* pile cover the two general classifications of concrete piles—namely, those moulded in place, and those moulded above ground and driven with a pile-driver. Fig. 58 shows the standard methods of producing the Simplex pile. In Fig. 58, *A* shows a cast-iron point which has been driven and imbedded in the ground, the concrete deposited, and the form partially withdrawn; while *B* shows the alligator point driving form. The only difference between the two forms shown in this figure, is that the alligator point is withdrawn and the cast-iron point remains in the ground. The concrete in either type is compacted by its own weight. As the form is removed, the concrete comes in contact with the soil and is bonded with it. A danger in using this type of pile is that, if a stream of water is encountered, the cement may be washed out of the concrete before it has a chance to set.

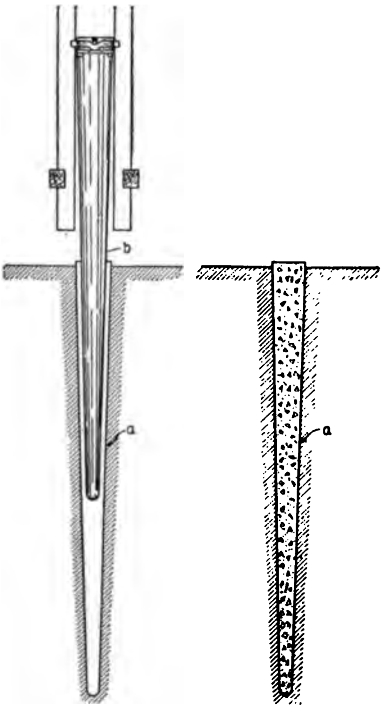
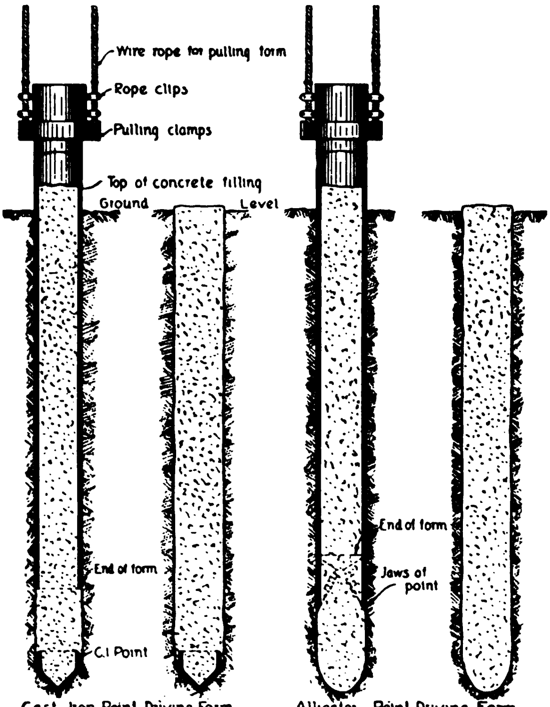


Fig. 57. Raymond Concrete Pile.

A shell pile and a moulded and driven pile are also produced by the same company which manufactures the Simplex, and are recommended for use under certain conditions. Any of these types of piles can be reinforced with steel. This company has driven piles 20 inches in diameter and 75 feet long.

212. **Steel Piles.** In excavating for the foundation of a 16-story building at 14th Street and 5th Avenue, New York, a pocket of quicksand was discovered with a depth of about 14 feet below the

bottom of the general excavation. A wall column of the building to be constructed was located at this point, with its center only 15 inches from the party line. The estimated load to be supported by this column was about 500 tons. It was finally decided to adopt steel piles which would afford the required carrying capacity in a small, compact cluster, and would transfer the load as well as the other foundations to the solid rock. These piles, 5 in number, were driven very close to an existing wall and without endangering it. Each pile was about 15 feet long, and was made with an outer shell consisting of a steel pipe $\frac{3}{8}$ inch thick and 12 inches inside diameter, filled with Portland cement concrete, reinforced with four vertical steel bars 2 inches in diameter. This gave a total cross-sectional area of 27.2 square inches of steel, with an allowed load of 6,000 pounds per square inch, and 100.5 square inches of concrete on which a unit-stress of 500 pounds was allowed. This utilizes the bearing strength of the external shell, and enables the concrete filling to be loaded to the maximum permitted by the New York Building Laws. The tubes and bars have an even bearing on hard bed-rock, to which the former were sunk by the use of a special air hammer and an inside hydraulic jet. The upper ends of the steel tubes and reinforcing bars were cut off after the piles were



Cast Iron Point Driving Form Operation / Finished Pile

A

Alligator Point Driving Form Operation / Finished Pile

B

Fig. 58. Standard Simplex Concrete Piles.

driven. The work was done with care, and a direct contact was secured between them and the finished lower surfaces of the cast-iron caps, without the intervention of steel shims.*

213. **Grillage.** A pile supports a load coming on an area of the foundation which is approximately proportional to the spacing between the piles. This area, of course, is several times the area of the top of the pile. It is therefore necessary to cap at least a group of the piles with a platform or grillage which not only will support any portion of the load located between the piles, but which also will tend to prevent a concentration of load on one pile and will distribute the load more or less uniformly over all the piles. Sometimes such a platform is made of heavy timbers, especially if timber is cheap; but this should never be done unless the grillage will be always under water; and even under such conditions the increasing cost of timber usually makes it preferable to construct the grillage of concrete. A concrete grillage is usually laid with its lower surface a foot or two below the tops of the piles. The piles are thus firmly anchored together at their tops. The thickness of the grillage is roughly proportional to the load per square foot to be carried. No close calculations are possible; a thickness of from 2 to 5 feet is usually made. When reinforced-concrete structures are supported on piles or other concentrated points of support, the heads of the piles are usually connected by reinforced-concrete beams, which will be described in Part III.

214. **Cushing Pile Foundation.** A combination of steel, concrete, and wooden piles is known as the *Cushing pile foundation.* A cluster of piles is driven so that it may be surrounded by a wrought-iron or steel cylinder, which is placed over them, and which is sunk into the soil until it is below any chance of scouring action on the part of any current of water. The space between the piles and the cylinder is then surrounded with concrete. Although the piles are subject to decay above the water line, yet they are so thoroughly surrounded with concrete that the decay is probably very slow. The steel outer casing is likewise subject to deterioration, but the strength of the whole combination is but little dependent on the steel. If such foundations are sunk at the ends of the two trusses of a bridge, and

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*Condensed from *Engineering Record*, February 22, 1908.

are suitably cross-braced, they form a very inexpensive and yet effective pier for the end of a truss bridge of moderate span. The end of such a bridge can be connected with the shore bank by means of light girders, and by this means the cost of a comparatively expensive masonry abutment may be avoided.

215. **Cost.** In comparing the cost of timber piles and concrete or reinforced-concrete piles, the former are found to be much cheaper per linear foot than the latter. As already stated, however, there are many cases where the economy of the concrete pile as compared with the wooden pile is worth considering. In general, the requirements of the work to be done should be carefully noted before the type of pile is selected.

The cost of wooden piles varies, depending on the size and length of the piles, and on the section of the country in which the piles can be bought. Usually piles can be bought of lumber dealers at 10 to 20 cents per linear foot for all ordinary lengths; but very long piles will cost more. The cost of driving piles is variable, ranging from 2 or 3 cents to 12 or 15 cents per linear foot. A great many piles have been driven for which the contract price ranged from 20 cents to 30 cents per linear foot of pile driven. The *length of the pile driven* is the full length of the pile left in the work after cutting off the pile at the desired level of the cap.

The contract price for concrete piles, about 16 inches in diameter and 25 to 30 feet long, is approximately $1.00 per linear foot. When a price of $1.00 per linear foot is given for a pile of this size and length, the price will generally be somewhat reduced for a longer pile of the same diameter. Concrete piles have been driven for 70 cents per linear foot, and perhaps less; and again, they have cost much more than the approximate price of $1.00 per linear foot.

216. **Piles for the Charles River Dam.** The first piles driven for the Cambridge (Mass.) conduit of the Charles River dam were on the Cambridge shore. On January 1, 1907, 9,969 piles had been driven in the Boston and Cambridge cofferdams, amounting to 297,000 linear feet. Under the lock, the average length of the piles, after being cut off, was 29 feet; and under the sluices, 31 feet 4 inches. The specifications called for piles to be winter-cut from straight, live trees, not less than 10 inches in diameter at the butt when cut off in

the work, and not less than 6 inches in diameter at the small end. The safe load assumed for the lock foundations was 12 tons per pile, and for the sluices 7 tons per pile.

The *Engineering News formula* was used in determining the bearing power of the piles. The piles under the lock walls were driven very close together; and as a result, many of them rose during the driving of adjacent piles, and it was necessary to redrive these piles.*

217. **Pile Foundation for Sea-Wall at Annapolis.** The piles for constructing the new sea-wall at Annapolis, Md., ranged in length from 70 feet to 110 feet. On the outer end of the breakwater, piles 70 feet to 85 feet were used. These piles were in one length, single sticks. Toward the inner end of the breakwater, lengths of 100 feet to 110 feet were required. Single sticks of this length could not be secured, and it was therefore necessary to resort to splicing (see Fig. 59). After a trial of several methods, it was found that a splice made by means of a 10-inch wrought-iron pipe was most satisfactory. When the top of the first pile had been driven to within three feet of the water, it was trimmed down to 10 inches in diameter. On this end was placed a piece of 10-inch wrought-iron pipe 10 inches long. The lower end of the top pile was trimmed the same as the top of the first pile, and, when raised by the leads, was fitted into the pipe and driven until the required penetration was reached. The piles were cut off 4½ feet below the surface of the water, by a circular saw mounted on a vertical shaft.†

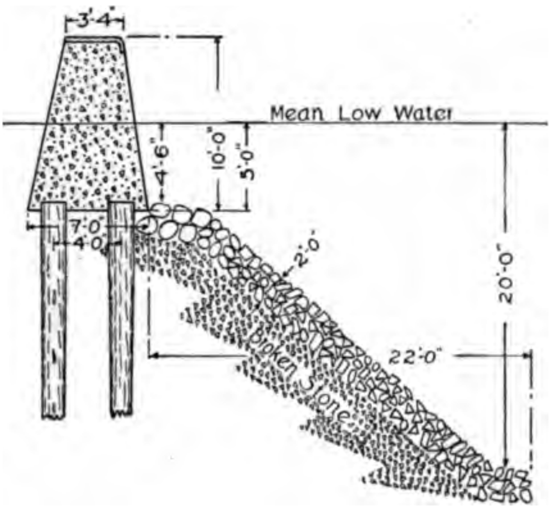

Fig. 59. Section of New Sea-Wall, Annapolis, Maryland.

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*See *Engineering-Contracting*, February 19, 1908.

†Proceedings of the Engineers' Club of Philadelphia, Vol. XXIII, No. 3.

COFFERDAMS AND CAISSONS

218. **Cofferdams.** Foundations are frequently constructed through shallow bodies of water by means of cofferdams. These are essentially walls of clay confined between wooden frames, the walls being sufficiently impervious to water so that all water and mud within the walled space may be pumped out and the soil excavated to the desired depth. It is seldom expected that a cofferdam can be constructed which will be so impervious to water that no pumping will be required to keep it clear; but when a cofferdam can be kept clear with a moderate amount of pumping, the advantages are so great that its use becomes advisable. A dry cofferdam is most easily obtained when there is a firm soil, preferably of clay, at a moderate depth (say 5 feet to 10 feet), into which sheet piling may be driven. The sheet piles are driven as closely together as possible. The bottom of each pile (when made of wood) is beveled so as to form a wedge which tends to force it against the pile previously driven (see Fig. 49). In this way a fairly tight joint between adjacent piles is obtained. Larger piles (see Fig. 60, *a*) made of squared timber are first driven to act as guide-piles. These are connected by waling strips (Fig. 60, *b*), which are bolted to the guide-piles and which serve as guides for the sheet piling (Fig. 60, *c*). The space between the two rows of sheet piling is filled with puddle, which ordinarily consists chiefly of clay. It is found that if the puddling material contains some gravel, there is less danger that a serious leak will form and enlarge. Numerous cross-braces or tie-rods (Fig. 60, *d*) must be used to prevent the walls of sheet piling from spreading when the puddle is

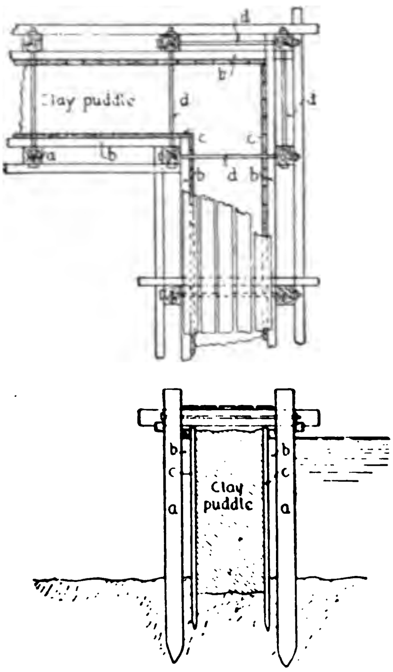


Fig. 60. Plan and Cross-Section of a Cofferdam.

being packed between them. The width of the puddle wall is usually made to vary between three feet and ten feet, depending upon the depth of the water. When the sheet piling obtains a firm footing in the subsoil, it is comparatively easy to make the cofferdam watertight; but when the soil is very porous so that the water soaks up from under the lower edge of the cofferdam, or when, on the other hand, the cofferdam is to be placed on a bare ledge of rock, or when the rock has only a thin layer of soil over it, it becomes exceedingly difficult to obtain a water-tight joint at the bottom of the dam. Excessive leakage is sometimes reduced by a layer of canvas or tarpaulin which is placed around the outside of the base of the cofferdam, and which is held in place by stones laid on top of it. Brush, straw, and similar fibrous materials are used in connection with earth for stopping the cracks on the outside of the dam, and are usually effective, provided they are not washed away by a swift current.

Although cofferdams can readily be used at depths of 10 feet, and have been used in some cases at considerably greater depth, the difficulty of preventing leakage, on account of the great water pressure at the greater depths, usually renders some other method preferable when the depth is much, if any, greater than 10 feet.

219. **Cribs.** A crib is essentially a framework (called a *bird-cage* by the English) which is made of timber, and which is filled with stone to weight it down. Such a construction is used only when the entire timber work will be perpetually under water. The timber framework must, of course, be designed so that it will safely support the entire weight of the structure placed upon it. The use of such a crib necessarily implies that the subsoil on which the crib is to rest is sufficiently dense and firm so that it will withstand the pressure of the crib and its load without perceptible yielding. It is also necessary for the subsoil to be leveled off so that the crib itself shall not only be level but shall also be so uniformly supported that it is not subjected to transverse stresses which might cripple it. This is sometimes done by dredging the site until the subsoil is level and sufficiently firm. Some of this dredging may be avoided through leveling up low spots by depositing loose stones which will imbed themselves in the soil and furnish a fairly firm subsoil. Although such methods may be tolerated when the maximum unit-loading is not great (as for a breakwater or a wharf), it is seldom that a satisfactory

foundation can be thus obtained for heavy bridge piers and similar structures.

220. **Open Caissons.** A *caisson* is literally a *box*; and an *open caisson* is virtually a huge box which is built on shore and launched in very much the same way as a vessel, and which is sunk on the site of the proposed pier. (See Fig. 61.) The box is made somewhat larger than the proposed pier, which is started on the bottom of the box. The sinking of the box is usually accomplished by the building of the pier inside of the box, the weight of the pier lowering it until it reaches the bed prepared for it on the subsoil. The preparation of this bed involves the same difficulties and the same objections as those already referred to in the adoption of cribs. The bottom of the box is essentially a large platform made of heavy timbers and planking. The sides of the caissons have sometimes been made so that they are merely tied to the bottom by means of numerous tie-rods extending from the top down to the extended platform at the bottom, where they are hooked into large iron rings. When the pier is complete above the water line so that the caisson is no longer needed, the tie-rods may be loosened by unscrewing nuts at the top. The rods may then be unhooked, and nearly all the timber in the sides of the caisson will be loosened and may be recovered.

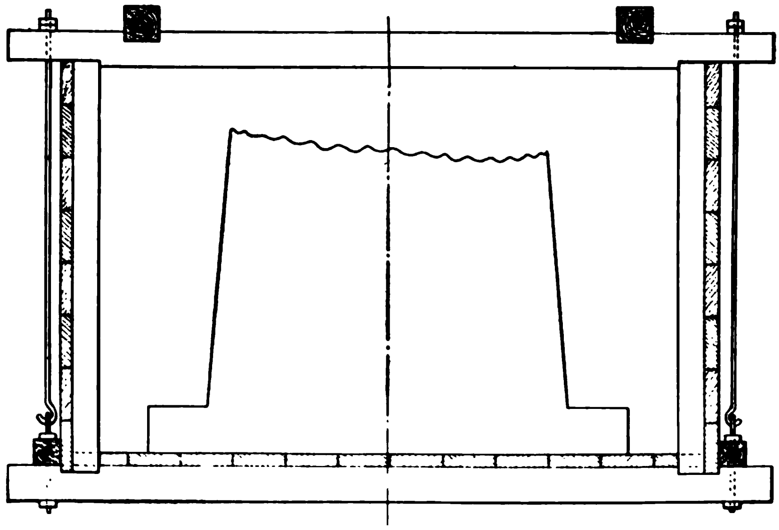


Fig. 61. Section of Open Caisson.

221. Hollow Cribs or Caissons. The foundation for a pier is sometimes made in the form of a box with walls several feet in thickness, but with a large opening or well through the center. Such piers may be sunk in situations where there is a soft soil of considerable depth through which the pier must pass before it can reach the firm subsoil. In such a case, the crib or caisson, which is usually made of timber, may be built on shore and towed to the site of the proposed pier. The masonry work may be immediately started; and as the pier sinks into the mud, the masonry work is added so that it is always considerably above the water line. (See Fig. 62.) The deeper the pier sinks, the greater will be the resistance of the subsoil, until, finally, the weight of the uncompleted pier is of itself insufficient to cause it to sink further. At this stage, or even earlier, dredging may be commenced by means of a *clam-shell* or *orange-peel* dredging bucket, through the interior well. The removal of the earth

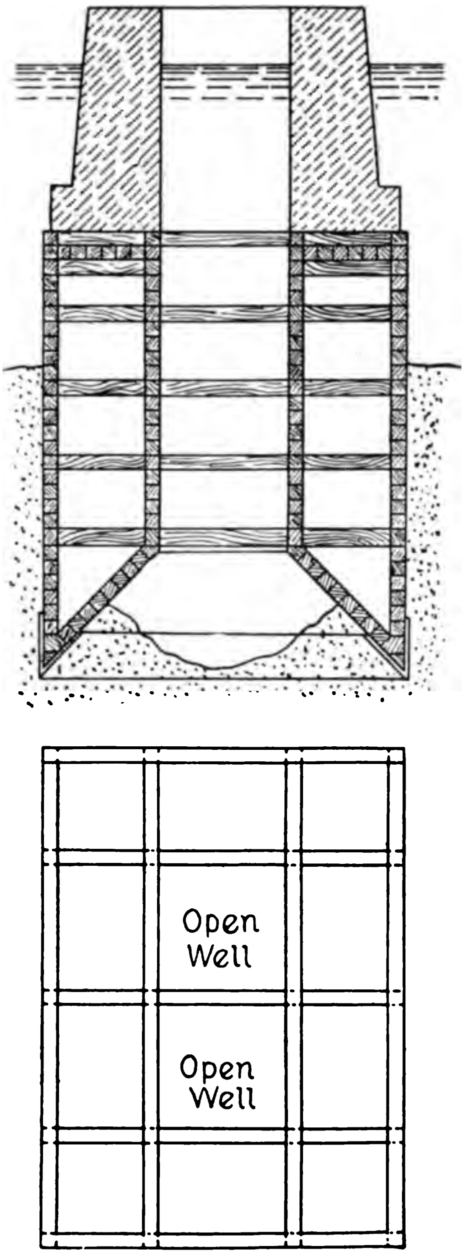


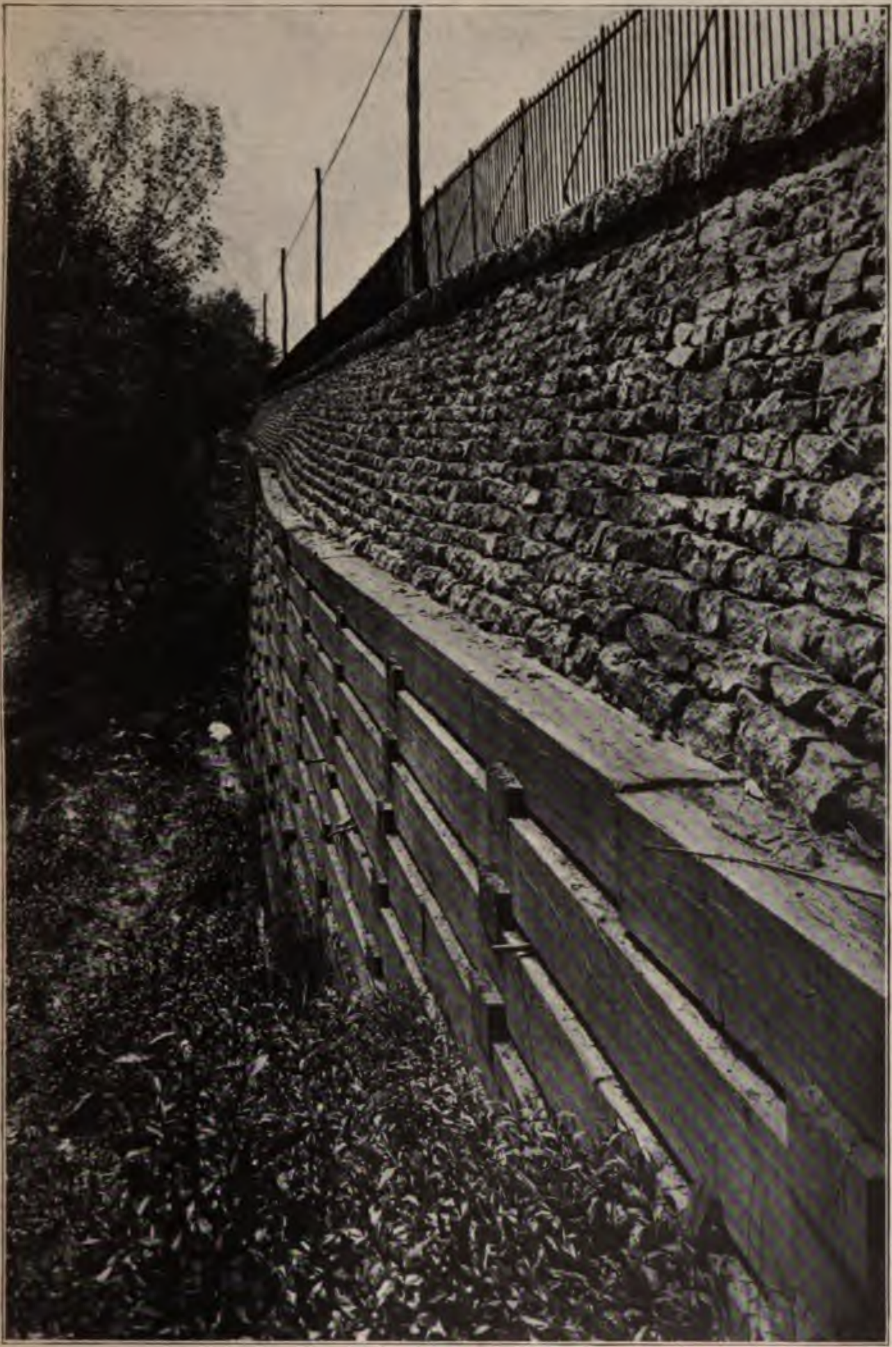
Fig. 62. Hollow Crib Material.

from the center of the subsoil on which the pier is resting, will cause the mud and soft soil to flow toward the center, where it is within reach of the dredge. The pressure of the pier accomplishes this. The deeper the pier sinks, the greater is its weight and the greater its pressure on the subsoil, although this is somewhat counteracted by the constantly increasing friction of the soil around the outside of the pier. Finally the pier will reach such a depth, and the subsoil will be so firm, that even the pressure of the pier is not sufficient to force any more loose soil toward the central well. The interior well may then be filled solidly with concrete, and thus the entire area of the base of the pier is resting on the subsoil, and the unit-pressure is probably reduced to a safe figure for the subsoil at that depth.

This principle was adopted in the Hawkesbury bridge in Australia, which was sunk to a depth of 185 feet below high water—a depth which would have been impracticable for the pneumatic caisson method described later. In this case, the caissons were made of iron, elliptical in shape, and about 48 feet by 20 feet. There were three tubes 8 feet in diameter through each caisson. At the bottom, these tubes flared out in bell-shaped extensions which formed sharp cutting edges with the outside line of the caisson. These bell-mouthed extensions thus forced the soil toward the center of the wells until the material was within reach of the dredging buckets.

This method of dredging through an opening is very readily applicable to the sinking of a comparatively small iron cylinder. As it sinks, new sections of the cylinder can be added; while the dredge, working through the cylinder, readily removes the earth until the subsoil becomes so firm that the dredge will not readily excavate it. Under such conditions the subsoil is firm enough for a foundation, and it is then only necessary to fill the cylinder with concrete to obtain a solid pier on a good and firm foundation.

One practical difficulty which applies to all of these methods of cribs and caissons, is the fact that the action of a heavy current in a river, or the meeting of some large obstruction such as a boulder or large sunken log, may deflect the pier somewhat out of its intended position. When such a deflection takes place, it is difficult if not impossible to force the pier back to its intended position. It therefore becomes necessary to make the pier somewhat larger than the



STOPPING THE SLIDING OF A RETAINING WALL, LOUISVILLE, KENTUCKY

Wall rests on a stratum of clay, plastic when wet, overlying sand and gravel. Its lower part is a series of stone-filled timber cribs, under which, at highest part of wall, wooden piles were driven to prevent sliding. Above cribs to street grade, wall is of stone laid dry. Repeated floods in creek running parallel with wall, probably rotted piles, allowing wall to slide forward under pressure of its earth backing. This was finally stopped by building a retaining wall of reinforced concrete, of inverted T shape, on 15-ft. reinforced concrete piles, about 10 ft. in front of old wall, the intermediate space filled with earth paved on top with dry block stone to prevent washing.

Courtesy of J. P. Claybrook, Chief Engineer, Dept. of Public Works, Louisville, Ky

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strict requirements of the superstructure would demand, so that the superstructure may have its intended alignment, even though the pier is six inches or even a foot out of its intended position.

222. **Pneumatic Caissons.** A pneumatic caisson is essentially a large inverted box on which a pier is built, and inside of which work may be done because the water is forced out of the box by compressed air. If an inverted tumbler is forced down into a bowl of water, the large air space within the tumbler gives some idea of the possibilities of working within the caisson. If the tumbler is forced to the bottom of the bowl, the possibilities of working on a river bottom are somewhat exemplified. It is, of course, necessary to have a means of communication between this working chamber and the surface; and it is likewise necessary to have an *air-lock* through which workmen (and perhaps materials) may pass.

The process of sinking resembles in many points that described in the previous section. The caisson is built on shore, is launched, and is towed to its position. Sometimes, for the sake of economy (provided timber is cheap), that portion of the pier from the top of the working chamber to within a few feet below the low-water line, may be built as a timber crib and filled with loose stone or gravel merely to weight it down. This method is usually cheaper than masonry; and the timber, being always under water, is durable. As in the previous section, the caisson sinks as the material is removed from the base, the sinking being assisted by the additional weight on the top. The only essential difference between the two processes consists in the method of removing the material from under the caisson. The greatest depth to which such a caisson has ever been sunk is about 110 feet below the water line. This depth was reached in sinking one of the piers for the St. Louis bridge. At such depths the air pressure per square inch is about 48 pounds, which is between three and four times the normal atmospheric pressure. Elaborate precautions are necessary to prevent leakage of air at such a pressure. Only men with strong constitutions and in perfect health can work in such an air pressure, and even then four hours work per day in two shifts of two hours each is considered a good day's work at these depths. The workmen are liable to a form of paralysis which is called *caisson disease*, and which, especially in those of weak constitution or intemperate habits, will result in partial or permanent disablement and even death.

In Fig. 63 is shown an outline, with but few details, of the pneumatic caisson used for a large bridge over the Missouri River near Blair, Nebraska. The caisson was constructed entirely of timber, which was framed in a fashion somewhat similar to that shown in greater detail in Fig. 62. The soil was very soft, consisting chiefly of sand and mud, which was raised to the surface by the operation of *mud pumps* that would force a stream of liquid mud and sand through the smaller pipes, which are shown passing through the pier. The larger pipes near each side of the pier, were kept closed during the process of sinking the caisson, and were opened only after the pier had been sunk to the bottom, and the working chamber was being filled with concrete, as described below. These extra openings facilitated the filling of the working chamber with concrete. Near the center of the pier, is an air-lock, with the shafts extending down to the working chamber and up to the surface. The ends of three trusses, which were made part of the construction of

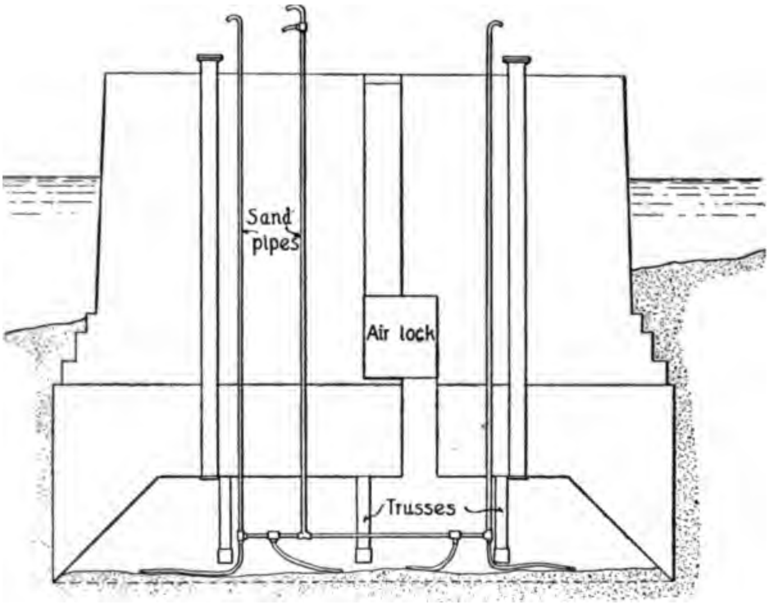


Fig. 63. Outline of Pneumatic Caisson.

the caisson in order to resist any tendency to collapse, are also shown.

A caisson is necessarily constructed in a very rigid manner, the timbers being generally 12 by 12-inch, and laid crosswise in alternate layers, which are thoroughly interlocked. An irregularity in the settling may often be counteracted by increasing the rate of excavation under one side or the other of the caisson, so that the caisson will be guided in its descent in that direction.

A great economy in the operation of the compressed-air locks is afforded by combining the pneumatic process with the open-well process described in the previous section, by maintaining a pit in the center of the caisson. A draft tube which is as low as the cutting edge of the caisson prevents a *blow-out* of air into the central well. The material dug by the workmen in the caisson is thrown loosely into the central well or *sump*, from which it is promptly raised by the dredging machinery (see Fig. 64). By the adoption of this plan, the air-lock needs to be used only for the entrance and exit of the workmen to and from the working chamber.

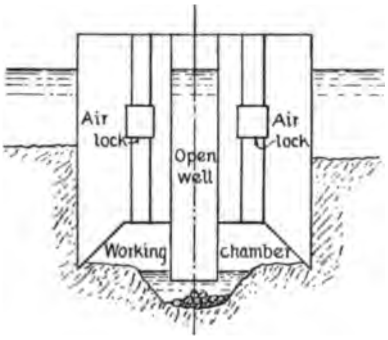


Fig. 64. Combination of Pneumatic Caisson and Open-Well Methods.

When the caisson has sunk to a satisfactory subsoil, and the bottom has been satisfactorily cleaned and leveled off, the working chamber is at once filled with concrete. As soon as sufficient concrete has been placed to seal the chamber effectively against the entrance of water, the air-locks may be removed, and then the completion of the filling of the chamber and of the central shaft is merely open-air work.

RETAINING WALLS

223. A retaining wall is a wall built to sustain the pressure of a vertical bank of earth. The stability of the wall is a comparatively simple matter when three quantities have been determined:

(1) The intensity of the earth pressure;
(2) The point of application of the resultant of the earth pressure;
(3) The line of action of this pressure.

Unfortunately, earthy material is very variable in its action in these respects, depending on its condition. It is not only true that different grades of earthy material act quite differently in these respects, but it is also true that the same material will act differently under varying physical conditions, especially in regard to its saturation with water. On these accounts it is impracticable, even by experiment, to determine values which are reliable for all conditions.

It is also comparatively easy to formulate a theory regarding the pressure of earthwork which shall be based on certain theoretical assumptions. One of these assumptions is that the so-called *plane of rupture* is a plane surface—or, in other words, that the line *a b* (Fig. 65) is a straight line. There is considerable evidence, and even theoretical grounds, for considering not only that the line *a b* is a curved line, but that the curve is variable, depending on the physical conditions. It is also assumed that the earthy material acts virtually the same as a liquid with a density considerably greater than water; but there is ground for believing that even this assumption is not strictly warranted. Theoretically the problem is also very much complicated by the question of the earth pressure which may be produced by a surcharged wall. A ***surcharge*** is a bank of earth which is built above the height of the top of the retaining wall and sloping back from it. It certainly adds to the pressure on the earth immediately back of the wall itself and increases the pressure on the wall.

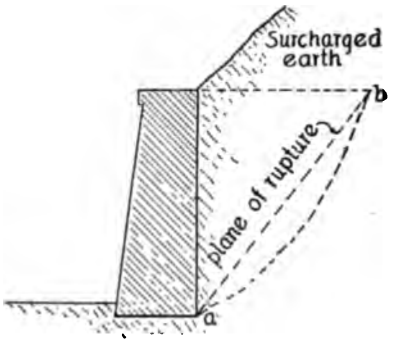


Fig. 65. Typical Retaining Wall.

224. **Theoretical Formulæ.** In spite of the unreliability of theoretical formulæ, for the reasons given above, certain formulæ which are here quoted without demonstration are sometimes used for lack of better formulæ as a guide in determining the thickness of a wall. For simplicity it is assumed that the rear face of the wall is vertical. The variation in the theory by attempting to allow for a slope of the rear face, merely complicates the theory; while the effect of such a variation from the vertical as is ever adopted is usually so

small that it is utterly swallowed up by the unavoidable uncertainties in the practical application of the theory. In Fig. 66,

Let E = Total pressure against rear face of wall on a unit-length of wall;
w = Weight of a unit-volume of the earth;
h = Height of wall;
ϕ = The *angle of repose* with the horizontal—that is, the angle at which that kind of earth will remain without further sliding.

Then, when the upper surface of the earth is horizontal, we have the equation:

$$E = \tan^2 \left(45° - \frac{\phi}{2}\right) \frac{wh^2}{2} \quad \ldots\ldots\ldots (6)$$

If the upper surface of the earth is surcharged with a bank of earth at a natural slope, or if the angle of slope of the surcharge = ϕ, then the equation becomes:

$$E = \frac{1}{2} \cos \phi \, w \, h^2 \quad \ldots\ldots\ldots\ldots (7)$$

An inspection of Equation 6 will show that the pressure E is greater for small values of ϕ. The angle of repose for various materials is not only variable for different grades of material, but is variable for the same grade of material under various conditions of saturation. A value of ϕ which is frequently adopted is 30°. This is considerably lower than the usual true value of ϕ for dry material, and is usually a safe value of ϕ for any material (except quicksand) either wet or dry. The adoption of this value, therefore, generally means that the result is safe, and that the factor of safety is merely made somewhat larger.

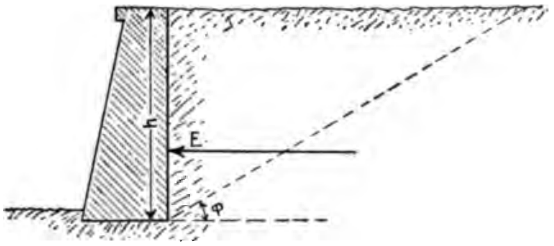


Fig. 66. Pressure on Retaining Wall.

225. *Example.* What will be the pressure per foot of length of the wall, for a wall 18 feet high, the angle of repose for the earth being assumed at 30°?

Solution. Here h = 18 feet, and ϕ = 30°. The weight of earth (w) is quoted as varying from 70 to 120 pounds per cubic foot according to the degree of saturation and density of packing. When the earth is densely packed, its angle of repose is greater; therefore we are safe in assuming a weight of 100 pounds per cubic foot for an angle (ϕ) of 30°. Substituting these values in Equation 6, we have:

$$
\begin{aligned}
E &= \tan^2 \left(45° - \frac{30°}{2}\right) \frac{100 \times 324}{2} \\
&= \tan^2 30° \times 16{,}200 \\
&= 5{,}400 \text{ pounds.}
\end{aligned}
$$

Using this value of $\phi = 30°$ gives us the simple relation that $E = \frac{1}{6} w h^2$, or one-sixth of the unit-weight of the earth times the square of the height, for a wall without a surcharge.

226 **Methods of Failure.** There are four distinct ways in which failure of a retaining wall may come about:

(1) A retaining wall may fail by sliding bodily on its foundations or on any horizontal joint. This may occur when the wall is resting on a soft soil, and especially when the foundation is not sunk sufficiently deep into the subsoil so that it is anchored. The failure of the wall on a horizontal joint is very improbable when the masonry work and its bonding are properly done. Perfectly flat, continuous joints should be avoided.

(2) A retaining wall may fail by crushing the toe of the wall. This may occur provided that the resultant of the weight of the wall and of the overturning pressure comes so near the toe of the wall, and the intensity of that pressure is so great, that the masonry is crushed. The method of calculating such pressure will be given later.

(3) The wall may fail by tipping over. This may occur provided that the resultant pressure (described later) passes *outside* the toe of the wall

(4) The same effect occurs provided that the subsoil is unable to withstand the concentrated pressure on the toe of the wall, and yields, while the masonry of the wall may nevertheless remain intact and the wall itself be properly proportioned.

227. **Determining the Resultant Pressure.** Assume a very simple numerical case, as in Fig. 67. The weight of the wall and its line of action are very readily determined with accuracy. The base of the wall has been made $\frac{4}{10}$ of the height, or 7.2 feet. The batter of the outer face is at the rate of 1 in 5, or is 3.6 feet in the total height of the wall, leaving 3.6 feet as the thickness at the top. The area of the cross-section $= \frac{1}{2}(3.6 + 7.2)\,18 = 97.2$ square feet. On the

basis that this masonry weighs 140 pounds per cubic foot, a section of this wall one foot long will weigh 13,608 pounds. To find the line of application of the weight, we must find the center of gravity of the trapezoid, and for this purpose we may divide the trapezoid into a rectangle and a right-angled triangle. The rectangle has an area of 64.8 square feet, and its center of gravity is 1.8 feet from the rear face. The center of gravity of the triangle (whose area equals $\frac{1}{2} \times 18 \times 3.6 = 32.4$ square feet) is at one-third of the base of the triangle from its right-hand vertical edge, or at a distance of 4.8 feet from the rear of the wall. The center of gravity of the trapezoid is then found numerically as follows:

$$\begin{array}{l} 64.8 \times 1.8 = 116.64 \\ \underline{32.4} \times 4.8 = \underline{155.52} \\ 97.2 \qquad\qquad\quad 272.16 \\ 272.16 \div 97.2 = 2.80 \text{ feet,} \end{array}$$

which is the distance of the center of gravity of the trapezoid from the rear face of the wall. The pressure of the earth on the rear wall, as stated above, is very uncertain; a value for it has already been computed (in the example in section 225) as 5,400 pounds. This value is probably excessive, except under the most unfavorable conditions. The point of application of the resultant of this pressure, as well as the direction of that resultant, is also uncertain, and has been the subject of much theoretical controversy. If the soil were merely a liquid which had no internal friction, there would be no uncertainty. In this case, the point of application of the pressure would be at one-third the height of the wall from the base, and its direction would be perpendicular to the rear face of the wall. This is the most unfavorable condition for stability which could be assumed; and on this account, calculations are sometimes made on this basis, with the knowledge that if the wall is stable under these most unfavorable conditions, it will certainly be stable no matter what the real conditions may be. On this basis we have the resultant pressure against the rear of the wall as indicated by the arrow in Fig. 67. The resultant pressure on the base of the wall is therefore a line the direction of which is indicated by the diagonal of a parallelogram whose two sides are parallel to the two forces, the sides being proportional to those forces. The amount of this pressure equals the square root of the sum of the squares of 5,400 and 13,608, or 14,640 pounds. The intersection of this line of pressure with the base is evidently at a distance from the intersection of the line of vertical pressure, equal to:

$$6 \text{ feet} \times \frac{5,400}{13,608} = 2.38 \text{ feet}.$$

That point is therefore 5.18 feet from the rear of the wall, or 2.02 feet from the toe. This point represents the *center of pressure* of the pressure on the subsoil. The pressure is most intense at the toe of the wall, and is there assumed to be twice as intense as the average pressure. It is also assumed that the pressure diminishes toward the rear, until, at a distance back from the center of pressure equal to twice the distance from the center of pressure to the toe, the pressure is zero. This would mean that the pressure varies as the ordinates of a triangle (as illustrated in Fig. 68), the triangle having a base of 3 × 2.02 = 6.06 feet. The average pressure would equal 14,640 ÷ 6.06 = 2,415 pounds per square foot. The maximum pressure at the toe would therefore equal twice this average pressure, or 4,830 pounds per square foot, or about 34 pounds per square inch. This unit-pressure is so far within that allowable for stone masonry, that there is no danger of the crushing of the masonry at the toe.

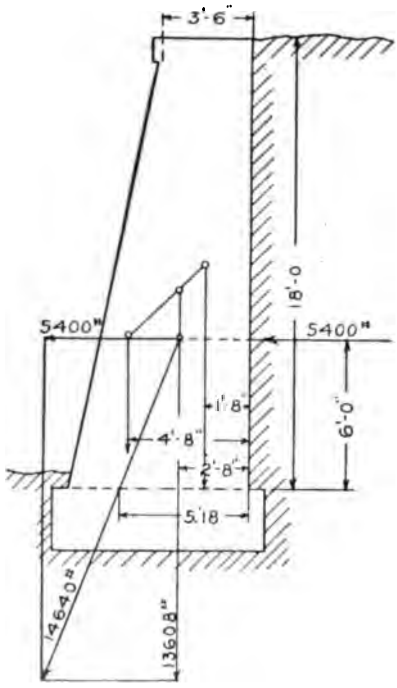


Fig. 67. Resultant Pressure of Retaining Wall.

The pressure on the subsoil, which is less than 2½ tons per square foot, is less than that usually allowable on a good subsoil. There is therefore but little danger that the subsoil will be crushed and that the wall will tip over bodily on account of the failure of the subsoil. Since the line of pressure is likewise two feet back of the toe of the wall, there is no danger that the wall will tip over around its toe. The accuracy with which these calculations have been carried out should not lead to the idea that the pressures will necessarily be exactly as stated, since the calculations are based on assumptions which are at the best very doubtful, but which, as previously stated, are probably excessively safe.

The form chosen for this wall is also so simple that a purely numerical calculation was the easiest and most satisfactory method. If the shape of the wall had been more irregular, it would have been easier to adopt the graphical method for the determination both of the center of gravity of the wall and of the resultant pressure on the subsoil. For instance, if the rear face of the wall had been inclined, the line of pressure would have been drawn perpendicular to the rear face and through a point at one-third the height of the wall. The position of the center of gravity of the wall would have been determined by the purely graphical method of determining the center of gravity of a trapezoid; and then the amount, direction, and intersection of the resultant with the base of the wall would have been determined by purely graphical methods.

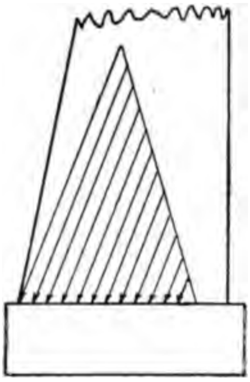


Fig. 68. Variation of Intensity of Pressure on Base.

228. **Empirical Rules.** On account of the unsatisfactory nature of theoretical calculations, retaining walls are usually built by the application of purely empirical rules. Trautwine recommends that for a wall of cut stone or of first-class large ranged rubble in mortar, the thickness should be .35 of its vertical height. For a good common mortar rubble or brick, the thickness should be .4, and for a dry wall .5, of the height. Military engineers who have a very extensive experience in constructing retaining walls as a feature of fortification work, use a rule giving much less thickness than this, and make it depend on the batter of the wall. The thickness at the base in proportion to the height, is as follows:

Gen. Fanshawe's Rule for Thickness of Retaining Walls

BATTER	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{24}$	Vertical
BASE ÷ HEIGHT	24%	25%	26%	27%	28%	30%	32%

The fact that experience has shown that the above proportions are usually safe, provided that the subsoil is sufficiently hard, is another proof that the assumptions made in the problem worked out above are excessively safe, since Fanshawe's rule would have required a ratio of base to height of only 24 per cent, while the ratio chosen for the problem was 40 per cent.

229. Failure of Retaining Walls. It is a significant fact that a retaining wall may apparently withstand the pressure against it for a period of several years, and may then slowly and gradually fail. This is sometimes due to the action of frost on the soil behind the wall. The water accumulates behind the wall in the early winter, and, if it is unable to drain away, may freeze, expand, and exert a pressure on the wall which forces it out. One great precautionary feature in the construction of retaining walls is to place drain-pipes through the wall at sufficient intervals so that water cannot accumulate and remain behind the wall. The gradual failure of walls may also be due to the undermining and weakening of the subsoil, which makes it unable to resist the concentrated pressure on the toe of the wall. Faulty construction and the violation of the ordinary rules of good masonry work—the latter being sometimes done with the idea that anything is good enough for a retaining wall—are also responsible for some failures, since they prevent the body of the wall from acting as a unit in resisting a tendency to overturn.

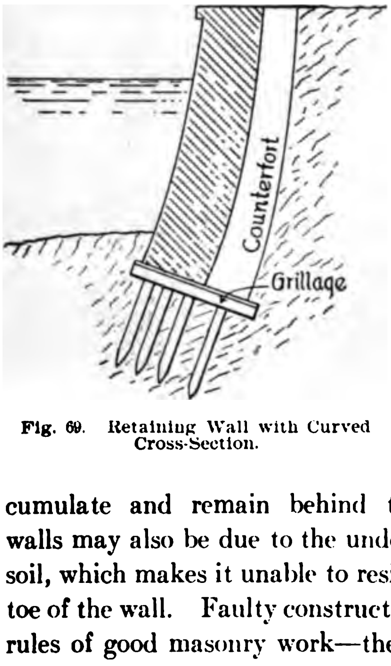


Fig. 69. Retaining Wall with Curved Cross-Section.

The tendency to slide outward at the bottom, and even the tendency to overturn, may be materially resisted by making the lower course with the joints inclined toward the rear. This method of construction is all the more logical, since it makes the joints nearly perpendicular to the line of pressure. In fact, the line of pressure is really a curved line which is more nearly vertical toward the top of the wall, and more and more inclined to the horizontal toward the bottom of the wall. The recognition of this principle has sometimes resulted in designing retaining walls on the principle illustrated in Fig. 69, which is somewhat similar to a section of an arch set on end. Such curved outlines, of course, are more expensive, and are sometimes inconvenient, and for that reason are but seldom adopted.

A detail which is frequently adopted in the design of retaining walls, is to use what is virtually a batter to the rear face of the wall, but to accomplish this by a series of steps on the rear of the wall. This not only permits the use of rectangular blocks of stone and the employment of vertical joints, but also adds considerably to the stability of the wall, since the vertical pressure of the earth on the horizontal steps adds considerably to the resistance to overturning. In Fig. 70 is shown a design for a retaining wall made to support a railway embankment in a location where the natural surface was so steep that the embankment would not readily obtain sufficient support. Although this use of a retaining wall is somewhat special, the general outline of the design not only conforms to the standards on that railroad, but represents good practice and is an illustration of many of the points referred to above. It should be noted that in this case the total width of the base of the wall is nearly one-half the height.

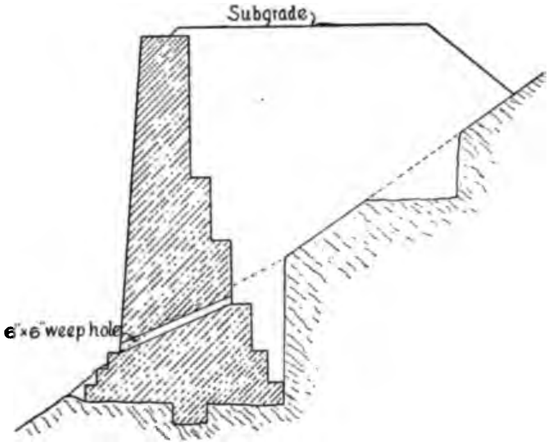


Fig. 70. Retaining Wall for Railroad Embankment.

BRIDGE PIERS AND ABUTMENTS

230. **Placing of Piers.** The outline design of a long bridge which requires several spans, involves many considerations:

(1) If the river is navigable, at least one deep and wide channel must be left for navigation. The placing of piers, the clear height of the spans above high water, and the general plans of all bridges

over navigable rivers, are subject to the approval of the United States Government.

(2) A long bridge always requires a solution of the general question of few piers and long spans, or more piers and shorter spans. No general solution of the question is possible, since it depends on the required clear height of the spans above the water, on the required depth below the water for a suitable foundation, and on several other conditions (such as swift current, etc.) which would influence the relative cost of additional piers or longer spans. Each case must be decided according to the particular circumstances of the case.

(3) Even the general location of the line of the bridge is often determined by a careful comparison, not only of several plans for a given crossing, but even a comparison of the plans for several locations.

231. **Usual Sizes and Shapes of Piers.** The requirements for the bridge seats for the ends of the two spans resting on a pier, are usually such that a pier with a top as large as thus required, and with a proper batter to the faces, will have all the strength necessary for the external forces acting on the pier. For example, the channel pier of one of the large railroad bridges crossing the Mississippi River was capped by a course of stonework 14 feet wide and 29 feet long, besides two semicircles with a radius of 7 feet. The footing of this pier was 30 feet wide by 70 feet long, and the total height from subsoil to top was about 170 feet. This pier, of course, was unusually large. For trusses of shorter span, the bridge seats are correspondingly smaller. The elements which affect stability are so easily computed that it is always proper, as a matter of precaution, to test every pier designed to fulfil the other usual requirements to see whether it is certainly safe against certain possible methods of failure. This is especially true when the piers are unusually high.

The requirements for supporting the truss are, fortunately, just such as give the pier the most favorable formation so that it offers the least obstruction to the flow of the current in the river. In other words, since the normal condition is for a bridge to cross a river at right angles, the bridge piers are always comparatively long (in the direction of the river) and narrow in a direction perpendicular to the flow of the current. The rectangular shape, however, is modified by making both the upper and the lower ends pointed. The pointing of the upper end serves the double purpose of deflecting the current, and thus offers less resistance to the flow of the water; and it also deflects the floating ice and timber, so that there is less danger of the formation of a jam during a freshet. The lower end should also be pointed in order to reduce the resistance to the flow of the water. The ends of the piers are sometimes made semicircular, but a better plan is to make them in the form of two arcs of circles which intersect at a point.

232. **Possible Methods of Failure.** The forces tending to cause a bridge pier to fail in a direction perpendicular to the line of the bridge, include the action of wind on the pier itself, on the trusses, and on a train which may be crossing the bridge. They will also include the maximum possible effect of floating ice in the river and of the current due to a freshet. It is not at all improbable that all of these causes may combine to act together simultaneously. The least favorable condition for resisting such an effect is that produced by the weight of the bridge, together with that of a train of *empty* cars, and the weight of the masonry of the pier above any joint whose stability is in question. The effects of wind, ice, and current will tend to make the masonry slide on the horizontal joints. They will also increase the pressure on the subsoil on the downstream end of the foundation of a pier. They will tend to crush the masonry on the downstream side, and will tend to tip the pier over.

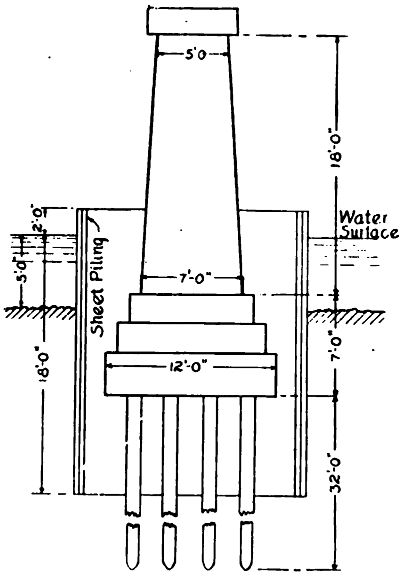


Fig. 71. Bridge Pier.

Another possible method of failure of a bridge pier arises from forces parallel with the length of the bridge. The stress produced on a bridge by the sudden stoppage of a train thereon, combined with a wind pressure parallel with the length of the bridge, will tend to cause the pier to fail in that direction (see Fig. 71). Although these forces are never so great as the other external forces, yet the resisting power of the pier in this direction is so very much less than that in the other direction, that the factor of safety against failure is probably less, even if there is no actual danger under any reasonable values for these external forces.

233. **Abutment Piers.** A pier is usually built comparatively thin in the direction of the line of the bridge, because the forces tending to produce overturning in that direction are usually very small. When a series of stone arches are placed on piers, the thrusts of the two arches on each side of a pier nearly balance each other, and it is only necessary for the pier to be sufficiently rigid to withstand the effect of an eccentric loading on the arches; but if, by any accident or failure, one arch is destroyed, the thrust on such a pier is unbalanced and the pier will probably be overturned by the unbalanced thrust of the adjoining arch. The failure of that arch would similarly cause the failure of the succeeding pier and arch. On this account a very long series of arches usually includes an *abutment pier* for every fourth or fifth pier. An abutment pier is one which has sufficient thickness to withstand the thrust of an arch, even though it is not balanced by the thrust of an arch on the other side of the pier. Abutment piers are chiefly for arch bridges; but all piers should have sufficient rigidity in the direction of the line of the bridge so that any possible thrust which may come from the action of a truss of the bridge may be resisted, even if there is no counterbalancing thrust from an adjoining truss.

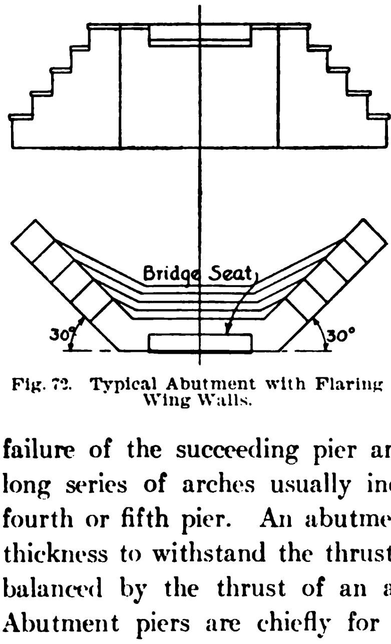
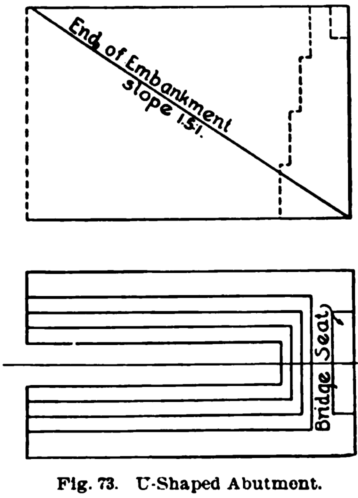


Fig. 72. Typical Abutment with Flaring Wing Walls.

234. **Abutments.** The term *abutment* usually implies not only a support for the bridge, but also what is virtually a retaining wall for the bank behind it. In the case of an arch bridge, the thrust of the arch is invariably so great that there is never any chance that the pressure of the earth behind the abutment will throw the abutment over, and therefore the abutment never needs to be designed as a

retaining wall in this case; but when the abutment supports a truss bridge which does not transmit any horizontal thrust through the bridge, the abutment must be designed as a retaining wall. The conditions of stability for such structures have already been discussed. This principle of the retaining wall is especially applicable if the abutment consists of a perfectly straight wall. There are other forms of abutments which tend to prevent failure as a retaining wall, on account of their design.

235. *Abutments with Flaring Wing Walls.* These are constructed substantially as shown in Fig. 72. The wing walls make an angle of about 30° to 45° with the face of the abutment, and the height decreases at such a rate that it will just catch the embankment formed behind it, the slopes of the embankment probably being at the rate of 1.5:1. If the bonding of the wing walls, and especially the bonding at the junction of the wing walls with the face of the abutment, are properly done, the wing walls will act virtually as counterforts and will materially assist in resisting the overturning tendency of the earth. The assistance given by these wing walls will be much greater as the angle between the wing walls and the face becomes larger.



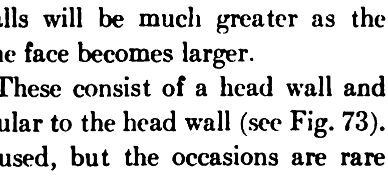


Fig. 73. U-Shaped Abutment.

236. *U-Shaped Abutments.* These consist of a head wall and two walls which run back perpendicular to the head wall (see Fig. 73). This form of wall is occasionally used, but the occasions are rare when such a shape is necessary or desirable.

237. *T-Shaped Abutments.* As the name implies, these consist of a head wall which has a core wall extending perpendicularly back from the center. The core wall serves to tie the head wall and prevent its overturning. Of course such an effect can be produced only by the adoption of great care in the construction of the wall, so that the bonding is very perfect and so that the wall has very considerable

tensile strength; otherwise the core wall could not resist the overturning tendency of the earth pressure against the rear face of the abutment.

CULVERTS

238. The term *culvert* is usually applied to a small waterway which passes under an embankment of a railroad or a highway. The term is confined to waterways which are so small that standard plans are prepared which depend only on the assumed area of waterway that is required. Although the term is sometimes applied to arches having a span of 10 or 15 feet, or even more, the fact that the structures are built according to standard plans justifies the use of the term culvert as distinguished from a structure crossing some perennial stream where a special design for the location is made. The term culvert therefore includes the drainage openings which may be needed to drain the hollow on one side of an embankment, even though the culvert is normally dry.

239. **Various Types of Culverts.** Culverts are variously made of cast iron, wrought iron, and tile pipe, wood, stone blocks with large cover-plates of stone slabs, stone arches, and plain and reinforced concrete; still another variety is made by building two side walls of stone and making a cover-plate of old rails.

240. Culverts made of wood should be considered as temporary, on account of the inevitable decay of the wood in the course of a few years. When wood is used, the area of the opening should be made much larger than that actually required, so that a more permanent culvert of sufficient size may be constructed *inside* of the wooden culvert before it has decayed. For present purposes, the discussion of the subject of Culverts will be limited to those built of stone and concrete.

241. **Stone Box Culverts.** The choice of stone as a material for culverts should depend on the possibility of obtaining a good quality of building stone in the immediate neighborhood. Frequently temporary trestles are used when good stone is unobtainable, with the idea that after the railroad is completed, it will be possible to transport a suitable quality of building stone from a distance and build the culvert under the trestle. The engineer should avoid the mistake of using a poor quality of building stone for the construction of even a culvert, simply because such a stone is readily obtainable.

ASTOR, LENOX
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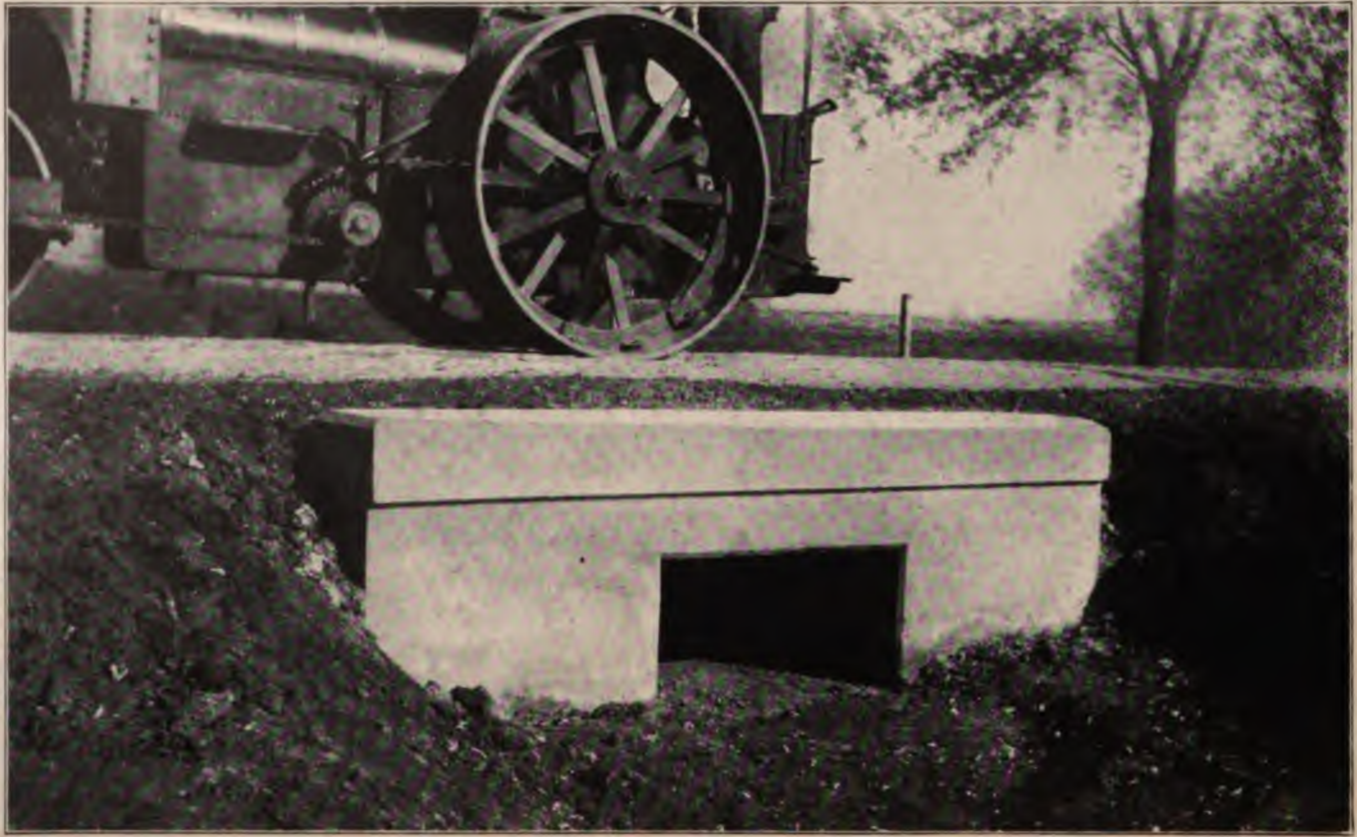


ROAD ROLLER TESTING A CONCRETE CULVERT

Since a culvert always implies a stream of water which will have a scouring action during floods, it is essential that the side walls of culverts should have an ample foundation, which is sunk to such a depth that there is no danger that it will be undermined. There are cases where a bed of quicksand has been encountered, and where the cost of excavating to a firmer soil would be very large. In such a case, it is generally possible to obtain a sufficient foundation by constructing a platform or grillage of timber which underlies the entire culvert, beneath the floor of the culvert. Of course, timber should not be used for the foundation, except in cases where it will always be underneath the level of the ground-water and will therefore always be wet. If the soil has a character such that it will be easily scoured, the floor of the culvert between the side walls should be paved with large pebbles, so as to protect it from scouring action. At both ends of the culvert, there should always be built a vertical wall which should run from the floor of the culvert down to a depth that will certainly be below any possible scouring influence, in order that the side walls and the flooring of the culvert cannot possibly be undermined.

The above specifications apply to all forms of stone culverts, and even to arch culverts, except that in the case of the larger arch culverts the precautions in these respects should be correspondingly observed. When stone culverts are built with vertical side walls which are from 2 to 4 feet apart, they are sometimes capped with large flagstones covering the span between the walls. The thickness of the cover-stone is sometimes determined by an assumption as to the transverse strength of the stone, and by applying the ordinary theory of flexure. The application of this theory depends on the assumption that the neutral axis for a rectangular section is at the center of depth of the stone, and that the modulus of elasticity for tension and compression is the same. Although these assumptions are practically true for steel and even wood, they are far from being true for stone. It is therefore improper to apply the theory of flexure to stone slabs, except on the basis of moduli of rupture which have been experimentally determined from specimens having substantially the same thickness as the thickness proposed. Also, on account of the variability of the actual strength of stones though nominally of the same quality, a very large factor of safety over the supposed ultimate strength of the stone should be used.

The maximum moment at the center of a slab one foot wide equals $\frac{1}{8}$ *Wl*, in which *W* equals the total load on the width of one foot of the slab, and *l* equals the span of the slab, in feet; but by the principles of Mechanics, this moment equals $\frac{1}{6}$ Rh^2, in which *R* equals the modulus of transverse strength, in *pounds per square foot*; and *h* equals the thickness of the stone, in *feet*. Placing these two expressions equal to each other, and solving for *h*, we find:

$$h^2 = \frac{6}{8} \cdot \frac{Wl}{R}$$

$$h = \sqrt{\frac{3}{4} \cdot \frac{Wl}{R}} \ldots\ldots\ldots\ldots\ldots (8)$$

242. *Example.* Assume that a culvert is covered with 6 feet of earth weighing 100 pounds per cubic foot. Assume a live load on top of the embankment equivalent to 500 pounds per square foot, in addition; or that the total load on the top of the slab is equivalent to 1,100 pounds per square foot of slab. Assume that the slab is to have a span (*l*) of 4 feet. Then the total load *W* on a section of the slab one foot wide, will be 1,100 × 4 = 4,400 pounds. Assume that the stone is sandstone, with an average ultimate modulus of 525 pounds per square inch (see Table XII), and that the *safe* value *R* is assumed to be 55 pounds per square inch, or 144 × 55 pounds per square foot. Substituting these values in the above equation for *h*, we find that *h* equals 1.29 feet, or 15.5 inches.

The above problem has been worked out on the basis of the live load which would be found on a railroad. For highways, this could be correspondingly decreased. It should be noted that in the above formula the thickness of the stone *h* varies as the *square root* of the span; therefore, for a span of 3 feet (other things being the same as above), the thickness of the stone *h* equals $15.5 \times \sqrt{\frac{3}{4}} = 13.4$ inches. For a span of 2 feet, the thickness should be $15.5 \times \sqrt{\frac{2}{4}} =$ 11.0 inches.

Owing to the uncertainty of the true transverse strength of building stone, as has already been discussed in the design of Offsets for Footings (see sections 181–183), no precise calculation is possible; and therefore many box culverts are made according to empirical rules, which dictate that the thickness shall be as follows:

For a 2-foot span, 10 inches;
For a 3-foot span, 13 inches;
For a 4-foot span, 15 inches.

These values are slightly less than those computed above.

Although a good quality of granite, and especially of bluestone flagging, will stand higher transverse stresses than those given above for sandstone, the rough rules just quoted are more often used, and are, of course, safer. When it is desired to test the safety of stone already cut into slabs of a given thickness, their strength may be computed from Equation 8, using the values for transverse stresses as already given in Table XII.

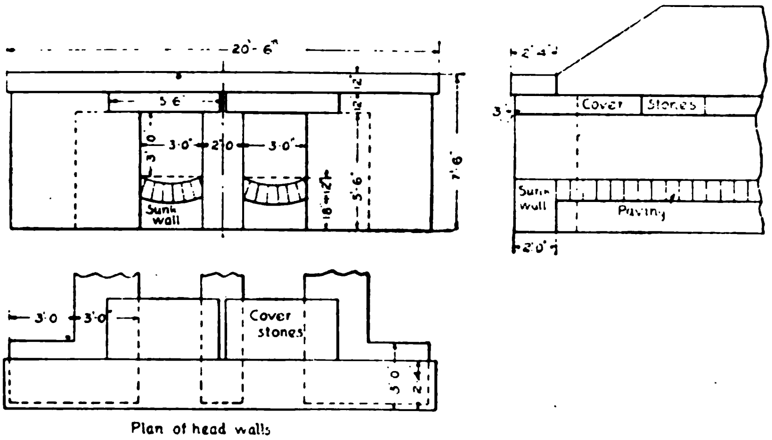


Fig. 74. Double Box Culvert.

243. **Double Box Culverts.** A box culvert with a stone top is generally limited by practice to a span of 4 feet, although it would, of course, be possible to obtain thicker stones which would safely carry the load over a considerably greater span. Therefore, when the required culvert area demands a greater width of opening than 4 feet, and when this type of culvert is to be used, the culvert may be made as illustrated in Fig. 74, by constructing an intermediate wall which supports the ends of the two sets of cover-stones forming the top. A section and elevation of a double box culvert of 3 feet span and a net height of 3 feet, is shown in Fig. 74. This figure also gives details of the wing walls and end walls. The double box culvert illustrated in Fig. 75 has two spans, each of 4 feet. The stone used

was a good quality of limestone. The cover-stones were made 15 inches thick.

244. **End Walls.** The ends of a culvert are usually expanded into end walls for the retention of the embankment. For the larger culverts, this may develop into two wing walls which act as retaining walls to prevent the embankment from falling over into the bed of the stream. An end wall is especially necessary on the upstream end of the culvert, so as to avoid the danger that the stream will scour the bank and work its way behind the culvert walls. The end wall is also carried up above the height of the top of the culvert, so as to guard still further against the washing of earth from the embankment over the end of the culvert into the stream below. All of these details are illustrated in the figures shown.

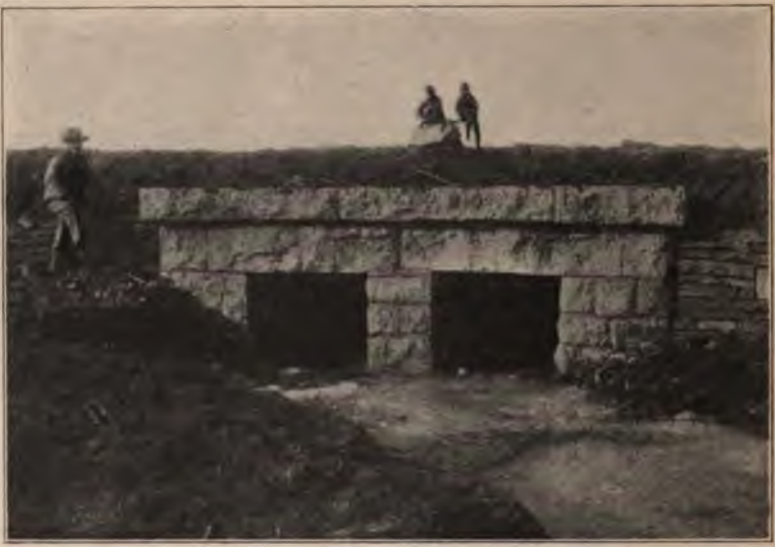


Fig. 75. Double Box Culvert, 4 by 3-Foot.

Box culverts are sometimes constructed as *dry* masonry—that is, without the use of mortar. This should never be done, except for very small culverts and when the stones are so large and regular that they form close, solid walls with comparatively small joints. A dry wall made up of irregular stones cannot withstand the thrusts which are usually exerted by the subsequent expansion of the earth embankment above it.

245. **Plain Concrete Culverts.** Culverts may be made of plain concrete, either in the box form or of an arched type, and having very much the same general dimensions as those already given for stone box culverts. They have a great advantage over stone culverts in that they are essentially monoliths. If the side walls and top are formed in one single operation, the joint between the side walls and top becomes a source of additional strength, and the culverts are therefore much better than similar culverts made of stone. The formula developed above (Equation 8) for the thickness of the concrete slab on top of a box culvert, may be used, together with the modulus of transverse strength as given for concrete in Table XII. This formula will apply, even though the slab for the cover of the culvert is laid after the side walls are built, and the slab is considered as merely resting on the side walls. If the side walls and top are constructed in one operation so that the whole structure is actually a monolith, it may be considered that there is that much additional strength in the structure; but it would hardly be wise to reduce the thickness of the concrete slab by depending upon the continuity between the top and the sides.

246. **Arch Culverts.** Stone arches are frequently used for culverts in cases where the span is not great, and in which the design of the culvert (except for some small details regarding the wing walls) depends only on the span of the culvert. The design of some arch culverts used on the Atchison, Topeka & Santa Fé Railroad (see Fig. 76, and also Fig. 74) is copied from a paper presented to the American Society of Civil Engineers by A. G. Allan, Asso. Mem. Am. Soc. C. E. The span of these arches is 14 feet, and the thickness at the crown is 18 inches. A photograph of one of these arch culverts, which shows also many other details, is reproduced in Fig. 77.

CONCRETE WALKS

247. **Drainage of Foundations.** The excavation should be made to a sufficient depth so as to get below the frost line. The ground should be tamped thoroughly, and the excavation filled with cinders, broken stone, gravel, or brickbat, to within four inches (or whatever thickness of slab is to be used) of the top of the grade. The foundation should be thoroughly rammed, and by using gravel or cinders to make the foundation, a very firm surface can be secured

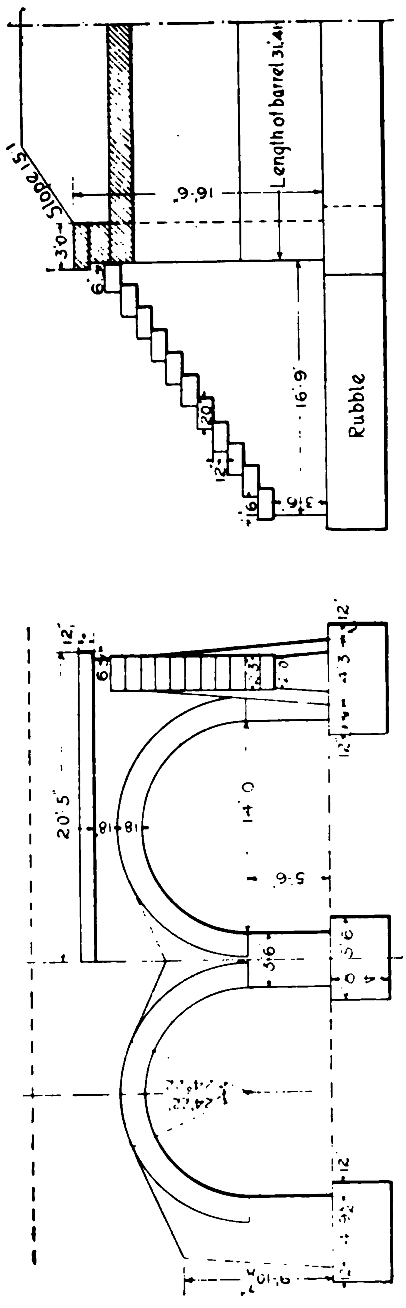


Fig. 76. Stone Arch Culvert. See Fig. 77 for Photographic View.

Side drains should be put in at convenient intervals where outlets can be secured. The foundation is sometimes omitted, even in cold climates, if the soil is porous. Walks laid on the natural soils have proven in many cases to be very satisfactory.

At the Convention of the National Cement Users' Association, held at Buffalo, N. Y., January 21 to 23, 1908, the Committee on Sidewalks, Streets, and Floors presented the following specifications for sidewalk foundations:

"The ground base shall be made as solid and permanent as possible. Where excavations or fills are made, all wood or other materials which will decompose shall be removed, and replaced with earth or other filling like the rest of the foundation. Fills of clay or other material which will settle after heavy rains or deep frost, should be tamped, and laid in layers not more than six inches in thickness, so as to insure a solid embankment which will remain firm after the walk is laid. Embankments should not be less than 2½ feet wider than the walk which is to be laid. When porous materials, such as coal ashes, granulated slag, or gravel, are used, underdrains of tile should be laid to the curb drains or gutters, so as to prevent water accumulating and freezing under the walk and breaking the block."

248. Concrete Base. The concrete for the base of walks is usually composed of 1 part Portland cement, 3 parts sand, and 5 parts stone or gravel. Sometimes, however, a richer mixture is used, consisting of 1 part cement, 2 parts sand, and 4 parts broken stone; but this mixture seems to be richer than what is generally required. The concrete should be thoroughly mixed and rammed, and cut into uniform blocks. See Fig. 78. The size of the broken stone or gravel should not be larger than one inch, varying in size down to ¼ inch, and free from fine screenings or soft stone. All stone or gravel under ⅛ inch is considered sand.

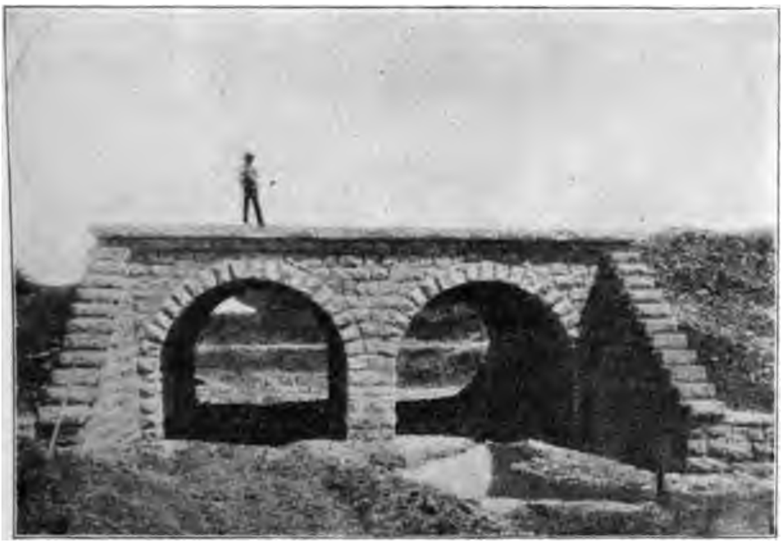


Fig. 77. Double Arch Culvert, 14 by 5½-Foot.

The thickness of the concrete base will depend upon the location, the amount of travel, or the danger of being broken by frost. The usual thickness in residence districts is 3 inches, with a wearing thickness of 1 inch, making a total of 4 inches. In business sections, the walks vary from four to six inches in total thickness, in which the finishing coat should not be less then 1¼ inches thick. The concrete base is cut into uniform blocks.

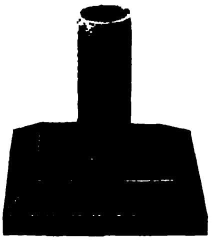


Fig. 78. Square Tamper.

The lines and grades given for walks by the Engineer, should be carefully followed. The mould strips should be firmly blocked and kept perfectly straight to the height of the grade given. The walks usually are laid with a slope of ¼ inch to the foot toward the curb.

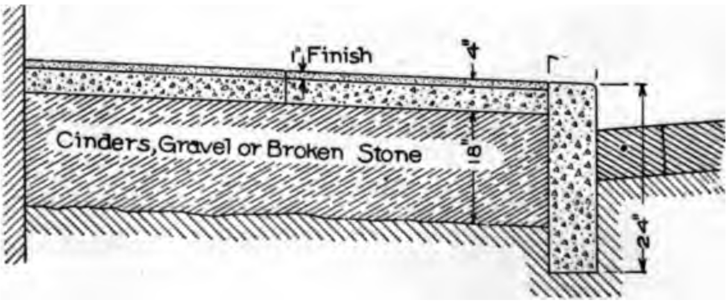


Fig. 79. Concrete Sidewalk and Curb.

The blocks are usually from four to six feet square, but sometimes they are made much larger than these dimensions. The joints made by cutting the concrete should be filled with dry sand, and the exact location of these joints should be marked on the forms. The cleaver or spud that is used in making the joints should not be less than ⅛ of an inch or over ¼ of an inch in thickness.

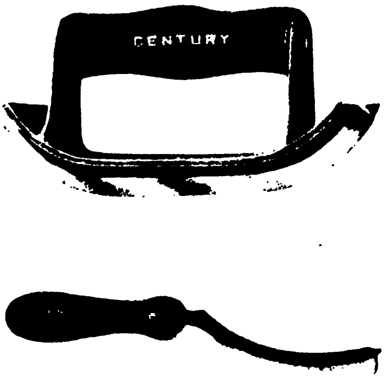


Fig. 80. Jointers.

249. **Top Surface.** The wearing surface usually consists of 1 part Portland cement and 2 parts crushed stone or good, coarse sand, all of which will pass through a ¼-inch mesh screen—thoroughly mixed so as to secure a uniform color. This mixture is then spread over the concrete base to a thickness of one inch, this being done before the concrete of the base has set or become covered with dust. The mortar is leveled off with a straight edge, and smoothed down with a float or trowel after the surface water has been absorbed. The exact time at which the surface should be floated depends upon the setting of the cement,

and must be determined by the workmen; but the final floating is not usually performed until the mortar has been in place from two to five hours and is partially set. This final floating is done first with a wooden float, and afterwards with a metal float or trowel. The top surface is then cut directly over the cuts made in the base, care being taken to cut entirely through the top and base all around each block. The joint is then finished with a jointer, Fig. 80, and all edges rounded or beveled. Care should be taken in the final floating or finishing, not to overdo it, as too much working will draw the cement to the surface, leaving a thin layer of neat cement, which is likely to peel off. Just before the floating, a very thin layer of *dryer* consisting of dry cement and sand mixed in the proportion of one to one, or even richer, is frequently spread over the surface; but this is generally undesirable, as it tends to make a glossy walk. A dot roller or line roller, Figs. 81 and 82, may be employed to relieve the smoothness.

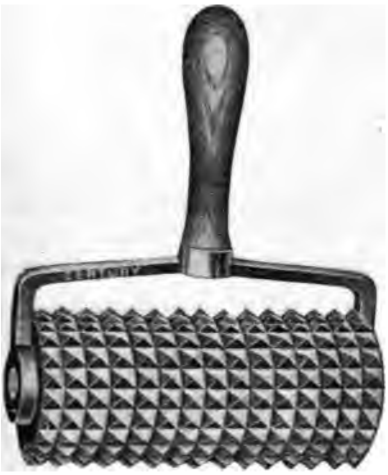


Fig. 81. Brass Dot Roller.

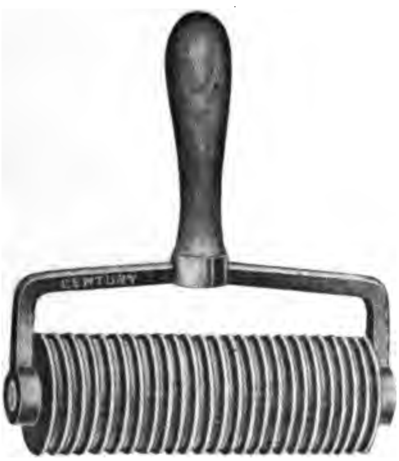


Fig. 82. Brass Line Roller.

At the meeting of the National Cement Users' Association already referred to, the Committee on Sidewalks, Floors, and Streets recommended the following specifications for the *top coat*:

> "Three parts high-grade Portland cement and five parts clean, sharp sand, mixed dry and screened through a No. 4 sieve. In the top coat, the amount of water used should be just enough so that the surface of the walk

> can be tamped, struck off, floated, and finished within 20 minutes after it is spread on the bottom coat; and when finished, it should be solid and not quaky."

In the January, 1907, number of *Cement*, Mr. Albert Moyer, Assoc. M. Am. Soc. C. E., discussing the subject of cement sidewalk pavements, gives specifications for monolithic slab for paving purposes. For an example of this construction, he gives the pavement around the Astor Hotel, New York:

> "As an alternative, and instead of using a top coat, make one slab of selected aggregates for base and wearing surface, filling in between the frames concrete flush with established grade. Concrete to be of selected aggregates, all of which will pass through a ¾-inch mesh sieve; hard, tough stones or pebbles, graded in size; proportions to be 1 part cement, 2½ parts crushed hard stone screenings or coarse sand, all passing a ¾-inch mesh, and all collected on a ¼-inch mesh. Tamped to an even surface, prove surface with straight edge, smooth down with float or trowel, and in addition a natural finish can be obtained by scrubbing with a wire brush and water while concrete is 'green,' but after final set."

250. **Seasoning.** The wearing surface must be protected from the rays of the sun by a covering which is raised a few inches above the pavement so as not to come in contact with the surfaces. After the pavement has set hard, sprinkle freely two or three times a day for a week or more.

251. **Cost.** The cost of concrete sidewalks is variable. The construction at each location usually requires only a few days work; and the time and expense of transporting the men, tools, and materials make an important item. One of the skilled workmen should be in charge of the men, so that the expense of a foreman will not be necessary. The amount of walk laid per day is limited by the amount of surface that can be floated and troweled in a day. If the surfacers do not work overtime, it will be necessary to stop concreting in the middle of the afternoon, so that the last concrete placed will be in condition to finish during the regular working hours. The work of concreting may be continued considerably later in the afternoon if a dryer concrete is used in mixing the top coat, and only enough water is used so that the surface can be floated and finished soon after being placed. The men who have been mixing, placing, and ramming concrete can complete their day's work by preparing and ramming the foundations for the next day's work.

The contract price for a well-constructed sidewalk 4 to 5 inches in thickness, with a granolithic finish, will vary from 15 cents to 30 cents per square foot.

CONCRETE CURB

252. The curb is usually built just in advance of the sidewalk. The foundation is prepared similarly to that of walks. The curb is divided into lengths similar to that of the walk; and the joints between the blocks, and also between the walk and the curb, are made similar to the joints between the blocks of the walk. The concrete is generally composed of 1 part Portland cement, 3 parts sand, and 5 parts stone, although a richer mixture is sometimes used. A facing, on the part exposed to wear, of mortar or granolithic finish, will improve the wearing qualities of the curb.

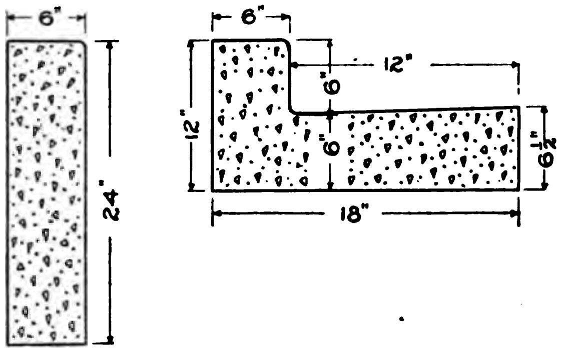


Fig. 83. Typical Curb Sections.

253. **Types of Curbing.** There are two general types of curb used—a curb rectangular in section, and a combined curb and gutter; both types are shown in Fig. 83. The foundation for either type is constructed in the same manner. Both these types of curb are made in place or moulded and set in place like stone curb, but the former method is preferable. A metal corner is sometimes laid in the exposed edge of the curb to protect it from wear.

254. **Construction.** The construction of the rectangular section is a simple process, but requires care to secure a good job. This is usually about 6 inches wide and from 20 to 30 inches deep. After the foundation has been properly prepared, the forms are set in place. Fig. 84 shows the section of a curb 6 inches wide and 24 inches deep, and the forms as they are often used. The forms for the front and back each consist of three planks 1⅞ inches thick and 8 inches wide, and are surfaced on the side next the concrete. They are held in place at the bottom by the two 2 by 4-inch stakes, and at the top the

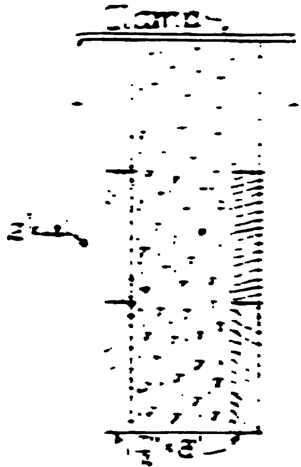


Fig. 84. Forms for [illegible]

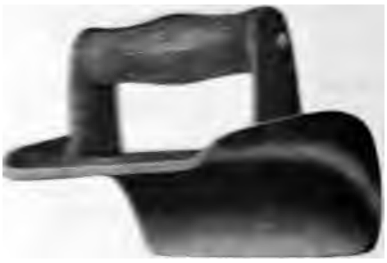


Fig. 85. Curb Edger.

stakes are kept from spreading by a clamp. A sheet-iron plate ¼ inch thick is inserted every 6 feet, or at whatever distance the joints are made. After the concrete has been placed and rammed, and has set hard enough to support itself, the plate and front forms are removed, and the surface and top are finished smooth with a trowel; the corner being rounded with an edger as shown in Fig. 85. The joint is usually plastered over, and acts as an expansion joint. The forms on the back are not removed until the concrete is well set. If a mortar or granolithic finish is used, a piece of sheet iron is placed in the form one inch from the facing, and mortar is placed between the sheet iron and the front form, and the coarser concrete is placed back of the sheet iron (Fig. 86). The sheet iron is then withdrawn and the two concretes thoroughly tamped.

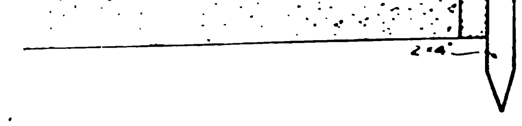


Fig. 86. Forms for Curb and Gutter.

Fig. 86 shows the section of a combined curb and gutter, and the forms that are necessary for its construction. This combination is often laid on a porous soil without any special foundation, with fair results. A 1⅞-inch plank 12 inches wide is used for the back form, and is held in place at the bottom by pegs. The front form consists of a plank 1⅞ by 6 inches, and is held in place by pegs. Before the concrete is placed, two sheet-iron plates, cut as shown in the figure, are placed in the forms, six feet to eight feet apart. After the concrete for the gutter and the lower part of the curb is placed and rammed, a 1⅞-inch plank is placed against these plates and held in place by screw clamps (Fig. 86). The upper part of the curb is then moulded. When the concrete is set enough to stay in place, the front forms and plates are removed, and the surface is treated in the same manner as described for the other type of curb.

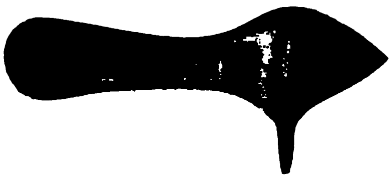


Fig. 87. Radius Tool.

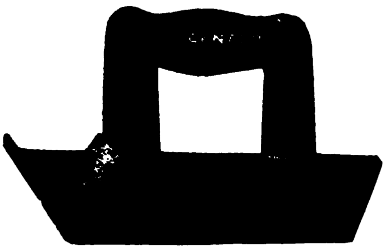
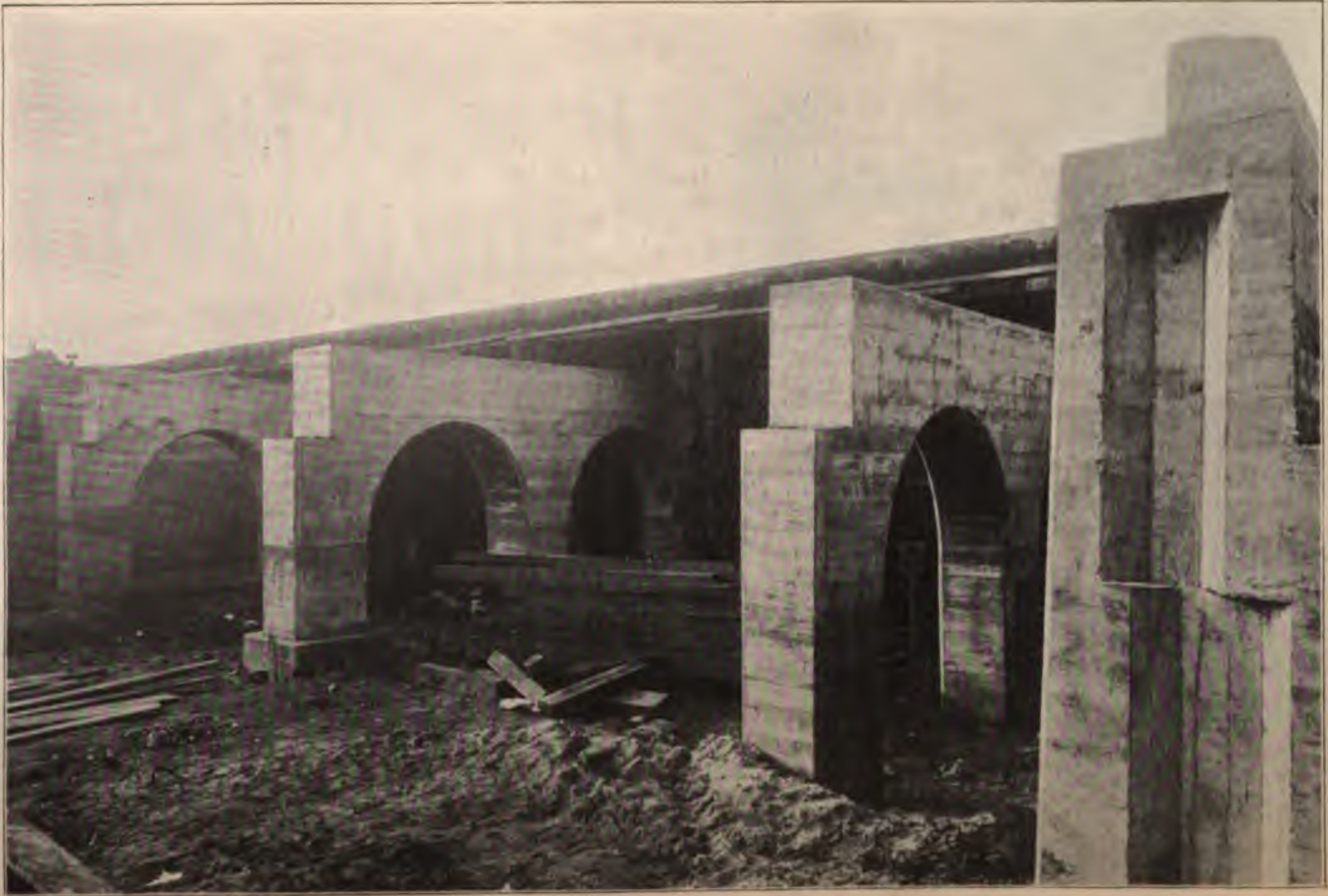


Fig. 88. Inside Angle Tool.

255. **Cost.** The cost of concrete curb will depend upon the conditions under which it is made. Under ordinary circumstances, the contract price for rectangular curbing 6 inches wide and 24 inches deep will be about $0.60 per linear foot; or $0.80 per linear foot for curb 8 inches wide and 24 inches deep. Under favorable conditions on large jobs, 6-inch curbing can be constructed for $0.40 or $0.45 per linear foot. These prices include the excavation that is required below the street grade.

The cost of the combined curb and gutter is about 10 to 20 per cent more than that of the rectangular curbing. In addition to having a larger surface to finish, the combined curb and gutter requires more material, and therefore more work, to construct it.



REINFORCED-CONCRETE BENTS FOR SUPPORTING REINFORCED-CONCRETE BRIDGE SLABS

Reinforced with "Johnson" corrugated bars. Track elevation of Illinois Central Railway, Chicago: A. S. Baldwin, Chief Engineer; R. E. Gaut, Bridge Engineer.

Courtesy of Expanded Metal & Corrugated Bar Co., St. Louis, Mo.

MASONRY AND REINFORCED CONCRETE

PART III

REINFORCED CONCRETE

GENERAL THEORY OF FLEXURE

256. **Introduction.** The theory of flexure in reinforced concrete is exceptionally complicated. A multitude of simple rules, formulæ, and tables for designing reinforced-concrete work have been proposed, some of which are sufficiently accurate and applicable *under certain conditions.* But the effect of these various conditions should be thoroughly understood. Reinforced concrete should not be designed by "rule-of-thumb" engineers. It is hardly too strong a statement to say that a man is criminally careless and negligent when he attempts to design a structure on which the safety and lives of people will depend, without thoroughly understanding the theory on which any formula he may use is based. The applicability of all formulæ is so dependent on the quality of the steel and of the concrete, and on many of the details of the design, that a blind application of a formula is very unsafe. Although the greatest pains will be taken to make the following demonstration as clear and plain as possible, it will be necessary to employ symbols, and to work out several algebraic formulæ on which the rules for designing will be based. The full significance of many of the terms mentioned below may not be fully understood until several subsequent paragraphs have been studied:

b = Breadth of concrete beam;

d = Depth from compression face to center of gravity of the steel;

A = Area of the steel;

p = Ratio of area of steel to area of concrete above the center of gravity of the steel, generally referred to as *percentage of reinforcement.*

$= \frac{A}{b\,d}$;

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E_s - Modulus of elasticity of steel;

E_c - *Initial* modulus of elasticity of concrete;

$r = \frac{E_s}{E_c}$ = Ratio of the moduli;

s - Tensile stress per unit of area in steel;

c - Compressive stress per unit of area in concrete at the outer fibre of the beam: this may vary from zero to c';

c' = Ultimate compressive stress per unit of area in concrete—the stress at which failure might be expected;

ϵ_s Deformation per unit of length in the steel:

ϵ_c " " " " " in outer fibre of concrete;

ϵ_c' " " " " " in outer fibre of concrete when crushing is imminent;

ϵ_c'' = Deformation per unit of length in outer fibre of concrete under a certain condition (described later):

$q = \frac{\epsilon_c}{\epsilon_c}$ Ratio of deformations;

k - Ratio of depth from compressive face to the neutral axis to the total effective depth d;

x - Distance from compressive face to center of gravity of compressive stresses;

ΣX Summation of horizontal compressive stresses;

M Resisting moment of a section.

257. Statics of Plain Homogeneous Beams. As a preliminary to the theory of the use of reinforced concrete in beams, a very brief discussion will be given of the statics of an ordinary homogeneous beam. Let $A B$ (Fig. 89) represent a beam carrying a uniformly distributed load W; then the beam is subjected to transverse stresses. Let us imagine that *one-half* of the beam is a "free body" in space, and is acted on by exactly the same external forces; we shall also assume the forces C and T (acting on the exposed section), which are just such forces as are required to keep that half of the beam in equilibrium.

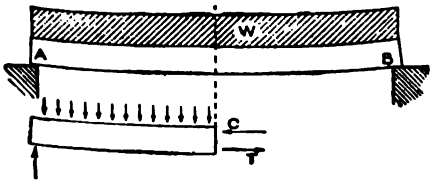
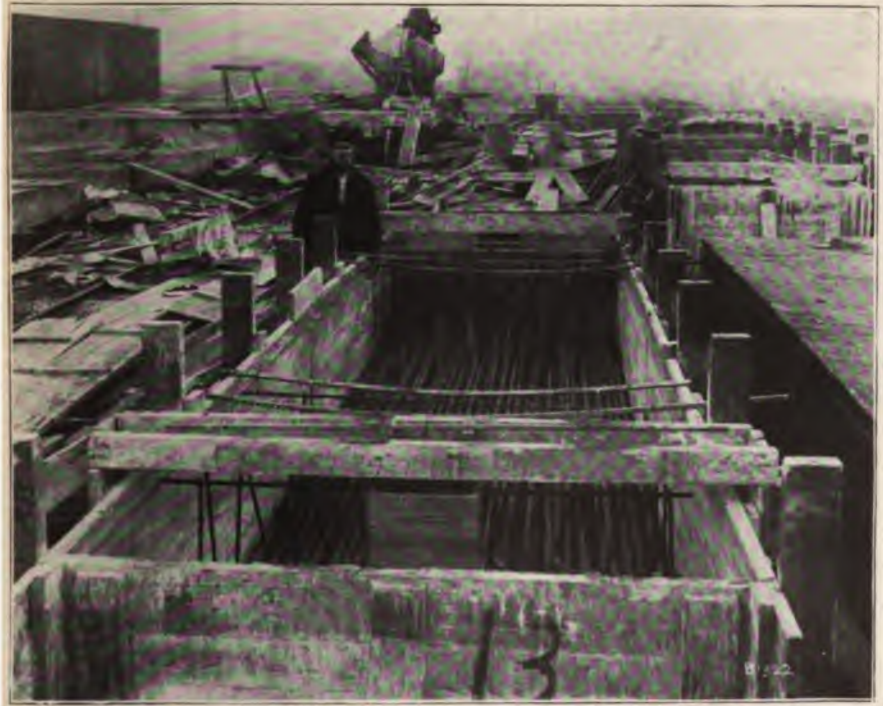


Fig. 89. Beam Carrying Uniformly Distributed Load.

These forces, and their direction, are represented in the lower diagram by arrows. The load W is represented by the series of small, equal, and equally spaced vertical arrows pointing downward. The reaction of the abutment *against the beam* is an *upward* force,

Reinforced-Concrete Bridge Slabs in Storage Yard of Illinois Central Railway, Chicago.



Form Box, with Reinforcing Bars, for Construction of Slabs Shown Above.

CONSTRUCTION OF REINFORCED-CONCRETE BRIDGE SLABS

Slabs measure 25 ft. by 6 ft. 2 in. by 2 ft. 10 in., reinforced with "Johnson" corrugated bars; used on 25-ft. spans for track elevation work. Designed by R. E. Gaut, Bridge Engineer, and A. S. Baldwin, Chief Engineer, Illinois Central Railway.

shown at the left. The forces acting on a *section* at the center are the equivalent of the two equal forces *C* and *T*.

The force *C*, acting at the top of the section, must act toward the left, and there is therefore compression in that part of the section. Similarly, the force *T* is a force acting toward the right, and the fibres of the lower part of the beam are in tension. For our present purpose we may consider that the forces *C* and *T* are in each case the resultant of the forces acting on a very large number of "fibres." The stress in the outer fibres is of course greatest. At the center of the height, there is neither tension nor compression. This is called the *neutral axis* (see Fig. 90).

Let us consider for simplicity a very narrow portion of the beam, having the full length and depth, but so narrow that it includes only one set of fibres, one above the other, as shown in Fig. 91. In the case of a plain, rectangular, homogeneous beam, the stresses in the fibres would be as given in Fig. 90; the neutral axis would be at the center of the height, and the stress at the bottom and the top would be equal but opposite. If the section were at the center of the beam, with a uniformly distributed load (as indicated in Fig. 89), the *shear* would be zero.

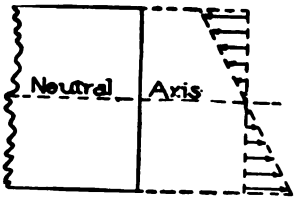


Fig. 90. Position of Neutral Axis.

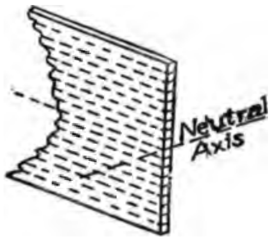


Fig. 91. Neutral Axis in Narrow Beam.

A beam *may* be constructed of plain concrete; but its strength will be very small, since the tensile strength of concrete is comparatively insignificant. Reinforced concrete utilizes the great tensile strength of steel, in combination with the compressive strength of concrete. It should be realized that the essential qualities are *compression* and *tension*, and that (other things being equal) the cheapest method of obtaining the necessary compression and tension is the most economical.

258. **Economy of Concrete for Compression.** The ultimate compressive strength of concrete is generally 2,000 pounds or over per square inch. With a factor of safety of four, a working stress of 500 pounds per square inch may be considered allowable. We may estimate that the concrete costs twenty cents per cubic foot, or $5.40 per cubic yard. On the other hand, we may estimate that the steel, placed in the work, costs about three cents per pound. It will weigh 480 pounds per cubic foot; therefore the steel costs $14.40 per cubic foot, or 72 times as much as an equal *volume* of concrete or an equal *cross-section* per unit of length. But the steel can safely withstand a compressive stress of 16,000 pounds per square inch, which is 32 times the safe working load on concrete. Since, however, a given volume of steel costs 72 times an equal volume of concrete, the cost of a given compressive resistance in steel is $\frac{72}{32}$ (or 2.25) times the cost of that resistance in concrete. Of course, the above assumed unit-prices of concrete and steel will vary with circumstances. The advantage of concrete over steel for compression may be somewhat greater or less than the ratio given above, but the advantage is almost invariably with the concrete. There are many other advantages in addition, which will be discussed later.

259. **Economy of Steel for Tension.** The ultimate tensile strength of ordinary concrete is rarely more than 200 pounds per square inch. With a factor of safety of four, this would allow a working stress of only 50 pounds per square inch. This is generally too small for practical use, and certainly too small for economical use. On the other hand, steel may be used with a working stress of 16,000 pounds per square inch, which is 320 times that allowable for concrete. Using the same unit-values for the cost of steel and concrete as given in the previous section, even if steel costs 72 times as much as an equal volume of concrete, its real tensile value economically is $\frac{320}{72}$ (or 4.44) times as great. Any reasonable variation from the above unit-values cannot alter the essential truths of the economy of steel for tension and of concrete for compression. In a reinforced-concrete beam, the steel is placed in the tension side of the beam. Usually it is placed from one to two inches from the outer face, with the double purpose of protecting the steel from corrosion or fire, and also to better insure the union of the concrete and the steel. But the concrete below the steel is not considered in the numerical calcu-

lations. Even the concrete which is between the steel and the neutral axis (whose position will be discussed later), is chiefly useful in transmitting the tension in the steel to the concrete. Although such concrete is theoretically subject to tension, and does actually contribute its share of the tension when the stresses in the beam are small, the proportion of the necessary tension which the concrete can furnish when the beam is heavily loaded, is so very small that it is usually ignored, especially since such a policy is on the side of safety, and also since it greatly simplifies the theoretical calculations and yet makes very little difference in the final result. We may therefore consider that in a unit-section of the beam, as in Fig. 92, the concrete above the neutral axis is subject to compression, and that the tension is furnished entirely by the steel.

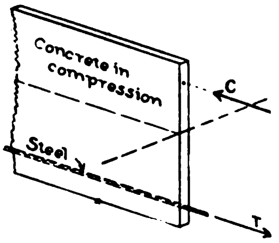


Fig. 92. Transmission of Tension in Steel to Concrete

260. Elasticity of Concrete in Compression. In computing the transverse stresses in a wooden beam or steel I-beam, it is assumed that the modulus of elasticity is uniform for all stresses within the elastic limit. Experimental tests have shown this to be so nearly true that it is accepted as a mechanical law. This means that if a force of 1,000 pounds is required to stretch a bar .001 of an inch, it will require 2,000 pounds to stretch it .002 of an inch. Similar tests have been made with concrete, to determine the law of its elasticity. Unfortunately, concrete is not so uniform in its behavior as steel. The results of tests are somewhat contradictory. Many engineers have argued that the elasticity is so nearly uniform that it may be considered to be such within the limits of practical use. But all experimenters who have tested concrete by measuring the proportional compression produced by various pressures, agree that the additional shortening produced by an additional pressure, say of 100 pounds per square inch, is greater at higher pressures than at low pressures.

A test of this sort may be made substantially as follows: A square or circular column of concrete at least one foot long is placed in a testing machine. A very delicate micrometer mechanism is fastened to the concrete by pointed screws of hardened steel. These points are originally at a known distance apart—say 8 inches. When the concrete is compressed, the distance between these points will be slightly less. A very delicate mechanism will permit this distance to be measured as closely as the ten-thousandth part of an inch, or to about $\frac{1}{100,000}$ of the length. Suppose that the various pressures per square inch, and the proportionate compressions, are as given in the following tabular form:

PRESSURE PER SQUARE INCH	PROPORTIONATE COMPRESSION
200 pounds	.00010 of total length
400 "	.00020 " " "
600 "	.00032 " " "
800 "	.00045 " " "
1,000 "	.00058 " " "
1,200 "	.00062 " " "
1,400 "	.00090 " " "
1,600 "	.00112 " " "

We may plot these pressures and compressions as in Fig. 93, using any convenient scale for each. For example, for a pressure of 800 pounds per square inch, we select the vertical line which is at the horizontal distance from the origin *O* of 800, according to the scale adopted. Scaling off on this vertical line the ordinate .00045, according to the scale adopted for compressions, we have the position of one point of the curve. The other points are obtained similarly. Although the points thus obtained from the testing of a single block of concrete would not be considered sufficient to establish the law of the elasticity of concrete in compression, a study of the curves which may be drawn through the series of

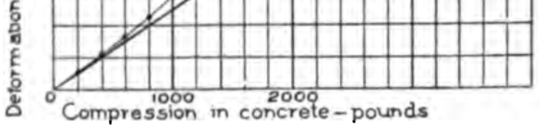


Fig. 93. Curve of Pressures and Compressions.

points obtained for each of a large number of blocks, shows that these curves will average very closely to parabolas that are tangent to the initial modulus of elasticity, which is here represented in the diagram by a straight line running diagonally across the figure.

It is generally considered that the axis of the parabola will be a horizontal line when the curve is plotted according to this method. The position of the vertex of the parabola cannot be considered as definitely settled. Professor Talbot has computed the curve as if the vertex were at the point of the ultimate compression of the concrete, although he conceded that the vertex might be in an imaginary position corresponding to a compression in the concrete higher than that which the concrete could really endure. Mr. A. L. Johnson, another noted authority, bases his computation of formulæ on the assumption that the ultimate compressive strength of the concrete is two-thirds of the value which would be required to produce that amount of compression in case the initial modulus of elasticity were the true value for all compressions. In other words, looking at Fig. 93, if *o c* is a line representing the initial modulus of elasticity, then, if the elasticity were uniform throughout, it would require a force of about 2,340 pounds (or *d f*) to produce a proportionate compression of .00132 of the length (represented by *o d*). Actually that compression will be produced when the pressure equals *d e*, which is $\frac{2}{3}$ of *d f*. It should not be forgotten that the above numerical values are given merely for illustrative purposes. They would, if true, represent a rather weak concrete. The following theory is therefore based on the assumption that the stress-strain curve is represented by the parabolic curve *o e* (see Fig 93); and that the ultimate stress per square inch in the concrete c' is represented by *d e*, which is $\frac{2}{3}$ of the compressive stress that would be required to produce that proportionate compression if the modulus of elasticity of the concrete were uniformly maintained at the value it has for very low pressures.

261. **Theoretical Assumptions.** The theory of reinforced-concrete beams is based on the usual assumptions that:

(*a*) The loads are applied at right angles to the axis of the beam. The usual vertical gravity loads supported by a horizontal beam fulfil this condition.

(*b*) There is no resistance to free horizontal motion. This condition is seldom, if ever, exactly fulfilled in practice. The more rigidly the beam is held at the ends, the greater will be its strength above that computed by the simple theory. Under ordinary conditions the added strength is quite inde-

[illegible] appreciation that it adds indef- [illegible] together without breaking the bond [illegible] beam before bending is [illegible]

[illegible] exaggerated form the essential [illegible] section [illegible] in the unstrained condition is changed to the [illegible] d when the load is applied. The compression at the top $= a\,a' =$ [illegible] The neutral axis is unchanged. The concrete at the bottom is stretched an amount $=$ [illegible] $= c\,d'$, while the stretch in the steel equals [illegible] The compression in the concrete between the neutral axis and the top is proportional to the distance from the neutral axis.

[illegible] is given a side view of the beam with special reference to the deformation of the fibres. Since the fibres between the neutral axis and the compressive face are compressed proportionally, then if $c\,c$ represents the linear compression of the outer fibre, the shaded lines represent [illegible] the same [illegible] fibres.

[illegible] would be in the outer [illegible] to all stresses. But [illegible] compression $c\,c$ is proportion- [illegible] the lesser compressions, the [illegible] may be represented by a line *v n*, and the pressure on the intermediate fibres by the ordinates to the curve *v N*.

In Fig. 96, *a* and *b*, are shown a pair of figures corresponding with those of Fig. 95, except that the compressive deformation of the concrete in the outer fibre *a a′* is only *one-half* of the value in Fig. 95. But it will require about three-fourths as much pressure to produce one-half as much compression. In Fig. 96, *v′ n′* is therefore three-fourths of *v n* in Fig. 95. The student should note that *k′* here differs slightly from *k*, which means that the position of the neutral axis varies with the conditions.

262. **Summation of the Compressive Forces.** The summation of the compressive forces is evidently indicated by the area of the shaded portion in Fig. 97. The curve *v N* is a portion of a parabola. The area of the shaded portion between the curve *v N* and the straight line *v N*, equals one-third of the area of the triangle *m N v*. The area of the triangle *v n N* = ½ *c kd*. Therefore, for the total shaded area, we have:

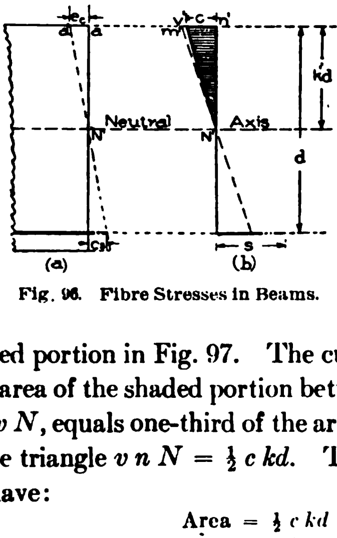


Fig. 96. Fibre Stresses in Beams.

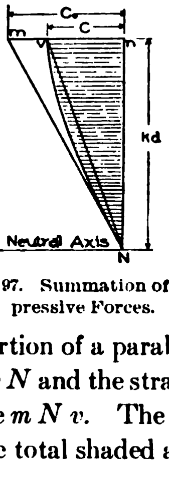


Fig. 97. Summation of Compressive Forces.

$$
\begin{aligned}
\text{Area} &= \tfrac{1}{2}\, c\, kd + \tfrac{1}{3}\,(c_0 - c)\,\tfrac{1}{2}\, kd,\\
&= \tfrac{1}{2}\, kd\,(c + \tfrac{1}{3}\, c_0 - \tfrac{1}{3}\, c),\\
&= \tfrac{1}{2}\, kd\,(\tfrac{2}{3}\, c + \tfrac{1}{3}\, c_0).
\end{aligned}
$$

But in this case, $c_0 = E_c \epsilon_c$; therefore,

$$\text{Area} = \tfrac{1}{2}\, kd\,(\tfrac{2}{3}\, c + \tfrac{1}{3}\, E_c \epsilon_c) \cdots\cdots\cdots\cdots (9)$$

In Fig. 98 has been redrawn the parabola of Fig. 93, in which *o* is the vertex of the parabola. Here c'' is the force which would produce a compression of ϵ_c'' provided the concrete could endure such a pressure without rupture. If the initial modulus of elasticity applied to all stresses, the required force would be the line $E_c\, \epsilon_c''$. And $c'' = \tfrac{1}{2}\, E_c\, \epsilon_c''$.

It is one of the well-known properties of the parabola that abscissæ are proportional to the squares of the ordinates, or that (in this case):

$$k'd : m\,n :: \overline{o\,k}^2 : \overline{o\,m}^2$$

Transforming to the symbols, we have:

$$(c'' - c) : c'' :: (\epsilon_c'' - \epsilon_c)^2 : \epsilon_c''^2 ;$$

$$c'' - c = c'' \frac{(\epsilon_c'' - \epsilon_c)^2}{\epsilon_c''^2}$$

$$c'' - c = c''(1 - q)^2, \text{ since } \frac{\epsilon_c}{\epsilon_c''} = q.$$

$$c = c'' \left\{ 1 - (1 - q)^2 \right\} ;$$

$$= c''(2q - q^2) ;$$

$$= \tfrac{1}{2} E_c \epsilon_c''(2q - q^2), \text{ since } c'' = \tfrac{1}{2} E_c \epsilon_c'' ; \text{ and also, since } \epsilon_c'' = \frac{\epsilon_c}{q}$$

$$= E_c \epsilon_c (1 - \tfrac{1}{2} q) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{(10)}$$

Substituting this value of c in Equation 9, we have:

$$\text{Area} = \tfrac{1}{2} kd \left\{ \tfrac{2}{3} E_c \epsilon_c (1 - \tfrac{1}{2} q) + \tfrac{1}{3} E_c \epsilon_c \right\}$$

$$= \tfrac{1}{2} kd \left\{ E_c \epsilon_c (1 - \tfrac{1}{3} q) \right\}$$

The summation of the horizontal forces (ΣX) within the shaded area, is evidently expressed by the above "area" multiplied by the breadth of the beam (b). Therefore,

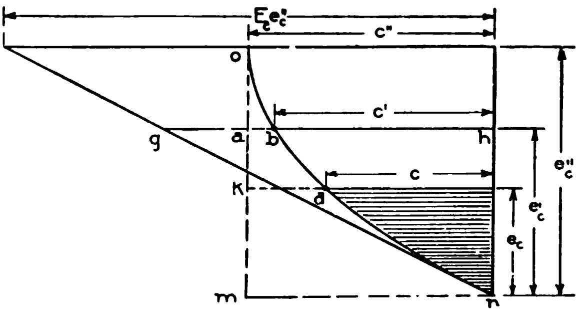


Fig. 98. Analysis of Compressive Stresses.

$$\Sigma X = \tfrac{1}{2} (1 - \tfrac{1}{3} q) E_c \epsilon_c b\,kd \quad \ldots\ldots\ldots \quad \mathbf{(11)}$$

In order to avoid the complication resulting from the attempt to develop formulæ which are applicable to all kinds of assumptions, it will be at once assumed, as previously referred to, that the ultimate compressive strength of the concrete is $\frac{2}{3}$ of the value which would be required to produce that amount of compression in case the initial modulus of elasticity were the true value for all compressions.

The proof that q will equal $\frac{2}{3}$ under these conditions, is perhaps determined most easily by computing the ratio of $b\,h$ to $g\,h$ (see Fig. 98) when $o\,a$ is *assumed* to be $\frac{1}{3}$ of $o\,m$. In this case, from the properties of the parabola, $a\,b = \frac{1}{9} m\,n$; $c' = \frac{8}{9} m\,n = \frac{8}{9} c'' = \frac{4}{9} E_c \epsilon_c''$.

But when $o\,a = \frac{1}{3}$ of om, $g\,h = \frac{2}{3}\,E_c\,\epsilon_c'' = \frac{4}{9}\,E_c\,\epsilon_c''$.

Therefore $c' = \frac{2}{3}\,g\,h$. But when $o\,a = \frac{1}{3}$ of $o\,m$, $\frac{\epsilon_c{}'}{\epsilon_c{}''} = \frac{2}{3}$.

Therefore, when $c' = \frac{2}{3}\,gh$, $q = \frac{2}{3}$.

It has already been shown that $c'' = \frac{1}{2}\,E_c\,\epsilon_c''$, and also that $\epsilon_c'' = \frac{\epsilon_c}{q}$. Therefore $\frac{1}{2}\,E_c\,\epsilon_c = c''q$. It has also been shown that $c' = \frac{8}{9}\,c''$, or that $c'' = \frac{9}{8}c'$. Therefore $\frac{1}{2}\,E_c\,\epsilon_c = \frac{9}{8}\,c'q$.

Substituting this value in Equation 11, we have for the summation of the compressive forces above the neutral axis, under such conditions:

$$\Sigma X = \tfrac{9}{8}\,(1 - \tfrac{1}{3}\,q)\,q\,c'b\,kd \,\ldots\ldots\ldots\, \textbf{(12)}$$

Substituting the further condition that $q = \frac{2}{3}$, we have:

$$\Sigma X = \tfrac{7}{12}\,c'b\,kd \,\ldots\ldots\ldots\ldots\, \textbf{(13)}$$

263. **Center of Gravity of Compressive Forces.** This is also called the *centroid of compression.* The theoretical determination of this center of gravity is virtually the same as the determination of the center of gravity of the shaded area shown in Figs. 96 and 97. The general method of determining this center of gravity requires the use of differential calculus, and is a very long and tedious calculation. But the final result may be reduced to a surprisingly simple form, as expressed in the following equation:

$$x = kd\,\frac{4 - q}{12 - 4q}\,.$$

Assuming, as explained above, the value of $q = \frac{2}{3}$, this reduces to:

$$x = .357\,kd \,\ldots\ldots\ldots\ldots\, \textbf{(14)}$$

When q equals zero, the value of x equals $.333\,kd$; and, at the other extreme, when $q = 1$, $x = .375\,kd$.

There is, therefore, a very small range of inaccuracy in adopting the value of $q = \frac{2}{3}$ for *all* computations.

264. **Position of the Neutral Axis.** According to one of the fundamental laws of mechanics, the sum of the horizontal tensile forces must be equal and opposite to the sum of the compressive forces. Ignoring the very small amount of tension furnished by the concrete below the neutral axis, the tension in the steel $= As = pbds =$ the total compression in the concrete. Therefore, applying Equation 11,

$$p\,b\,d\,s = \tfrac{1}{2}\,(1 - \tfrac{1}{3}\,q)\,E_c\,\epsilon_c\,k\,b\,d$$

But $s = E_s\,\epsilon_s$; therefore,

$$p\,E_s\,\epsilon_s = \tfrac{1}{2}\,(1 - \tfrac{1}{3}\,q)\,E_c\,\epsilon_c k$$

But $\frac{E_s}{E_c} = r$; and by proportional triangles, as shown in Fig. 96,

$$\frac{\epsilon_c}{kd} = \frac{\epsilon_s}{d - kd}\ ; \text{ or } \epsilon_c = \epsilon_s \frac{k}{1-k}\,.$$

Making these substitutions, we have:

$$p\,r = \tfrac{1}{2}(1 - \tfrac{1}{3}q)\,\frac{k^2}{1-k} \quad \ldots\ldots\ldots \textbf{(15)}$$

Solving this quadratic for k, we have:

$$k = \sqrt{\frac{2\,p\,r}{(1 - \frac{1}{3}q)} + \frac{p^2r^2}{(1 - \frac{1}{3}q)^2}} - \frac{p\,r}{(1 - \frac{1}{3}q)} \quad \ldots\ldots \textbf{(16)}$$

Equation **16** is a perfectly general equation which depends for its accuracy only on the assumption that the law of compressive stress to compressive strain is represented by a parabola. The equation shows that k, the ratio determining the position of the neutral axis, depends on three variables—namely, the percentage of the steel (p), the ratio of the moduli of elasticities (r), and the ratio of the deformations in the concrete (q). These must all be determined more or less accurately before we can know the position of the neutral axis.

On the other hand, if it were necessary to work out Equation **16**, as well as many others, for every computation in reinforced concrete, the calculations would be impracticably tedious. Fortunately the extreme range in k for any one ratio of moduli of elasticities, is only a few per cent, even when q varies from 0 to 1. We shall therefore simplify the calculations by using the constant value $q = \frac{2}{3}$, as explained above.

Substituting $q = \frac{2}{3}$ in Equation **16**, we have:

$$k = \sqrt{\frac{18}{7}\,p\,r + \frac{81}{49}\,p^2r^2} - \frac{9}{7}\,p\,r \quad \ldots\ldots \textbf{(17)}$$

The various values for the ratio of the moduli of elasticity (r) are discussed in the succeeding section. The values of k for various values of r and p, and for the uniform value of $q = \frac{2}{3}$, have been computed in the following tabular form. Five values have been chosen for r, in conjunction with nine values of p, varying by 0.2 per cent and covering the entire practicable range of p, on the basis of which values k has been worked out in the tabular form. Usually the value of k can be determined directly from the table. By interpolating between two values in the table, any required value within the limits of ordinary practice can be determined with all necessary accuracy.

TABLE XIII

Values of *k* for Various Values of *r* and *p* (Parabolic Formulæ)

	p								
r	.020	.018	.016	.014	.012	.010	.008	.006	.004
10	.505	.487	.468	.446	.422	.395	.362	.323	.274
12	.536	.517	.497	.475	.450	.422	.388	.348	.295
15	.574	.555	.535	.513	.488	.458	.422	.379	.323
20	.623	.604	.583	.561	.535	.505	.468	.421	.362
40	.736	.718	.700	.678	.654	.623	.584	.535	.468

265. **Ratio of Moduli.** Theoretically there is an indefinite number of values of *r*, the ratio of the moduli of elasticity of the steel and the concrete. The modulus for steel is fairly constant at about 29,000,000 or 30,000,000. The value of the *initial* modulus for concrete varies according to the quality of the concrete, from 1,500,000 to 3,000,000 for stone concrete. An average value for cinder concrete is about 750,000. Some experimental values for stone concrete have fallen somewhat lower than 1,500,000, while others have reached 4,000,000 and even more. We may probably use the following values with the constant value of 29,000,000 for the steel.

TABLE XIV

Modulus of Elasticity of Some Grades of Concrete

KIND OF CONCRETE	MIXTURE	E_c	*r*
Cinder	1:2:5	750,000	40
Broken Stone	1:6:12	1,450,000	20
" "	1:3:5	2,400,000	12
" "	1:2:4	2,900,000	10

The value given above for 1:6:12 concrete is mentioned only because the value $r = 20$ is sometimes used with the weaker grades of concrete, and the value of approximately 1,450,000 for the elasticity of such concrete has been found by experimenters. The use of such a lean concrete is hardly to be recommended, because of its unreliability. Considering the variability in cinder concrete, the even value of $r = 40$ is justifiable, rather than the precise value 38.67.

266. **Percentage of Steel.** The previous calculations have been made as if the percentage of the steel might be varied almost in-

definitely. While there is considerable freedom of choice, there are limitations beyond which it is useless to pass; and there is always a most economical percentage, depending on the conditions. We have already determined that:

$$\frac{\epsilon_c}{\epsilon_s} = \frac{k}{1-k}.$$

But $\epsilon_c = \frac{c}{E_c\,(1-\frac{1}{2}q)}$; (see Equation 10), and

$$\epsilon_s = \frac{s}{E_s}; \text{ therefore,}$$

$$\frac{\epsilon_c}{\epsilon_s} = \frac{c\,E_s}{sE_c\,(1-\frac{1}{2}q)} = \frac{c\,r}{s(1-\frac{1}{2}q)} = \frac{k}{1-k}.$$

Solving for k, we have:

$$k = \frac{c\,r}{c\,r + s\,(1-\frac{1}{2}q)}.$$

Using as before the value of $q = \frac{2}{3}$, the equation becomes:

$$k = \frac{c\,r}{c\,r + .667\,s}.$$

Using the same value of q in Equation **15**, and solving for p, we have:

$$p\,r = \frac{7\,k^2}{18\,(1-k)}.$$

Substituting the above value of k in this equation, we have, after considerable reduction:

$$p = \frac{7}{12}\,\frac{c}{s}\,\frac{c\,r}{(c\,r + .667\,s)} \quad \ldots\ldots\ldots\ldots \quad \textbf{(18)}$$

The above equation shows that we cannot select the percentage of steel at random, since it evidently depends on the selected stresses for the steel and concrete, and also on the ratio of their moduli. For example, consider a high-grade concrete (1:2:4) whose modulus of elasticity is considered to be 2,900,000, and which has a limiting compressive stress of 2,700 pounds (c'), which we may consider in conjunction with the limiting stress of 55,000 pounds in the steel. The values of c, s, and r are therefore 2,700, 55,000, and 10 respectively. Substituting these values in Equation **18**, we compute $p = .012$.

Example. What percentage of steel would be required for ordinary stone concrete, with $r = 15$, $c = 2{,}000$, and $s = 55{,}000$? Ans. 0.95 per cent.

267. Resisting Moment. The moment which resists the action of the external forces is evidently measured by the product of the distance from the center of gravity of the steel to the centroid of compression of the concrete, times the total compression of the concrete, or, otherwise, times the tension in the steel. The compression in the concrete and the tension in the steel are equal, and it is therefore only a matter of convenience to express this product in terms of the tension in the steel. Therefore, adopting the notation already mentioned, we may write the formula:

$$M = A\,s\,(d-x) \quad \ldots\ldots\ldots\ldots (19)$$

But since the computations are frequently made in terms of the dimensions of the concrete and of the percentage of the reinforcing steel, it may be more convenient to write the equation:

$$M = p\,b\,d\,s\,(d-x) \quad \ldots\ldots\ldots\ldots (20)$$

From Equation **12** we have the total compression in the concrete. Multiplying this by the distance from the steel to the centroid of compression ($d - x$), we have another equation for the moment:

$$M_0 = \frac{9}{8}\left(1 - \frac{1}{3}q\right) qc'bkd\,(d-x) \quad \ldots\ldots (21)$$

This equation is perfectly general, except that it depends on the assumption as to the form of the stress-strain diagram as described in Article 260. On the assumption that $q = \frac{2}{3}$ for ultimate stresses in the concrete, the equation becomes:

$$M_0 = \frac{7}{12} c'bkd\,(d-x) \quad \ldots\ldots\ldots\ldots (22)$$

When the percentage of steel used agrees with that computed from Equation **18**, then Equations **20** and **22** will give identically the same results; but when the percentage of steel is selected arbitrarily, as is frequently done, then the proposed section should be tested by both equations. When the percentage of steel is larger than that required by Equation **18**, the concrete will be compressed more than is intended before the steel attains its normal tension. On the other hand, a lower percentage of steel will require a higher unit-tension in the steel before the concrete attains its normal compression. When the discrepancy between the percentage of steel assumed and the true economical value is very great, the stress in the steel (or the concrete) *may* become dangerously high when the stress in the other element (on which the computation may have been made) is only normal.

268. *Example* 1. What is the ultimate resisting moment of a concrete beam made of 1:3:5 concrete, which is 7 inches wide, 10 inches deep to the

reinforcement, and which uses 1.2 per cent of reinforcement? The concrete is supposed to have a ratio for the moduli of elasticity (r) of 15. The ultimate strength of the concrete (c') is assumed as 2,000.

Answer. From Table XIII, $p = .012$, and $r = 15$, $k = .490$; $x = .357\,kd = .175\,d$; $d - x = .825\,d$. From Equation **22** we have:

$$M_0 = \frac{7}{12} \times 2{,}000 \times .490 \times 7 \times 10 \times 8.25 = 330{,}137 \text{ inch-pounds.}$$

The total compression in the concrete is the continued product of all the factors except the last, and equals 40,017. But this equals the tension in the steel, whose area $= pbd = .012 \times 7 \times 10 = .84$ square inch. Therefore the unit-stress in the steel would equal $40{,}017 \div .84 = 47{,}640$ pounds per square inch. This is considerably less than the usual ultimate of 55,000, and shows that the percentage of steel is considerably in excess of the normal value.

From Equation **20**, assuming $s = 55{,}000$, we have:

$$M_0 = .012 \times 7 \times 10 \times 55{,}000 \times 8.25 = 372{,}900 \text{ inch-pounds.}$$

If the beam were actually stressed with this moment, the total compression in the concrete would equal $372{,}900 \div 8.25 = 45{,}200$ pounds. From Equation 13 we have:

$$45{,}200 = \frac{7}{12}\,c'\,bkd = \frac{7}{12}\,c' \times 7 \times .490 \times 10.$$

Solving for c',

$$c' = 45{,}200 \div 20.008 = 2{,}205 \text{ pounds,}$$

which is considerably more than that assumed—2,000.

The practical interpretation of the above is that if the beam is tested by Equation **22**, indicating an ultimate moment of 330,137 inch-pounds, and the actual proposed loading, multiplied by its factor of safety, does not have a moment which exceeds this value, the compression in the concrete will not be more than 2,000 pounds per square inch, while the tension in the steel will be not greater than 47,640 pounds per square inch (*ultimate* value), which is safe but uneconomical. On the other hand, if Equation **20** were employed, indicating an ultimate moment of 372,900 pounds, and the ultimate loading of the beam seemed to require this moment, the steel would be all right, but the concrete would have an ultimate compression of 2,205 pounds, which would be dangerous for that grade of concrete. Therefore, as a *general rule*, whenever the percentage of steel has been assumed, both equations (**20** and **22**) should be tested. The ***lowest*** ultimate moment should be the limit which should not be exceeded by the ultimate moment of the actual loading, for the use of the higher value will mean an excessive stress in either the concrete or the steel.

Example 2. What will be the ultimate resisting moment of a 5-inch slab made of a high quality of concrete (1:2:4), using the most economical percentage of steel?

Answer. For this quality of concrete, $r = 10$; the ultimate compressive strength of the concrete is 2,700; and the ultimate tension in the steel is assumed at 55,000. Substituting these values in Equation **18**, we find that the economical percentage of steel is 1.21. Interpolating this value of p in Table XIII, considering that $r = 10$, we have $k = .424$. Substituting this value of k in Equation **14**, we find that $x = .151\,d$. In the case of the 5-inch slab, we shall assume that the center of gravity of the steel is placed 1 inch from the bottom of the slab. Therefore $d = 4$ inches. For a slab of indefinite width, we shall assume that $b = 12$ inches. Therefore our computed value for the ultimate resisting moment gives the moment of a strip of the slab one foot wide, and the computed amount of the steel is the amount of steel per foot of width of the slab.

Substituting these various values in Equation **20**, we find as the value of the ultimate resisting moment:

$$M_o = .0121 \times 12 \times 4 \times 55{,}000 \times .849 \times 4 = 108{,}482 \text{ inch-pounds}$$

The area of steel required for each foot of width is:

$$A = .0121 \times 12 \times 4 = .5808 \text{ square inch.}$$

This equals .0484 square inch per inch of width. Since a ½-inch square bar has an area of .25 square inch, we may provide the reinforcement by using ½-inch square bars spaced $\frac{.25}{.0484} = 5.17$ inches, or, say, 5¼ inches.

Example 3. A very instructive comparison may be made by considering a 5-inch slab with $d = 4$ inches, but made of 1:3:5 concrete. In this case we call $r = 12$; $c = 2{,}000$; and s (as before) $= 55{,}000$. By the same method as before, we obtain $p = .0084$; $k = .395$; and therefore $x = .141\,d$. Substituting these values in Equation **20**, we have:

$$M_o = .0084 \times 12 \times 4 \times 55{,}000 \times .859 \times 4 = 76{,}197 \text{ inch-pounds.}$$

The area of steel per foot of width is:

$$A = .0084 \times 12 \times 4 = .4032 \text{ square inch.}$$

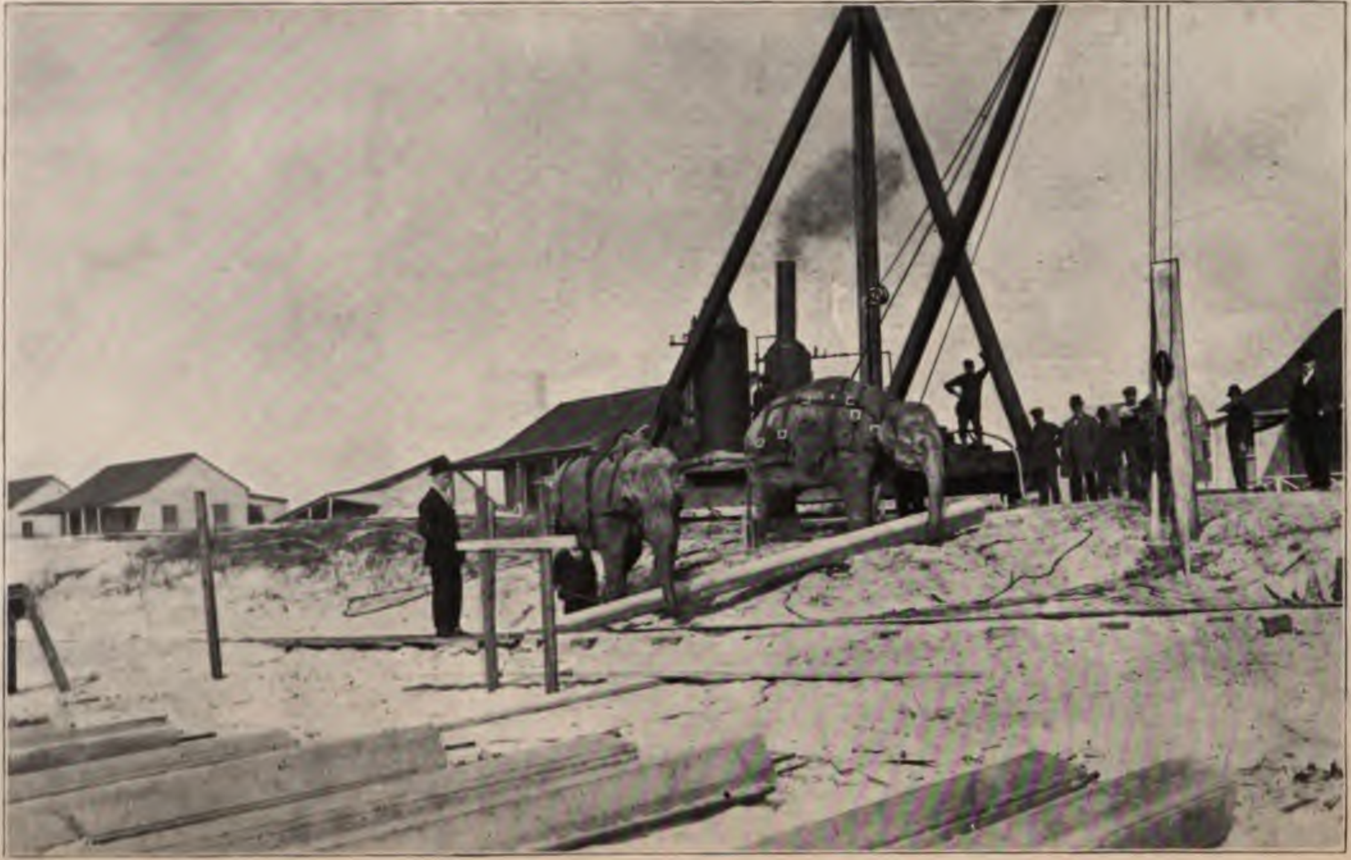
This would require ½-inch square bars spaced 7.33 inches. Although the amount of steel required in this slab is considerably less than was required in the previous case, the ultimate moment of the slab is also very much less. In fact the reduction of strength is very nearly in proportion to the reduction in the amount of steel. Therefore, it must be observed that, although the percentage of steel used with high-grade concrete is considerably higher, the thickness of the concrete will be considerably less; and in spite of the fact that the *percentage* of steel may be higher, its absolute amount for a slab of equal strength may be approximately the same.

Example 4. Another instructive principle may be learned by determining the required thickness of a slab made of 1:3:5 concrete, which shall have the same ultimate strength as the high-grade concrete mentioned in example 2. In other words, its ultimate moment per foot of width must equal 108,482 inch-pounds. The values of r, c, and s are the same as in example 3, and therefore the value of p must be the same as in example 3; therefore $p = .0084$. Since r and p are the same as in example 3, k again equals .395, and therefore $x = .141\,d$. We therefore have from Equation **20**:

$$M_0 = 108{,}482 = .0084 \times 12 \times d \times 55{,}000 \times .859 \times d.$$

Solving this equation for d, we find $d^2 = 22.78$; and $d = 4.77$. The area of the steel $A = p\,b\,d = .0084 \times 12 \times 4.77 = .481$. This is considerably less than the area of steel per foot of width as computed in example 2 for a slab of equal strength. On the other hand, the slab of 1:3:5 concrete will require about 15 per cent more concrete. It will also weigh about 10 pounds per square foot more than the thinner slab, which will reduce by that amount the permissible live load. The determination of the relative economy of the two kinds of concrete will therefore depend somewhat on the relative price of the concrete and the steel. The difference in the total cost of the two methods is usually not large; and abnormal variation in the price of cement or steel may be sufficient to turn the scale one way or the other.

269. **Determination of Values for Frequent Use.** The above methods of calculation may be somewhat simplified by the determi-



ELEPHANTS CARRYING CONCRETE PILES FOR CONSTRUCTION OF BOARD-WALK AT LONG BEACH, LONG ISLAND, N. Y.

These elephants were brought from India for this work.

nation, once for all, of constants which are in frequent use. For example, a very large amount of work is being done, using 1:3:5 concrete. Sometimes engineers will use the formulæ developed on the basis of 1:3:5 concrete, even when it is known that a richer mixture will be used. Although such a practice is not economical, the error is on the side of safety; and it makes some allowance for the fact that a mixture which is nominally richer *may* not have any greater strength than the values used for the 1:3:5 mixture, on account of defective workmanship or inferior cement or sand. Some of the constants for use with 1:3:5 mixture and 1:2:4 mixture will now be worked out.

For the 1:3:5 mixture, $r = 12$; $c = 2{,}000$; and we shall assume $s = 55{,}000$. On the basis of such values, the economical *percentage* of steel is .84 per cent. Under these conditions, k will always be .395; and x will equal .141 d. Therefore the term $(d - x)$ will always equal .859 d, or, say, .86 d, which is close enough for a working value. Since the above values for c and s represent the ultimate values, the resulting moment is the ultimate moment, which we shall call M_u. Therefore, for 1:3:5 concrete, we have the constant values:

$$\left.\begin{aligned} M_u &= .0084 \times bd \times 55{,}000 \times .86d \\ &= 397\, bd^2 \\ A &= .0084\, bd \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$
$$(d-x) = .86d$$

Similarly we can compute a corresponding value for 1:2:4 concrete, using the values previously allowed for this grade:

$$\left.\begin{aligned} M_u &= .565\, bd^2 \\ A &= .0121\, bd \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$
$$(d-x) = .86d$$

Numerical Example. A flooring with a live-load capacity of 150 pounds per square foot, is to be constructed on I-beams spaced 6 feet center to center, using 1:3:5 concrete. What thickness of slab will be required, and how much steel must be used?

Answer. Using the approximate estimate, based on experience, that such a slab will weigh *about* 50 pounds per square foot, we can compute the ultimate load by multiplying the live load, 150, by four, and the dead load, 50, by two, and obtain a total ultimate load of 700 pounds per square foot. A strip 1 foot wide and 6 feet long (between the beams) will therefore carry a total load of 700 × 6 = 4,200 pounds. Considering this as a simple beam, we have:

$$M_0 = \frac{W_0 l}{8} = \frac{4{,}200 \times 6 \times 12}{8} = 37{,}800 \text{ inch-pounds.}$$

Placing this numerical value of $M_0 = 397\,b\,d^2$, as in Equation **23**, we have $37{,}800 = 397\,b\,d^2$. In this case, $b = 12$ inches. Substituting this value of b, we solve for d^2, and obtain $d^2 = 7.93$, and $d = 2.82$ inches. Allowing an extra inch below the steel, this will allow us to use a 4-inch slab. Theoretically we could make it a little less. Practically this figure should be chosen. The required steel, from Equation **23**, equals $.0084\,bd$. Taking $b = 1$, we have the required steel per *inch* of width of the slab $= .0084 \times 2.82 = .0237$ square inch. If we use ½-inch square bars which have a cross-sectional area of .25 square inch, we may space the bars $\frac{.25}{.0237} = 10$ inches. This reinforcement could also be accomplished by using ⅜-inch square bars, which have an area of .1406. The spacing may therefore be $\frac{.1406}{.0237} = 6.0$ inches. As referred to later, there should also be a few bars laid perpendicular to the main reinforcing bars, or parallel with the I-beams, so as to prevent shrinkage. The required amount of this steel is not readily calculable. Since the I-beams are 6 feet apart, if we place two lines of ⅜-inch square bars spaced 2 feet apart, parallel with the I-beams, there will then be reinforcing steel in a direction parallel with the I-beams at distances apart not greater than 2 feet, since the I-beams themselves will prevent shrinkage immediately around them.

270. **Straight-Line Formulæ.** The working unit-compressions for even the best grade of concrete are seldom allowed to exceed 600 pounds per square inch. An inspection of Fig. 93 will show that the curve from the point o to the point indicating a pressure of 600 pounds, although really a parabola, is so nearly a straight line that there is but little error in considering it to be straight. On this account, many formulæ for the strength of reinforced concrete have been developed on the basis of a uniform modulus of elasticity for the concrete. This is virtually the same as assuming that q equals zero in Equation **16**. The other equations which are derived from equations involving q, must also be correspondingly modified.

Adopting the same notation as in Article 256, we may say that the triangle mnN in Fig. 97 represents the compressive forces; that the area of the triangle measures the summation of those forces; and, assuming that in this case $c = mn$, the summation is:

$$\Sigma X = \frac{1}{2} cbkd \quad \text{. . . . . (25)}$$

The center of gravity of the triangle, which is the centroid of compression of the concrete, is at ⅓ of the height of the triangle (kd) from the compression face of the concrete. The same value is obtained by making $q = 0$ in the equation above Equation **14**, which gives us:

$$x = \frac{1}{3} kd \quad \text{. . . . . (26)}$$

Making $q = 0$ in Equation **16**, we have:

$$k = \sqrt{2pr + p^2r^2} - pr \quad \text{. . . . . (27)}$$

From this equation we may deduce Table XV, which corresponds to Table XIII.

TABLE XV

Value of *k* for Various Values of *r* and *p*
(Straight-Line Formulæ)

r	*p* .020	.018	.016	.014	.012	.010	.008	.006	.004	.003
10	.464	.446	.427	.407	.385	.358	.328	.292	.246	.216
12	.493	.476	.457	.436	.412	.385	.353	.314	.266	.235
15	.531	.513	.493	.471	.446	.418	.384	.343	.291	.258
20	.580	.562	.542	.519	.493	.463	.428	.384	.328	.292
40	.698	.679	.659	.637	.611	.579	.542	.493	.428	.384

From an equation in Article 266, by calling $q = 0$, we may write: $k = \frac{cr}{cr + s}$. By making $q = 0$ in Equation **15**, we may write $pr = \frac{1}{2}\frac{k}{1-k}$. By eliminating k from these two equations, we may write:

$$p = \frac{1}{2}\frac{c}{s}\frac{cr}{(cr + s)} \quad \text{. . . . . (28)}$$

The similarity of this equation to Equation **18** is readily apparent, the difference being due only to the elimination of the effect of q.

The moment of resistance of a beam equals the total tension in the steel, or the total compression in the concrete (which are equal), *times* $(d - x)$. Therefore we have the choice of two values (as before):

$$\left.\begin{aligned} M_c &= \frac{1}{2} cbkd\,(d-x) \\ M_s &= As\,(d-x) = pbds\,(d-x) \end{aligned}\right\} \quad \ldots\ldots \quad (29)$$

If the economical percentage p has already been determined from Equation **28**, then either equation may be used, as most convenient, since they will give identical results. If the percentage has been arbitrarily chosen, then the *least* value must be determined, as was described in Article 267.

271. **Determination of Values for Frequent Use.** For 1:3:[illegible] concrete, using as before $r = 12$, and with a working value for c = 500, and $s = 16{,}000$, we find from Equation **28** that the economical percentage of steel equals:

$$p = \frac{1}{2}\,\frac{500}{16{,}000}\;\frac{500 \times 12}{(500 \times 12 + 16{,}000)} = .0043$$

From Table XV we find by interpolation that, for $r = 12$, and $p = .0043$, $k = .273$. Then (see Equation **26**):

$$x = \frac{1}{3}\,kd = .091d; \text{ and } (d-x) = .909d.$$

Substituting these values in either formula of Equation **29**, we have:

$$M = 62\,bd^2.$$

The percentage of steel computed from Equation **28** has been called *the most economical percentage,* because it is the percentage which will develop the maximum allowed stress in the concrete and the steel *at the same time,* or by the loading of the beam to some definite maximum loading. The real meaning of this is best illustrated by a numerical example using another percentage. Assume that the percentage of steel is exactly doubled, or that $p = 2 \times .0043 = .0086$. From Table XV, for $r = 12$, and $p = .0086$, we find $k = .362$; $x = .121d$; and $(d-x) = .879d$. Substituting these values in both forms of Equation **29**, we have:

$$M_c = 80\,bd^2; \text{ and,}$$
$$M_s = 121\,bd^2.$$

The interpretation of these two equations, and also of the equation found above ($M = 62\ bd^2$), is as follows: Assume a beam of definite dimensions b and d, and made of concrete whose modulus of elasticity is $\frac{1}{12}$ that of the modulus of elasticity of the reinforcing steel; assume that it is reinforced with steel having a cross-sectional area $= .0043\ bd$. Then, when it is loaded with a load which will develop a moment of 62 bd^2, the tension in the steel will equal 16,000 pounds per square inch, and the compression in the concrete will equal 500 pounds per square inch at the outer fibre. Assume that the area of the steel is exactly doubled. One effect of this is to *lower* the neutral axis (k is increased from .273 to .362), and more of the concrete is available for compression. The load may be increased about 29 per cent, or until the moment equals 80 bd^2, before the compression in the concrete reaches 500 pounds per square inch. Under these conditions the steel has a tension of about 10,600 pounds per square inch, and its full strength is not utilized. If the load were increased until the moment was 121 bd^2, then the steel would be stressed to 16,000 pounds per square inch, but the concrete would be compressed to over 750 pounds, which would of course be unsafe with such a grade of concrete. If the compression in the concrete is to be limited to 500 pounds per square inch, then the load must be limited to that which will give a moment of 80 bd^2. Even for this the steel is doubled in order to increase the load 29 per cent. Whether this is justifiable, depends on several circumstances—the relative cost of steel and concrete, the possible necessity for keeping the dimensions of the beam within certain limits, etc. Usually a much larger ratio of steel than 0.43 per cent is used; 1.0 per cent is far more common; but when such is used, it means that the strength of the steel cannot be fully utilized unless the concrete can stand high compression. A larger value of r will indicate higher values of k, which will indicate higher moments; but r cannot be selected at pleasure. It depends on the character of the concrete used; and, with E_s constant, a large value of r means a small value for E_c, which also means a small value for c, the permissible compression stress. Whenever the percentage of steel is greater than the *economical percentage*, as is usual, then the upper of the two formulæ of Equation 29 should be used. When in doubt, both should be tested, and that one giving the lower moment should be used.

TABLE XVI

Ultimate Load on Slabs of "Average" Concrete (1:3:5) in Pounds per Square Foot
Weight of Slab Included

Effective Thickness of Slab d	Area of Steel in 12-In. Width	Spacing of Bars		Span in Feet (L)											
		$\frac{3}{8}$-In. Sq.	$\frac{1}{2}$-In. Sq.	4	5	6	7	8	9	10	11	12	13	14	15
2.5	.252	$6\frac{3}{4}$ in.	12 in.	1,241	794	551	405	310	245	198	.....	.....	.....	.....	.....
3.0	.302	$5\frac{1}{2}$ "	10 "	1,786	1,143	794	583	446	353	286	236	198	.....	.....	.....
3.5	.353	$4\frac{3}{4}$ "	$8\frac{1}{2}$ "	2,432	1,556	1,080	793	608	480	389	322	270	230	198	.....
4.0	.403	$4\frac{1}{4}$ "	$7\frac{1}{2}$ "	3,176	2,033	1,411	1,037	794	627	508	420	353	300	259	226
4.5	.454	$3\frac{3}{4}$ "	$6\frac{3}{4}$ "	4,020	2,573	1,786	1,312	1,005	794	643	531	446	380	328	286
5.0	.504	$3\frac{3}{8}$ "	6 "	4,962	3,176	2,206	1,620	1,241	980	794	656	551	470	405	353
5.5	.554	3 "	$5\frac{1}{2}$ "	6,005	3,843	2,669	1,960	1,501	1,186	960	794	667	569	490	427
6.0	.605	.....	5 "	.....	4,573	3,176	2,334	1,787	1,412	1,142	945	793	677	583	508
7.0	.706	.....	$4\frac{1}{4}$ "	.....	.....	4,323	3,176	2,432	1,921	1,556	1,286	1,080	921	794	692
8.0	.806	.....	$3\frac{3}{4}$ "	.....	.....	.....	4,148	3,176	2,509	2,033	1,680	1,410	1,203	1,037	904

Using $p = .0075$, and $r = 12$, we have $k = .343$; $x = .114d$; and $(d - x) = .886\ d$. Then, since p is greater than the *economical* value, we use the upper formula of Equation **29**, and have:

$$M = 76\ bd^2 \ldots\ldots\ldots\ldots\ldots \mathbf{(30)}$$

272. *Example* 1. What is the working moment for a slab with 5-inch thickness to the steel, the concrete having the properties described above?

Answer. Calling $b = 12$ inches, $M = 76 \times 12 \times 25 = 22{,}800$ inch-pounds, the permissible moment on a section 12 inches wide.

Example 2. A slab having a span of 8 feet is to support a load of 150 pounds per square foot. The concrete is to be as described above, and the percentage of steel is to be 0.75. What is the required thickness d of the steel?

Answer. A strip 12 inches wide has an area of 8 square feet, and carries a load of 1,200 pounds. The moment $= \frac{1}{8}\ Wl = \frac{1}{8} \times 1{,}200 \times 96 = 14{,}400$ inch-pounds. For a strip 12 inches wide, $b = 12$ inches, and $M = 76 \times 12 \times d^2 = 912d^2 = 14{,}400$; $d^2 = 15.79$; $d = 3.97$ inches—or, say, 4 inches.

273. **Table for Slab Computation.** The necessity of very frequently computing the required thickness of slabs, renders very useful a table such as is shown in Table XVI, which has been worked out on the basis of 1:3:5 concrete, and computed by solving Equation **23** for various thicknesses d, and for various spans L varying by single feet. It should be noted that the loads as given are *ultimate* loads *per square foot,* and that they therefore include the weight of the slab itself, which must be multiplied by its factor of safety, which is usually considered as 2.

For example, in the numerical case of Article 269, we computed that there would be a total load of 700 pounds on a span of 6 feet. In the column headed 6, we find 794 on the same line as the value of 3.0 in the column d. This shows that 3.0 is somewhat excessive for the value of d. We computed its precise value to be 2.82. On the same line, we find under "Spacing of Bars," that $\frac{3}{8}$-inch square bars spaced $5\frac{1}{2}$ inches will be sufficient. In the above more precise calculation, we found that the bars could be spaced 6 inches apart, as was to be expected, since the computed ultimate load is considerably less than the nearest value found in the table.

Example 1. What is the ultimate load that will be carried by a 5-inch slab on a span of 10 feet, using 1:3:5 concrete?

Answer. The 5 inches here represents the total thickness, and we shall assume that the effective thickness (d) is 1 inch less. Therefore $d = 4$ inches. On the line opposite $d = 4$ in Table XVI, and under the column $L = 10$, we have 508, which gives the ultimate load per square foot. A 5-inch slab will weigh approximately 60 pounds per square foot, allowing 12 pounds per square foot per inch of thickness. Using a factor of 2, we have 120 pounds, which, subtracting from 508, leaves 388 pounds; dividing this by 4, we have 97 pounds per square foot as the allowable working load. Such a load is heavier than that required for residences or apartment houses. It would do for an office building.

Example 2. The floor of a factory is to be loaded with a live load of 300 pounds per square foot, the slab to be supported on beams spaced 8 feet apart. What must be the thickness of the floor-slab?

Answer. With 1,200 pounds per square foot ultimate load for the live load alone, we notice in Table XVI, under $L = 8$, that 1,241 is opposite to $d = 5$. This shows that it would require a slab nearly 6 inches thick to support the live load alone. We shall therefore add another half-inch as an estimated allowance for the weight of the slab; and, assuming that a $6\frac{1}{2}$-inch slab having a weight of 78 pounds per square foot will do the work, we multiply 300 by 4, and 78 by 2, and have 1,356 pounds per square foot as the ultimate load to be carried. Under $L = 8$, in Table XVI, we find that 1,356 comes between 1,241 and 1,501, showing that a slab with an effective thickness d of about $5\frac{1}{4}$ inches will have this ultimate carrying capacity. The total thickness of the slab should therefore be about $6\frac{1}{4}$ inches. The table also shows that $\frac{1}{2}$-inch bars spaced about $5\frac{3}{4}$ inches apart will serve for the reinforcement. We might also provide the reinforcement by $\frac{3}{8}$-inch square bars spaced a little over 3 inches apart; but it would probably be better policy to use the half-inch bars, especially since the $\frac{3}{8}$-inch bars will cost somewhat more per pound.

274. **Practical Methods of Spacing Slab Bars.** It is too much to expect of workmen that bars will be accurately spaced when their distance apart is expressed in fractions of an inch. But it is a comparatively simple matter to require the workmen to space the bars evenly, provided it is accurately computed how many bars should be laid in a given width of slab. For example, in the above case, a panel of the flooring which is, say, 20 feet wide, should have a definite

number of bars; 20 feet = 240 inches, and 240 ÷ 5.75 = 41.7. We shall call this 42, and instruct the workmen to distribute 42 bars equally in the panel 20 feet wide. The workmen can do this without even using a foot-rule, and can adjust them to an even spacing with sufficient accuracy for the purpose.

275. **Table for Computation of Simple Beams.** In Table XVII has been computed for convenience the ultimate total load on rectangular beams made of average concrete (1:3:5) and with a width of 1 inch. For other widths, multiply by the width of the beam. Since $M_0 = \frac{1}{8} W_0 l$; and since by Equation **23**, for this grade of concrete, $M_0 = 397\ b\ d^2$; and since for a computation of beams 1 inch wide, $b = 1$, we may write $\frac{1}{8} W_0 l = 397\ d^2$. For l we shall substitute $12\ L$. Making this substitution and solving for W_0, we have $W_0 = 265\ d^2 \div L$. Since $b = 1$, A, the area of steel per inch of width of the beam $= .0084\ d$.

Example. What is the ultimate total load on a simple beam having a depth of 16 inches to the reinforcement, 12 inches wide, and having a span of 20 feet?

Answer. Looking in Table XVII, under $L = 20$, and opposite $d = 16$, we find that a beam 1 inch wide will sustain a total load of 3,392 pounds. For a width of 12 inches, the total ultimate load will be 12 × 3,392 = 40,704 pounds. At 144 pounds per cubic foot, the beam will weigh 3,840 pounds. Using a factor of 2 on this, we shall have 7,680 pounds, which, subtracted from 40,704, gives 33,024. Dividing this by 4, we have 8,256 lbs. as the allowable live load on such a beam.

276. **Resistance to the Slipping of the Steel in the Concrete.** The previous discussion has considered merely the tension and compression in the upper and lower sides of the beam. A plain, simple beam resting freely on two end supports, has neither tension nor compression in the fibres at the ends of the beam. The horizontal tension and compression, found at or near the center of the beam, entirely disappear by the time the end of the beam is reached. This is done by transferring the tensile stress in the steel at the bottom of the beam, to the compression fibres in the top of the beam, by means of the intermediate concrete. This is, in fact, the main use of the concrete in the lower part of the beam.

It is therefore necessary that the bond between the concrete and

TABLE XVII

Ultimate Total Load on Rectangular Beams of Average Concrete (1:3:5), One Inch Wide

For other widths, multiply by width of beam. Formula: $W = 265\,d^2 \div L$; $A = .0084\,d$. Ultimate compression in concrete 2,000 pounds per sq. in.; ultimate tension in steel 55,000 pounds per sq. in.

Effective Depth of Beam, d	Area of Steel per Inch of Width	Span in Feet (L)																	Twice Dead Load per Foot of Beam
		4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
4	.0336	1,060	848	707	606	530	471	424	385	353	326	303	283	265	249	236	223	212	10
5	.0420	1,656	1,324	1,104	946	828	736	662	602	552	510	473	441	414	390	368	349	331	12
6	.0504	2,385	1,908	1,590	1,363	1,192	1,060	954	867	795	734	681	636	596	561	530	502	477	15
7	.0588	3,246	2,596	2,164	1,855	1,623	1,443	1,298	1,180	1,082	999	927	865	812	764	721	683	649	17
8	.0672	4,240	3,392	2,827	2,423	2,120	1,884	1,696	1,542	1,413	1,305	1,211	1,131	1,060	998	942	893	848	20
9	.0756	5,366	4,292	3,577	3,066	2,683	2,385	2,146	1,951	1,789	1,651	1,533	1,431	1,341	1,263	1,193	1,130	1,073	22
10	.0840	6,625	5,300	4,417	3,786	3,312	2,944	2,650	2,409	2,208	2,038	1,893	1,767	1,656	1,559	1,472	1,395	1,325	24
11	.0924	8,016	6,412	5,344	4,581	4,008	3,563	3,206	2,915	2,672	2,466	2,290	2,137	2,004	1,886	1,781	1,688	1,603	26
12	.1008	9,540	7,632	6,360	5,451	4,770	4,240	3,816	3,469	3,180	2,935	2,726	2,544	2,385	2,245	2,120	2,008	1,908	28
13	.1092	11,196	8,957	7,464	6,398	5,598	4,976	4,478	4,071	3,732	3,445	3,199	2,986	2,799	2,634	2,488	2,357	2,239	30
14	.1176	12,985	10,388	8,657	7,420	6,492	5,771	5,194	4,722	4,328	3,995	3,710	3,463	3,246	3,055	2,886	2,734	2,597	32
15	.1260	14,906	11,924	9,937	8,518	7,453	6,625	5,962	5,420	4,969	4,586	4,259	3,975	3,726	3,508	3,312	3,138	2,981	34
16	.1344	16,960	13,568	11,307	9,691	8,480	7,538	6,784	6,167	5,653	5,218	4,845	4,523	4,240	3,991	3,769	3,571	3,392	36
17	.1428	19,146	15,317	12,764	10,941	9,573	8,509	7,658	6,962	6,382	5,891	5,470	5,106	4,786	4,505	4,255	4,031	3,829	38
18	.1512	21,465	17,172	14,310	12,266	10,732	9,540	8,586	7,805	7,155	6,605	6,133	5,724	5,366	5,051	4,770	4,519	4,293	40
19	.1596	23,916	19,133	15,944	13,666	11,958	10,629	9,566	8,697	7,972	7,359	6,833	6,378	5,979	5,627	5,315	5,035	4,783	42
20	.1680	26,500	21,200	17,667	15,143	13,250	11,778	10,600	9,636	8,833	8,154	7,571	7,067	6,625	6,235	5,889	5,579	5,300	44

For values in the lower left-hand corner of the table, possible failure by diagonal shear must be very carefully tested and provided for.

the steel shall be sufficiently great to withstand the tendency to slip. The required strength of this bond is evidently equal to the difference in the tension in the steel per unit of length. For example, suppose that we are considering a bar 1 inch square in the middle of the length of a beam. Suppose that the bar is under an actual tension of 15,000 pounds per square inch. Since the bar is 1 inch square, the actual total tension is 15,000 pounds. Suppose that, at a point 1 inch beyond, the moment in the beam is so reduced that the tension in the bar is 14,900 pounds instead of 15,000 pounds. This means that the difference of pull (100 pounds) has been taken up by the concrete. The surface of the bar for that length of one inch, is four square inches. This will require an adhesion of 25 pounds per square inch between the steel and the concrete, in order to take up this difference of tension. The adhesion between concrete and plain bars is usually considerably greater than this, and there is therefore but little question about the bond in the center of the beam. But near the ends of the beam, the change in tension in the bar is far more rapid, and it then becomes questionable whether the bond is sufficient.

Although there is no intention to argue the merits of any form of patented bar, this discussion would not be complete without a statement of the arguments in favor of *deformed* bars, or bars with *a mechanical bond,* instead of plain bars. The deformed bars have a variety of shapes; and since they are not prismatic, it is evident that, apart from adhesion, they cannot be drawn through the concrete without splitting or crushing the concrete immediately around the bars. The choice of form is chiefly a matter of designing a form which will furnish the greatest resistance, and which at the same time is not unduly expensive to manufacture. Of course, the deformed bars are necessarily somewhat more expensive than the plain bars. The main line of argument of those engineers who defend the use of plain bars, may be summed up in the assertion that the plain bars are "good enough," and that, since they are less expensive than deformed bars, the added expense is useless. The arguments in favor of a mechanical bond, and against the use of plain bars, are based on three assertions:

First: It is claimed that tests have apparently verified the assertion that the mere soaking of the concrete in water for several months is sufficient to reduce the adhesion from $\frac{1}{2}$ to $\frac{2}{3}$. If this contention is true, the adhesion of bars in concrete which is likely to be perpetually soaked in water, is unreliable.

Second: Microscopical examination of the surface of steel, and of concrete which has been moulded around the steel, shows that the adhesion depends chiefly on the roughness of the steel, and that the cement actually enters into the microscopical indentations in the surface of the metal. Since a stress in the metal even within the elastic limit necessarily reduces its cross-section somewhat, the so-called adhesion will be more and more reduced as the stress in the metal becomes greater. This view of the case has been verified by recent experiments by Professor Talbot, who used bars made of tool steel in many of his tests. These bars were exceptionally smooth; and concrete beams reinforced with these bars failed generally on account of the slipping of the bars. Special tests to determine the bond resistance, showed that it was far lower than the bond resistance of ordinary plain bars.

Third: There is evidence to show that long-continued vibration, such as is experienced in many kinds of factory buildings, etc., will destroy the adhesion during a period of years. Some failures of buildings and structures which were erected several years ago, and which were long considered perfectly satisfactory, can hardly be explained on any other hypothesis. Owing to the fact that there are comparatively few reinforced-concrete structures which have been built for a very long period of years, positive information as to the durability and permanency of adhesion is lacking. It must be conceded, however, that comparative tests of the bond between concrete and steel when the bars are plain and when they are deformed (the tests being made within a few weeks or months after the concrete is made), have comparatively little value as an indication of what that bond will be under some of the adverse circumstances mentioned above, which are perpetually occurring in practice. Non-partisan tests have shown that, even under conditions which are most favorable to the plain bars, the deformed bars have an actual hold in the concrete which is from 50 to 100 per cent greater than that of plain bars. It is unquestionable that age will increase rather than diminish the relative inferiority of plain bars.

277. **Computation of the Bond Required in Bars.** From

Equation **19** we have the formula that the resisting moment at any point in the beam equals the area of the steel, times the unit tensile stress in the steel, times the distance from the steel to the centroid of compression of the steel, which is the distance $d - x$. We may compute the moment in the beam at two points at a unit-distance apart. The area of the steel is the same in each equation, and $d - x$ is substantially the same in each case; and therefore the *difference* of moment, divided by $(d - x)$, will evidently equal the *difference* in the unit-stress in the steel, times the area of the steel. To express this in an equation, we may say, denoting the difference in the moment by $d\,M$, and the difference in the unit-stress in the steel by $d\,s$:

$$\frac{dM}{(d-x)} = A \times d\,s.$$

But $A \times d\,s$ is evidently equal to the actual difference in tension in the steel, measured in pounds. It is the amount of tension which must be transferred to the concrete in that unit-length of the beam. But the computation of the difference of moments at two sections that are only a unit-distance apart, is a comparatively tedious operation, which, fortunately, is unnecessary. Theoretical mechanics teaches us that the difference in the moment at two consecutive sections of the beam is measured by the *total vertical shear* in the beam at that point. The shear is very easily and readily computable; and therefore the required amount of tension to be transferred from the steel to the concrete can readily be computed. A numerical illustration may be given as follows: Suppose that we have a beam which, with its load, weighs 20,000 pounds, on a span of 20 feet. Using ultimate values, for which we multiply the loading by 4, we have an ultimate loading of 80,000 pounds. Therefore,

$$M_0 = \frac{W_0\,l}{8} = \frac{80{,}000 \times 240}{8} = 2{,}400{,}000.$$

Using the constants previously chosen for 1:3:5 concrete, and therefore utilizing Equation **23**, we have this moment equal to $397\,b\,d^2$. Therefore $b\,d^2 = 6{,}045$.

If we assume $b = 15$ inches; $d = 20.1$ inches; then $d - x = .86d = 17.3$ inches. The area of steel equals:

$$A = .0084\,b\,d = 2.53 \text{ square inches.}$$

We know from the laws of mechanics, that the moment diagram for a beam which is uniformly loaded is a parabola, and that the ordinate to this curve at a point one inch from the abutment will, in the above case, equal $(\frac{119}{120})^2$ of the ordinate at the abutment. This ordinate is measured by the maximum moment at the center, multiplied by the factor $(\frac{119}{120})^2 = \frac{14,161}{14,400} = .9834$; therefore the actual moment at a point one inch from the abutment $= (1.00 - .9834) = .0166$ of the moment at the center. But $.0166 \times 2,400,000 = 39,840$.

But our ultimate loading being 80,000 pounds, we know that the shear at a point in the middle of this one-inch length equals the shear at the abutment, minus the load on this first $\frac{1}{2}$ inch, which is $\frac{1}{240}$ of 40,000 (or 167) pounds. The shear at this point is therefore 40,000 − 167 (or 39,833) pounds. This agrees with the above value 39,840, as closely as the decimals used in our calculations will permit.

The value of $d - x$ is somewhat larger when the moment is very small than when it is at its ultimate value. But the difference is comparatively small, is on the safe side, and it need not make any material difference in our calculations. Therefore, dividing 39,840 by 17.3, we have 2,303 pounds as the difference in tension in the steel in the last inch at the abutment. Of course this does not literally mean the last inch in the length of the beam, since, if the net span were 20 feet, the actual length of the beam would be considerably greater. The area of the steel as computed above is 2.53 square inches. Assuming that this is furnished by five $\frac{3}{4}$-inch square bars, the surfaces of these five bars per inch of length equals 15 square inches. Dividing 2,303 by 15, we have 153 pounds per square inch as the required adhesion between the steel and the concrete. While this is not greater than the adhesion usually found between concrete and steel, it is somewhat risky to depend on this; and therefore the bars are usually bent so that they run diagonally upward, and thus furnish a very great increase in the strength of the beam, which prevents the beam from failing at the ends. Tests have shown that beams which are reinforced by bars only running through the lower part of the beam without being turned up, or without using any stirrups, will usually fail at the ends, long before the transverse moment, which they possess at their center, has been fully developed.

278. **Distribution of Vertical Shears.** Beams which are tested to destruction frequently fail at the ends of the beams, long before the transverse strength at the center has been fully developed. Even

if the bond between the steel and the concrete is amply strong for the requirements, the beam may fail on account of the shearing or diagonal stresses in the concrete between the steel and the neutral axis. The student must accept without proof some of the following statements regarding the distribution of the shear.

The intensity of the shear of various points in the height of the beam, may be represented by the diagram in Fig. 99. If we ignore the tension in the concrete due to transverse bending, the shear will be uniform between the steel and the neutral axis. Above the neutral axis, the shear will diminish toward the top of the beam, the curve being parabolic.

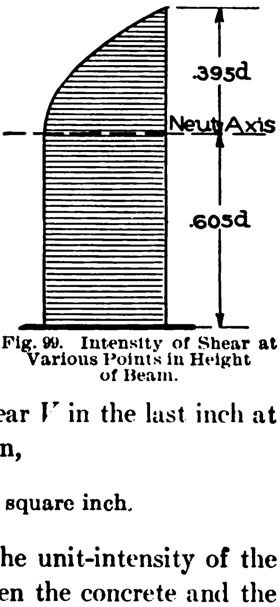


Fig. 99. Intensity of Shear at Various Points in Height of Beam.

If the distribution of the shear were uniform throughout the section, we might say that the shear per square inch would equal $V \div b\,d$. It may be proved that v, the intensity of the vertical shear per square inch, is:

$$v = \frac{V}{b\,(d-x)} \quad \ldots\ldots\ldots\ldots\ldots \quad (31)$$

In the above case, the ultimate total shear V in the last inch at the end of the beam, is 39,840 pounds. Then,

$$v = \frac{39{,}840}{15 \times 17.3} = 153.5 \text{ pounds per square inch.}$$

The agreement of this numerical value of the unit-intensity of the vertical shear with the required bond between the concrete and the steel, is due to the accidental agreement of the width of the beam (15 inches) with the superficial area of the bars per inch of length of the beam (15 square inches). If other bars of the same *cross-sectional* area, but with greater or less superficial surface, had been selected for the reinforcement, even this accidental agreement would not have been found.

The actual strength of concrete in shear is usually far greater than this. The failure of beams which fail at the ends when loaded with loads far within their capacity for transverse strength, is generally due to the *secondary stresses*. The computation of these stresses is a complicated problem in Mechanics; but it may be proved that if we ignore the tension in the concrete due to bending stresses, the diagonal tension per unit of area equals the vertical shear per unit of area (v). But concrete which may stand a shearing stress of 1,000 pounds per square inch will probably fail under a direct tension of 200 pounds per square inch. The diagonal stress has the nature of a direct tension. In the above case the beam probably would not fail by this method of failure, since concrete can usually stand a tension up to 200 pounds per square inch; but such beams, when they are not diagonally reinforced, frequently fail in that way before their ultimate loads are reached.

279. **Methods of Guarding against Failure by Shear or Diagonal Tension.** The failure of a beam by actual shear is almost unknown. The failures usually ascribed to shear are generally caused by diagonal tension. A solution of the very simple Equation **31** will indicate the intensity of the vertical shear.

The relation of crushing strength to shearing strength is expressed by the equation:

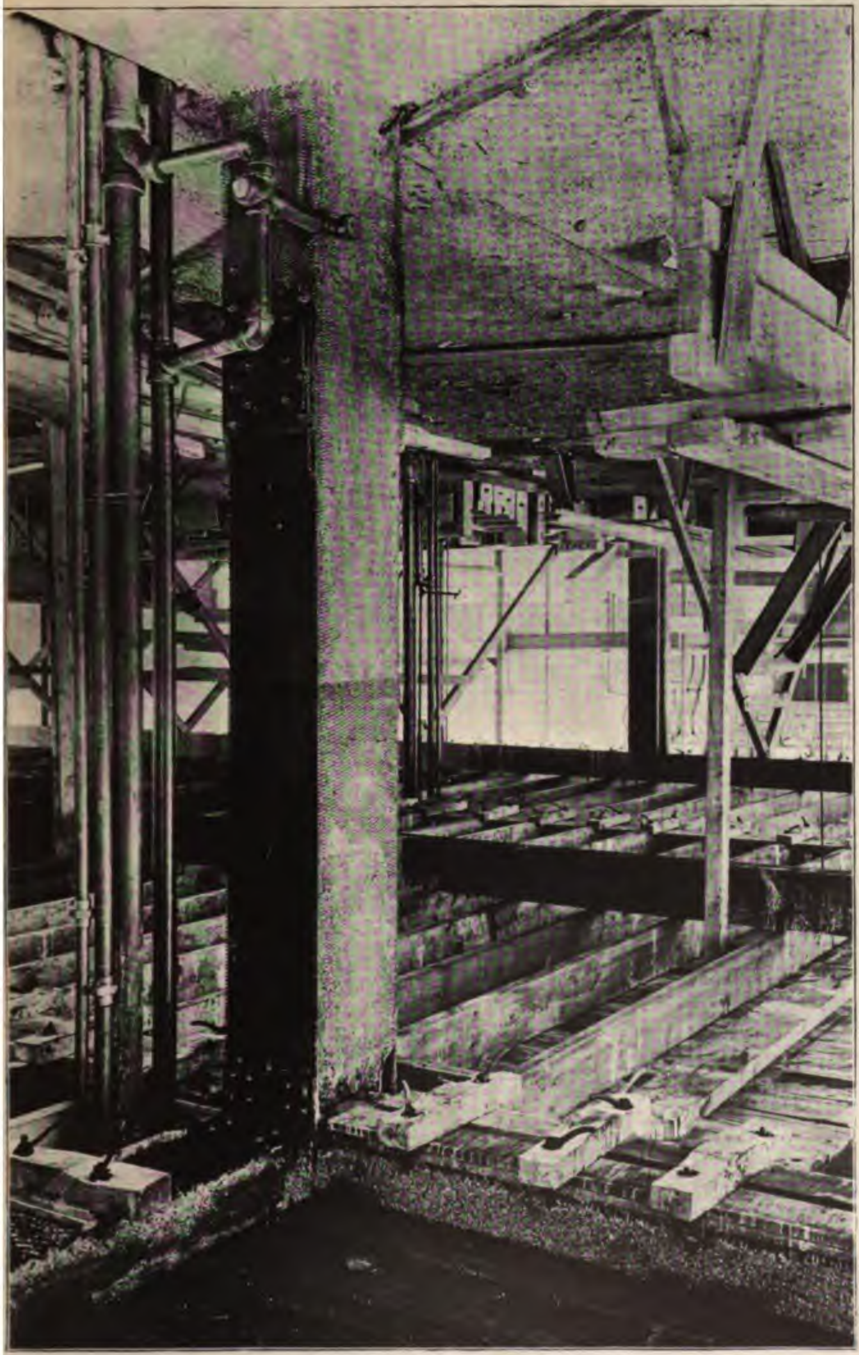
$$\text{Unit shearing strength } z = \frac{c'}{2 \tan \theta},$$

in which z is the unit shearing strength, and θ is the angle of rupture under direct compression. This angle is usually considered to be 60°; for such a value the shearing strength would equal $c' \div 3.464$. When $\theta = 45°$, the shearing strength would equal *one-half* of the crushing strength, and this agrees very closely with the results of tests made by Professor Spofford. But the shearing strength is considered to be a far less reliable quantity than the crushing strength; and therefore dependence is not placed on shear, even for *ultimate* loading, to a greater value than about one-half of the above value; or,

$$\text{Unit shearing strength } z = c' \div 6.928.$$

Usually the unit-intensity of the vertical shear (even for ultimate loads) is less than this. But this ignores the assistance furnished by the bars. Actual failure would require that the bars must crush the concrete under them. When, as is usual, there are bars passing obliquely through the section, a considerable portion of the shear is carried by direct tension in the bars.

It seems impracticable to develop a rational formula for the amount of assistance furnished by these diagonal bars, unless we make assumptions which are doubtful and which therefore vitiate



FIREPROOF COLUMN AND FLOOR CONSTRUCTION
Showing use of forms and expanded metal.

the reliability of the whole calculation. Therefore the *rules* which have been suggested for a prevention of this form of failure are wholly empirical. Mr. E. L. Ransome uses a rule for spacing vertical *stirrups*, made of wires or ¼-inch rods, as follows:

The first stirrup is placed at a distance from the end of the beam equal to one-fourth the depth of the beam; the second is at a distance of one-half the depth beyond the first stirrup; the third, three-fourths of the depth beyond the second; and the fourth, a distance equal to the depth of the beam beyond the third (see Fig. 100). This empirical rule agrees with the theory, in the respect that the stirrups are closer at the ends of the beam, where the shear is greatest. The four stirrups extend for a distance from the end equal to 2½ times the depth of the beam. Usually this is a sufficient distance; but some "systems" use stirrups throughout the length of the beam. On very short beams, the shear changes so rapidly that at 2½ times the depth from the end of the beam the shear is not generally so great as to produce dangerous stresses. With a very long beam, the change in the shear is correspondingly more gradual; and it is possible that stirrups or some other device must be used for a greater actual distance from the end, although for a less proportional distance.

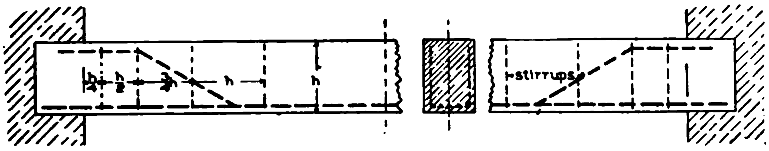


Fig. 100. Spacing of Stirrups.

When the diagonal reinforcement is accomplished by bending up the bars at an angle of about 45°, the bending should be done so that there is at all sections a sufficient area of steel in the lower part of the beam to withstand the transverse moment at that section. As fast as the bars can be spared from the bottom of the beam, they may be turned up diagonally so that there are at every section of the beam one or more bars which would be cut diagonally by such a section. On this account it is far better to use a larger number of bars, than a smaller number of the same area. For example, if it were required that there shall be 2.25 square inches of steel for the section at the middle of the beam, it would be far better to use nine ½-inch bars than four ¾-inch bars. In either case, the steel has the same area and the same weight. The nine ½-inch bars give a much better distribution of the metal. The superficial area of the nine ½-inch bars is 18 square inches per linear inch of the beam, while the area of the four ¾-inch bars is only 12 square inches per inch of length. But an even greater advantage is furnished by the fact that we have nine bars instead of four, which may be bent upward (and bent more easily than the ¾-inch bars) as fast as they can be spared from the bottom of the beam. In this way the shear near the end of the beam may be much more effectually and easily provided for.

Since the shear is greatest at the ends of the beam, more bars should be reserved for turning up near the ends. For example, in the above case of the nine bars, one or two bars might be turned up at about the quarter-points of the beam. One or two more might be turned up at a distance equal to, or a little less than, the depth of the beam from the quarter-points toward the abutments. Others would be turned up at intermediate points; at the abutments there should be at least two, or perhaps three, diagonal bars, to take up the maximum shear near the abutments. This is illustrated, although without definite calculations, in Fig. 101.

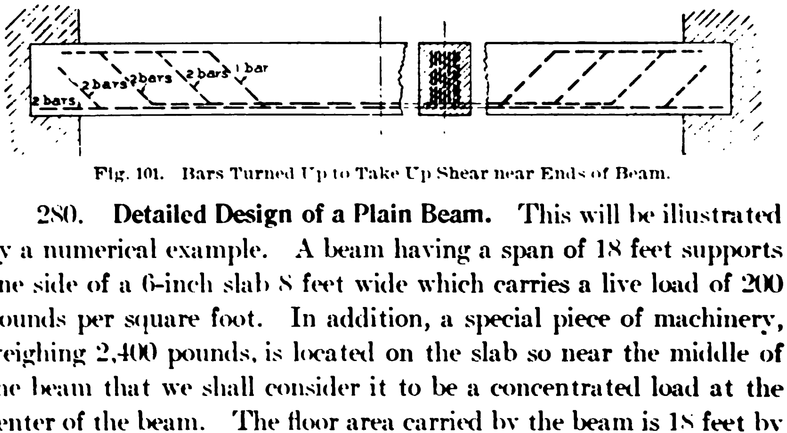


Fig. 101. Bars Turned Up to Take Up Shear near Ends of Beam.

280. Detailed Design of a Plain Beam. This will be illustrated by a numerical example. A beam having a span of 18 feet supports one side of a 6-inch slab 8 feet wide which carries a live load of 200 pounds per square foot. In addition, a special piece of machinery, weighing 2,400 pounds, is located on the slab so near the middle of the beam that we shall consider it to be a concentrated load at the center of the beam. The floor area carried by the beam is 18 feet by 4 feet = 72 square feet. Adding 3 inches to the 6 inches thickness of the slab as an allowance for the weight of the beam, we have 9 × 12 = 108 pounds per square foot for the dead weight of the floor. With a factor of 2 for dead load, this equals 216. Using a factor of 4 on the live load (200), we have 800 pounds per square foot. Then the ultimate load on the beam, due to these sources, is (216 + 800)

72 = 73,152 pounds. So far as its effect on moment is concerned, the concentrated load of 2,400 pounds at the center would have the same effect as 4,800 pounds uniformly distributed. As it is a piece of vibrating machinery, we shall use a factor of *six* (6), and thus have an ultimate effect of 6 × 4,800 = 28,800 pounds. Adding this to 73,152, we have 101,952 pounds as the equivalent, ultimate, uniformly distributed load. Then,

$$M_o = \tfrac{1}{8} W_u l = \tfrac{1}{8} \times 101{,}952 \times 216 = 2{,}752{,}704.$$

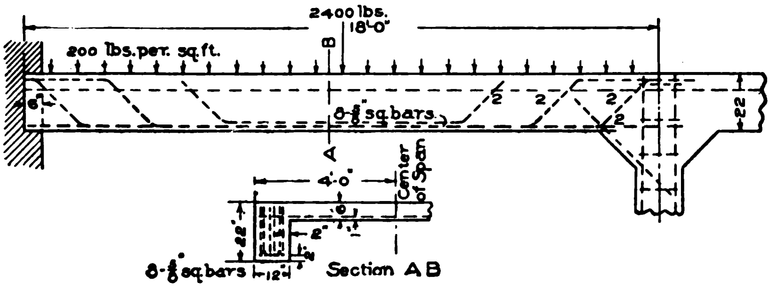


Fig. 102. Reinforced Beam.

In order to reduce as much as possible the size and weight of this beam, we shall use 1 : 2 : 4 concrete, and therefore apply Equation **24:**

$$2{,}752{,}704 = 565\, bd^2;$$
$$bd^2 = 4{,}872.$$

If $b = 16$ inches, $d^2 = 304.5$, and $d = 17.5$ inches.

A still better combination would be a deeper and narrower beam with $b = 12$ inches, and $d = 20.15$ inches. With this combination, the required area of the steel will equal:

$$A = .0121 \times bd = .0121 \times 12 \times 20.15 = 2.93 \text{ square inches}.$$

This can be supplied by eight bars ⅝ inch square.

The total ultimate load as determined above, is 101,952 pounds. One-half of this gives the maximum shear at the ends, or 50,976 pounds. Applying Equation **31,** we have, since $d - x = .85\, d = 17$ inches:

$$v = \frac{V}{b\,(d - x)} = \frac{50{,}796}{12 \times 17} = 249 \text{ pounds per square inch.}$$

As already discussed in previous cases, the ends of the beam must be reinforced against diagonal tension, since the above value of v is too great, even as an ultimate value, for such stress. Therefore the ends of the beam must be reinforced by turning the bars up, or by the use of stirrups. The beam must therefore be reinforced about as shown in Fig. 112. Although the concentrated center load in this case is comparatively too small to require any change in the design, it should not be forgotten that a concentrated load *may* cause the shear to change so rapidly that it might require special provision for it by means of stirrups in the center of the beam, where there is ordinarily no reinforcement which will assist shearing stresses.

281. **Effect of Quality of Steel.** There is one very radical difference between the behavior of a concrete-steel structure and that of a structure composed entirely of steel, such as a truss bridge. A truss bridge may be overloaded with a load which momentarily passes the elastic limit, and yet the bridge will not necessarily fail, nor cause the truss to be so injured that it is useless and must be immediately replaced. The truss might sag a little, but no immediate failure is imminent. On this account, the factor of safety on truss bridges is usually computed on the basis of the ultimate strength.

A concrete-steel structure acts very differently. As has already been explained, the intimate union of the concrete and the steel at *all* points along the length of the bar (and not merely at the ends), is an absolute essential for stability. If the elastic limit of the steel has been exceeded owing to an overload, then the union between the concrete and the steel has unquestionably been destroyed, provided that union depends on mere adhesion. Even if that union is assisted by a mechanical bond, the distortion of the steel has broken that bond to some extent, although it will still require a very considerable force to pull the bar through the concrete. It is therefore necessary that the elastic limit of the steel should be considered the virtual ultimate so far as the strength of the steel is concerned. It is accordingly considered advisable, as already explained, to multiply all working loads by the desired factor of safety (usually taken as 4), and then to proportion the steel and concrete so that such an ultimate load will produce crushing in the upper fibre of the concrete, and at the same time will stress the steel to its elastic limit. On this basis, economy in the use of steel requires that the elastic limit should be made as high as possible.

The manufacture of steel of very high elastic limit requires the use of a comparatively large proportion of carbon, which may make the steel objectionably brittle. The steel for this purpose must therefore avoid the two extremes—on the one hand, of being brittle; and on the other, of being so soft that its elastic limit is very low.

Several years ago, bridge engineers thought that a great economy in bridge construction was possible by using *very* high carbon steel, which has not only a high elastic limit but also a correspondingly high ultimate tensile strength. But the construction of such bridges requires that the material shall be punched, forged, and otherwise handled in a way that will very severely test its strength and perhaps cause failure on account of its brittleness. The stresses in a concrete-steel structure are very different. The steel is never punched; the individual bars are never subjected to transverse bending *after* being placed in the concrete. The direct shearing stresses are insignificant. The main use, and almost the only use, of the steel, is to withstand a direct tension; and on this account a considerably harder steel may be used than is usually considered advisable for steel trusses.

If the structure is to be subject to excessive impact, a somewhat softer steel will be advisable; but even in such a case, it should be remembered that the mere weight of the structure will make the effect of the shock far less than it would be on a skeleton structure of plain steel. The steel ordinarily used in bridge work, generally has an elastic limit of from 30,000 to 35,000. If we use even 33,000 pounds as the value for s on the basis of ultimate loading, we shall find that the required percentage of steel is very high. On the other hand, if we use a grade of steel in which the carbon is somewhat higher, having an ultimate strength of about 90,000 to 100,000 pounds per square inch, and an elastic limit of 55,000 pounds per square inch, the required percentage of steel is much lower.

282. **Slabs on I-Beams.** There are still many engineers who will not adopt reinforced concrete for the skeleton structure of buildings, but who construct the frames of their buildings of steel, using steel I-beams for floor-girders and beams, and then connect the beams with concrete floor-slabs (Fig. 103). These are usually computed on the basis of transverse beams which are free at the ends, instead of considering them as *continuous beams*, which will add about 50 per cent to their strength. Since it would be necessary to move the reinforcing steel from the lower part to the upper part of the slab when passing over the floor-beams, in order to develop the additional strength which

is theoretically possible with continuous beams, and since this is not usually done, it is by far the safest practice to consider all floor-slabs as being "free-ended." The additional strength which they undoubtedly have to some extent because they are continuous over the beams, merely adds indefinitely to the factor of safety. Usually the requirement that the I-beams shall be *fireproofed* by surrounding the beam itself with a layer of concrete such that the outer surface is at least 2 inches from the nearest point of the steel beam, results in having a shoulder of concrete under the end of each slab, which quite materially adds to its structural strength. But usually no allowance is made; nor is there any reduction in the thickness of the slab on account of this added strength. In this case also, the factor of safety is again indefinitely increased. The fireproofing around the beam must usually be kept in place by wrapping a small sheet of expanded metal or wire lath around the lower part of the beam before the concrete is placed.

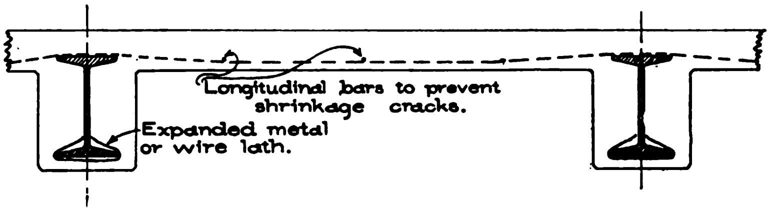

Longitudinal bars to prevent shrinkage cracks.
Expanded metal or wire lath.

Fig. 103. Concrete Floor-Slabs on I-Beam Girders.

Slabs Reinforced in Both Directions. When the floor-beams of a floor are spaced nearly equally in both directions, so as to form, between the beams, panels which are nearly square, a material saving can be made in the thickness of the slab by reinforcing it with bars running in both directions. The theoretical computation of the strength of such slabs is exceedingly complicated. It is usually considered that such slabs have twice the strength of a slab supported only on two sides and reinforced with bars in but one direction. The usual method of computing such slabs is to compute the slab thickness, and the spacing and size of the reinforcing steel, for a slab which is to carry *one-half* of the actual load. Strictly speaking, the slab should be thicker by the thickness of one set of reinforcing bars.

283. **Reinforcement against Temperature Cracks.** The modulus of elasticity of ordinary concrete is approximately 2,400,000 pounds

per square inch, while its ultimate tensional strength is about 200 pounds per square inch. Therefore a pull of about $\frac{1}{12,000}$ of the length would nearly, if not quite, rupture the concrete. The coefficient of expansion of concrete has been found to be almost identical with that of steel, or .0000065 for each degree Fahrenheit. Therefore, if a block of concrete were held at the ends with absolute rigidity, while its temperature were lowered about 12 degrees, the stress developed in the concrete would be very nearly, if not quite, at the rupture point. Fortunately the ends will not usually be held with such rigidity; but nevertheless it does generally happen that, unless the entire mass of concrete is permitted to expand and contract freely so that the temperature stresses are small, the stresses will usually localize themselves at the weak point of the cross-section, wherever it may be, and will there develop a crack, provided the concrete is not reinforced with steel. If, however, steel is well distributed throughout the cross-section of the concrete, it will prevent the concentration of the stresses at local points, and will distribute it uniformly throughout the mass.

Reinforced-concrete structures are usually provided with bars running in all directions, so that temperature cracks are prevented by the presence of such bars, and it is generally unnecessary to make any special provision against such cracks. The most common exception to this statement occurs in floor-slabs, which structurally require bars in only one direction. It is found that cracks parallel with the bars which reinforce the slab will be prevented if a few bars are laid perpendicularly to the direction of the main reinforcing bars. Usually ½-inch or ⅜-inch bars, spaced about 2 feet apart, will be sufficient to prevent such cracks.

Retaining walls, the balustrades of bridges, and other similar structures, which may not need any bars for purely structural reasons, should be provided with such bars in order to prevent temperature cracks. A theoretical determination of the amount of such reinforcing steel is practically impossible, since it depends on assumptions which are themselves very doubtful. It is usually conceded that if there is placed in the concrete an amount of steel whose cross-sectional area equals about ⅓ of 1 per cent of the area of the concrete, the structure will be proof against such cracks. Fortunately, this amount of steel is so small that any great refinement in its determination is of little importance. Also, since such bars have a value in tying the structure together, and thus adding somewhat to its strength and ability to resist disintegration owing to vibrations, the bars are usually worth what they cost.

STRENGTH OF T-BEAMS

284. When concrete beams are laid in conjunction with overlying floor-slabs, the concrete for both the beams and the slabs being laid in one operation, the strength of such beams is very much greater than their strength considered merely as plain beams, even though we compute the depth of the beams to be equal to the total depth from the bottom of the beam to the top of the slab. An explanation of this added strength may be made as follows:

If we were to construct a very wide beam with a cross-section such as is illustrated in Fig. 104, there is no hesitation about calculating such strength as that of a plain beam whose width is *b*, and whose effective depth to the reinforcement is *d*. Our previous study in plain beams has shown us that the steel in the bottom of the beam takes care of practically all the tension; that the neutral axis of the beam is somewhat above the center of its height; that the only work of the concrete below the neutral axis is to transfer the stress in the steel to the concrete in the top of the beam; and that even in this work it must be assisted somewhat by stirrups or by bending up the steel bars. If, therefore, we cut out from the lower corners of the beam two rectangles, as shown by the unshaded areas, we are saving a very large part of the concrete, with very little loss in the strength of the beam, provided we can fulfil certain conditions. The steel, instead of being distributed uniformly throughout the bottom of the wide beam, is concentrated into the comparatively narrow portion which we shall hereafter call the *rib* of the beam. The concentrated tension in the bottom of this rib must be transferred to the compression area at the top of the beam. We must also design the beam so that the shearing stresses in the plane (*mn*) immediately below the

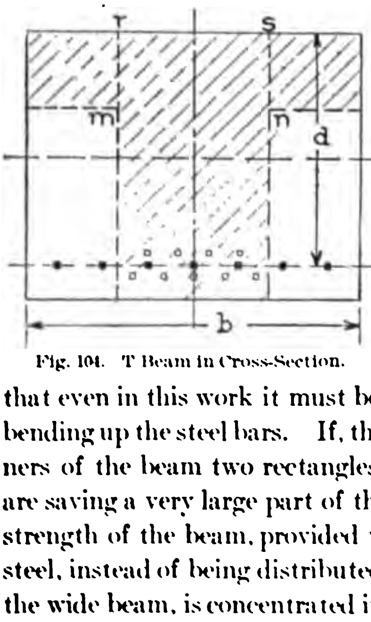


Fig. 104. T-Beam in Cross-Section.

slab shall not exceed the allowable shearing stress in the concrete. We must also provide that failure shall not occur on account of shearing in the vertical planes (*m r* and *n s*) between the sides of the beam and the flanges.

285. **Resisting Moments of T-Beams.** These will be computed in accordance with straight-line formulæ. There are three possible cases, according as the neutral axis is: (1) *below* the bottom of the slab (which is the most common case, and which is illustrated in Fig. 105); (2) *at* the bottom of the slab; or (3) *above* it. All possible effect of tension in the concrete is ignored. For Case 1, even the compression furnished by the concrete between the neutral axis and the under side of the slab is ignored. Such compression is of course zero at the neutral axis; its maximum value at the bottom of the slab is small; the summation of the compression is evidently small; the lever arm is certainly not more than $\frac{2}{3}y$; therefore the moment due to such compression is insignificant compared with the resisting moment due to the slab. The computations are much more complicated; the resulting error is a very small percentage of the true figure, and the error is on the side of safety.

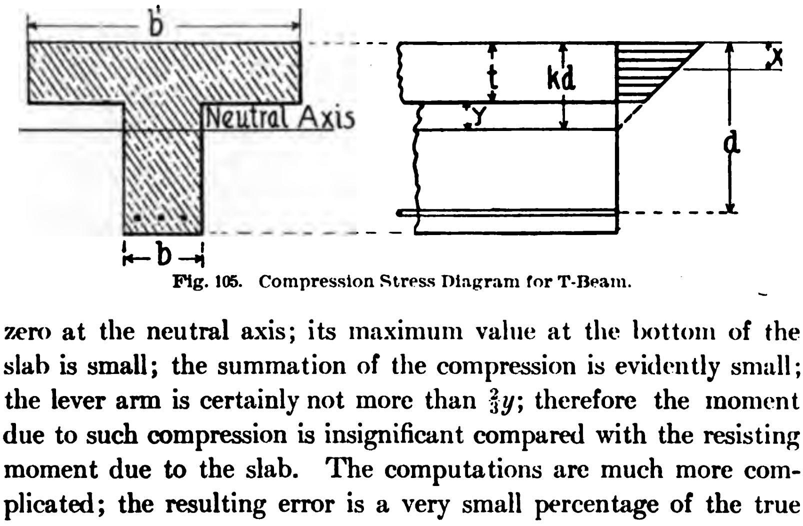


Fig. 105. Compression Stress Diagram for T-Beam.

286 *Case* 1. If c is the maximum compression at the top of the slab, and the stress-strain diagram is rectilinear, as in Fig. 105, then the compression at the bottom of the slab is $c\frac{kd-t}{kd}$. The average compression $= \frac{1}{2}\left(c + c\frac{kd-t}{kd}\right) = \frac{c}{kd}(kd - \frac{1}{2}t)$. The total compression equals the average compression multiplied by the area $b't$; or,

$$C = As = b't\,\frac{c}{kd}(kd - \tfrac{1}{2}t) \qquad \text{. . . . .(32)}$$

The center of gravity of the compressive stresses is evidently at the center of gravity of the trapezoid of pressures. The distance z of this center of gravity from the top of the beam is given by the formula

$$z = \frac{t}{3}\,\frac{3kd - 2t}{2kd - t} \quad \ldots\ldots\ldots\ldots (33)$$

It has already been shown in Article 264, that:

$$\frac{\epsilon_c}{\epsilon_s} = \frac{cr}{s} = \frac{kd}{d - kd}.$$

Combining this equation with Equation 32, we may eliminate $\frac{c}{s}$, and obtain a value for *kd*:

$$kd = \frac{Ard + \frac{1}{2}\,b't^2}{Ar + b't} \quad \ldots\ldots\ldots (34)$$

If the percentage of steel is chosen at random, the beam will probably be over-reinforced or under-reinforced. In general it will therefore be necessary to compute the moment with reference to the steel and also with reference to the concrete, and, as before with plain beams (Equation 29), we shall have a pair of equations:

$$\left.\begin{aligned} M_c &= C(d - x) = b't\,\frac{c}{kd}\,(kd - \tfrac{1}{2}t)(d - x) \\ M_s &= As(d - x) = pb'ds\,(d - x) \end{aligned}\right\} \ldots . (35)$$

287. *Case* 2. If we place $kd = t$ in the equation above Equation 34, and solve for d, we have a relation between d, c, s, r, and t, which holds when the neutral axis is just at the bottom of the slab. The equation becomes:

$$d = \frac{t(cr + s)}{cr} \quad \ldots\ldots\ldots\ldots (36)$$

A combination of dimensions and stresses which would place the neutral axis *exactly* in this position, is improbable, although readily possible; but Equation 36 is very useful to determine whether a given numerical problem belongs to Case 1 or Case 3. When the stresses s and c in the steel and concrete, the ratio r of the elasticities, and the thickness t of the slab are all determined, then the solution of Equation 36 will give a value of d which would bring the neutral axis at the bottom of the slab. But it should not be forgotten that the compression in the concrete (c) and the tension in the steel will not simultaneously have certain definite values (say $c = 500$, and $s = 16{,}000$) unless the percentage of steel has been so chosen as to give those simultaneous values. When, as is usual, some other percentage of steel is used, the equation is not strictly applicable, and it therefore should not be used to determine a value of d which will place the neutral axis at the bottom of the slab and thus simplify somewhat the numerical calculations. For example, for $c = 500$, $s = 16{,}000$, $r = 12$, and $t = 4$ inches, d will equal 14.67 inches. Of course this particular depth may not satisfy the requirements of the problem. If the proper value for d is *less* than that indicated by Equation **36**, the problem belongs to Case 3; if it is *more*, the problem belongs to Case 1.

288. *Case* 3. The diagram of pressure is very similar to that in Fig. 105, except that it is a triangle instead of a trapezoid, the triangle having a base c and a height kd which is less than t. The center of compression is at $\frac{1}{3}$ the height from the base, or $x = \frac{1}{3}\,kd$. Equations **25** to **29** are applicable to this case as well as to Case 2, which may be considered merely as the limiting case to Case 3. But it should be remembered that b' refers to the width of the flange or slab, and not to the width of the stem or rib.

289. **Width of Flange.** The width (b') of the flange is usually considered to be equal to the width between adjacent beams, or that it extends from the middle of one panel to the middle of the next. The chief danger in such an assumption lies in the fact that if the beams are very far apart, they must have corresponding strength to carry such a floor load, and the shearing stresses between the rib and the slab will be very great. The method of calculating such shear will be given later. It sometimes happens (as illustrated in Article 296), that the width of slab on each side of the rib is almost indefinite. In such a case we must arbitrarily assume some limit, and say that the compression in the slab which is due to the T-beam is confined to a strip which is (say) fifteen or twenty times the thickness of the slab. If the compression is computed for two cases, both of which have the same size of rib, same steel, same thickness of slab, but different slab widths, it is found, as might be expected, that for the narrower slab width the unit-compression is greater, the neutral axis is very slightly lower, and even the unit-tension in the steel is slightly greater. No demonstration has ever been made to determine any limitation of width of slab beyond which no compression would be developed by the transverse stress in a T-beam

rib under it. It is probably safe to assume that it extends for seven to ten times the thickness of the slab on *each* side of the rib. If the beam as a whole is safe on this basis, then it is still safer for any additional width to which the compression may extend.

290. **Width of Rib.** Since it is assumed that all of the compression occurs in the slab, the only work done by the concrete in the rib is to transfer the tension in the steel to the slab, to resist the shearing and web stresses, and to keep the bars in their proper place. The width of the rib is somewhat determined by the amount of reinforcing steel which must be placed in the rib, and whether it is desirable to use two or more rows of bars instead of merely one row. As indicated in Fig. 104, the amount of steel required in the base of a T-beam is frequently so great that two rows of bars are necessary in order that the bars may have a sufficient spacing between them so that the concrete will not split apart between the bars. Although it would be difficult to develop any rule for the proper spacing between bars without making assumptions which are perhaps doubtful, the following empirical rule is frequently adopted by designers: *The spacing between bars, center to center, should be two and a-quarter times the diameter of the bars.* Fire insurance and municipal specifications usually require that there shall be two inches clear outside of the steel This means that the beam shall be four inches wider than the net width from out to out of the extreme bars. The data given in Table XVIII will therefore be found very convenient, since, when it is desired to use a certain number of bars of given size, a glance at the table will show immediately whether it is possible to space them in one row; and, if this is not possible, the necessary arrangement can be very readily designed. For example, assume that six ⅞-inch bars are to be used in a beam. The table shows immediately that the required width of the beam (following the rule) will be 14.72 inches; but if, for any reason, a beam 11 inches wide is considered preferable, the table shows that four ⅞-inch bars may be placed side by side, leaving two bars to be placed in an upper row. Following the same rule regarding the spacing of the bars in vertical rows, the distance from center to center of the two rows should be 2.25 × .875 = 1.97 inches, showing that the rows should be, say, two inches apart center to center. It should also be noted that the plane of the center of gravity of this steel is at two-fifths of the distance between the bars

TABLE XVIII

Required Width of Beam, Allowing $2\frac{1}{4} \times d$, for Spacing Center to Center, and 2 Inches Clear on Each Side

Formula: $\text{Width} = (n-1)2.25d + d + 4 = 2.25nd - 1.25d + 4$

No. of Bars	½-In.	⅝-In.	¾-In.	⅞-In.	1-In.	1¼-In.
2	5.62 In.	6.03 In.	6.44 In.	6.84 In.	7.25 In.	8.06 In.
3	6.75 "	7.44 "	8.13 "	8.81 "	9.50 "	10.87 "
4	7.87 "	8.84 "	9.81 "	10.78 "	11.75 "	13.68 "
5	9.00 "	10.25 "	11.50 "	12.75 "	14.00 "	16.50 "
6	10.12 "	11.65 "	13.19 "	14.72 "	16.25 "	19.31 "
7	11.25 "	13.06 "	14.87 "	16.68 "	18.50 "	22.12 "
8	12.37 "	14.46 "	16.56 "	18.65 "	20.75 "	24.94 "
9	13.50 "	15.87 "	18.25 "	20.62 "	23.00 "	27.75 "
10	14.62 "	17.28 "	19.94 "	22.59 "	25.25 "	30.56 "

above the lower row, or that it is eight-tenths of an inch above the center of the lower row.

291. **Numerical Illustrations of T-Beams.** *Example* 1. Assume that a 5-inch slab is supporting a load on beams spaced 8 feet apart, the beams having a span of 20 feet. Assume that the moment of the beam has been computed as 900,000 inch-pounds. What will be the dimensions of the beam if the concrete is not to have a compression greater than 500 pounds per square inch and the tension of the steel is not to be greater than 16,000 pounds per square inch?

Answer. There are an indefinite number of solutions to this problem. There are several terms in Equation 35 which are mutually dependent; it is therefore impracticable to obtain directly the depth of the beam on the basis of assuming the other quantities; therefore it is only possible to assume figures which experience shows will give approximately accurate results, and then test these figures to see whether all the conditions are satisfied. Within limitations, we may assume the amount of steel to be used, and determine the depth of beam which will satisfy the other conditions together with that of the assumed area of steel. For example, we shall assume that six ⅞-inch square bars having an area of 4.59 square inches will be a suitable reinforcement for this beam. We shall also assume as a trial figure that $x = 1.5$. Substituting these values in the second formula of Equation 35, we may write the second formula:

$$900{,}000 = 4.59 \times 16{,}000\,(d - 1.5).$$

Solving for d, we find that $d = 13.75$. If we test this value by means of Equation **36**, we shall find that, substituting the values of t, c, r, and s in Equation **36**, the resulting value of d equals 18.33. This shows that if we make the depth of the beam only 13.75, the neutral axis will probably be within the slab, and the problem comes under Case 3, to which we must apply Equation **29**. Dividing the area of the steel, 4.50, by $(b' \times d)$, we have the value of p equals .00348. Interpolating with this value of p in Table XV, we find that when $r = 12$, $k = 2.50$; $kd = 3.44$; $x = 1.15$; and $d - x = 12.6$. Substituting these values in Equation **29**, we find that the moment $900{,}000 = 2{,}082c$, or that $c = 432$ pounds per square inch. This shows that the unit-compression of the concrete is safely within the required figure. Substituting the known values in the second part of Equation **29**, we find that the stress in the steel s equals about 15,500 pounds per square inch.

Example 2. Assume that a floor is loaded so that the total weight of live and dead load is 200 pounds per square foot; assume that the T-beams are to be 5 feet apart, and that the slab is to be 4 inches thick; assume that the span of the T-beams is 30 feet. We therefore have an area of 150 square feet to be supported by each beam, which will give a total load of 30,000 pounds on each beam. The moment at the center of such a beam will therefore be equal to the total load, multiplied by one-eighth of the span (expressed in inches), and the moment is therefore 1,350,000 inch-pounds. As a trial value, we shall assume that the beam is to be reinforced with six ¾-inch bars, which have an area of 3.37 square inches. Substituting this value of the area in the second part of Equation **35**, and assuming that $s = 16{,}000$ pounds per square inch, we find as an approximate value for $d - x$, that it will equal 25 inches. This is very much greater than the value of d that would be found from substituting the proper values in Equation **36**, so that we know at once that the problem must be solved by the methods of Case 1. For a 4-inch slab, the value of x must be somewhere between 1.33 and 2.0. As a trial value, we may call it 1.5, and this means that d will equal 26.5. Assuming that this slab is to be made of concrete using a value for $r = 12$, we know all the values in Equation **34**, and may solve for kd, which we find to equal 5.54 inches. As a check on the approximations made above, we may substitute this value of kd, and also the value of t in Equation **33**, and obtain a more precise value of x, which we find to equal 1.62. Substituting the value of the moment and the other known quantities in the upper formula of Equation **35**, we may solve for the value of c, and obtain the value that $c = 352$ pounds per square inch. This value for c is so very moderate that it would probably be economy to assume a lower value for the area of the steel, and increase the unit-compression in the concrete; but this solution will not be here worked out.

292. **Approximate Formulæ.** A great deal of T-beam computation is done on the basis that the center of pressure of the concrete is at the middle of the slab, and therefore that the lever-arm of the steel $= d - \frac{1}{2}t$. From these assumptions we may write the approximate formula:

$$M_s = As\left(d - \frac{1}{2}t\right) \quad \ldots\ldots\ldots\ldots (37)$$

If the values of M_s and s are known or assumed, we may assume a reasonable value for either A or $(d - \frac{1}{2}t)$ and calculate the corresponding value of the other. On the assumption that the slab takes *all* the compression, the distance between the steel and the center of compression of the concrete varies between $(d - \frac{1}{2}t)$ and $(d - .14t)$, which is the approximate value when the beam becomes so small that it merges into the slab. The smaller value $(d - \frac{1}{2}t)$ is the absolute limit which is never reached. Therefore the lever-arm is *always* larger than $(d - \frac{1}{2}t)$. Therefore, if we use Equation **37** to compute the area of steel A for a definite moment M_s and unit steel tension s, the resulting value of A for an assumed depth d, or the resulting value of d for an assumed area A, will be larger than necessary. In either case the result is safe, but uneconomically so.

As an illustration, using the values in Example 2, Article 291, of $M_s = 1{,}350{,}000$; $s = 16{,}000$; $(d - \frac{1}{2}t) = (26.5 - 2) = 24.5$, the resulting value of $A = 3.44$ square inches, which is larger than the more precise value previously computed.

Equation **37** is particularly applicable when the neutral axis is in the rib. Under this condition, the *average* pressure on the concrete of the slab is always greater than $\frac{1}{2}c$, or at least it is never less than $\frac{1}{2}c$. As before explained, the average pressure just equals $\frac{1}{2}c$ when the neutral axis is at the bottom of the slab. We may therefore say that the total pressure on the slab is *always greater* than ½ $c\,b\,t$. We therefore write the approximate equation:

$$M_c = \frac{1}{2} c\, b'\, t \left(d - \frac{1}{2} t\right) \cdots\cdots\cdots\cdots (38)$$

As before, the values obtained from this equation are safe, but are unnecessarily so. Applying them to Example 2, Article 291, by substituting M_c = 1,350,000, b' = 60, t = 4, and (d − ½t) = 24.5, we compute c = 459. But we know that this approximate value of c is greater than the true value; and if this value is safe, then the true value is certainly safe. The more accurate value of c, computed in Article 291, is 352. If the value of c in Equation 38 is assumed, and the value of d is computed, the result is a depth of beam unnecessarily great.

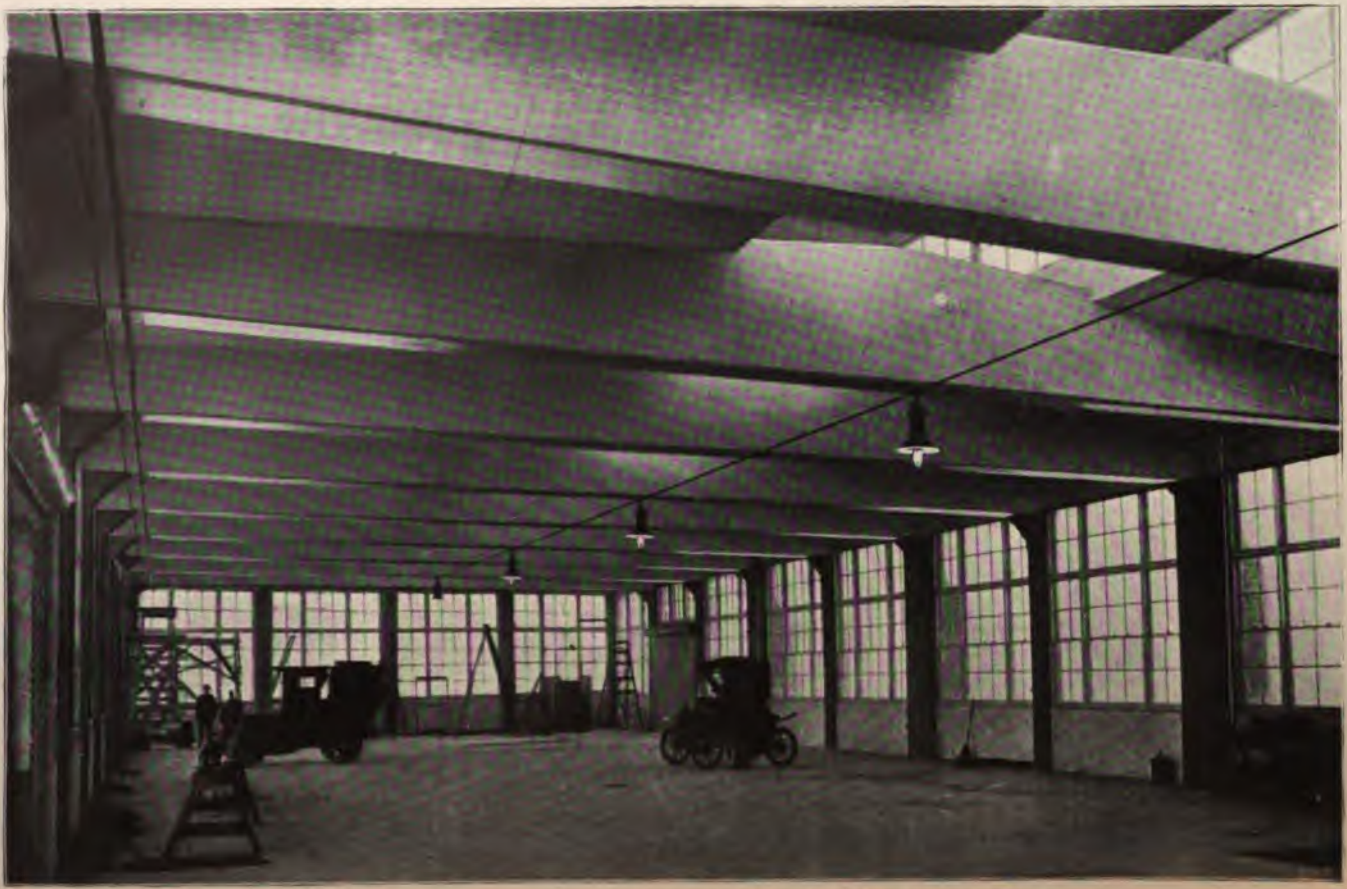
If the beam is so shallow that we may know, even without the test of Equation 36, that the neutral axis is *certainly* within the slab, then we may know that the center of pressure is certainly less than ⅓ t from the top of the slab, and that the lever-arm is certainly less than (d − ⅓t); and we may therefore modify Equation 37 to read:

$$M_s = A s \left(d - \frac{1}{3} t\right) \cdots\cdots\cdots\cdots (39)$$

Applying this to Example 1 of Article 291, and substituting M_s = 900,000, s = 16,000, (d − ⅓ t) = (13.75 − 1.67) = 12.08, we find that A = 4.65, instead of the 4.59 previously computed. This again illustrates that the formula gives an excessively safe value, although in this case the difference is small.

Equations 37 and 38 should be considered as a pair which are applied according as the steel or the concrete is the determining feature. When the percentage of steel is assumed (as is usual), both equations should be used to test whether the unit-stresses in both the steel and the concrete are safe. It is impracticable to form a simple approximate equation corresponding to Equation 39, which will express the moment as a function of the compression in the concrete. Fortunately it is unnecessary, since, when the neutral axis is within the slab, there is always an abundance of compressive strength.

293. **Shearing Stresses between Beam and Slab.** Every solution for T-beam construction should be tested at least to the extent



INTERIOR OF GARAGE OF THE GEORGE N. PIERCE COMPANY, BUFFALO, N. Y.

Note the 55-foot spans and the efficient lighting. Kahn System of Reinforced Concrete.

Courtesy of Trussed Concrete Steel Company, Detroit, Mich.

of knowing that there is no danger of failure on account of the shear between the beam and the slab, either on the horizontal plane at the lower edge of the slab, or in the two vertical planes along the two sides of the beam. Let us consider a T-beam such as is illustrated in Fig. 106. In the lower part of the figure is represented one-half of the length of the flange, which is considered to have been separated from the rib. Following the usual method of considering this as a free body in space, acted on by external forces and by such internal forces as are necessary to produce equilibrium, we find that it is acted on at the left end by the abutment reaction, which is a vertical force, and also by a vertical load on top. We may consider P' to represent the summation of all compressive forces acting on the flanges at the center of the beam. In order to produce equilibrium, there must be a shearing force acting on the under side of the flange. We represent this force by S_h. Since these two forces are the only horizontal forces, or forces with horizontal components, which are acting on this free body in space, P' must equal S_h. Let us consider z to represent the shearing force per unit of area. We know from the laws of mechanics, that, with a uniformly distributed load on the beam, the shearing force is maximum at the ends of the beam, and diminishes uniformly towards the center, where it is zero. Therefore the *average* value of the unit-shear for the half-length of the beam, must equal $\frac{1}{2}z$. As before, we represent the width of the rib by b. For convenience in future computations, we shall consider L to represent the length of the beam, measured in *feet*. All other dimensions are measured in inches. Therefore the total shearing force along the lower side of the flange, will be:

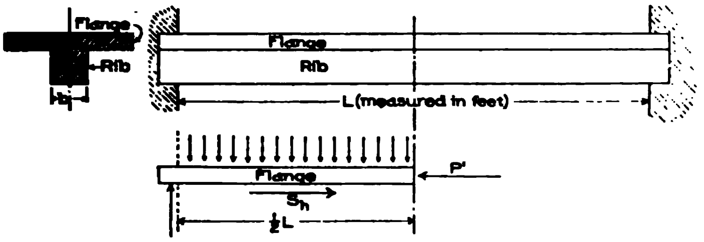


Fig. 106. Analysis of Stresses in T-Beam.

$$S_h = \frac{1}{2}z \times b \times \frac{1}{2}L \times 12 = 3bzL \quad \ldots\ldots (40)$$

There is also a possibility that a beam may fail in case the flange (or the slab) is too thin; but the slab is always reinforced by bars which are transverse to the beam, and the slab will be placed on both sides of the beam, giving two shearing surfaces.

294. **Numerical Illustration.** It is required to test the beam which was computed in Example 1 of Article 291. Here the total compressive stress in the flange $= \frac{1}{2}\,cbkd = \frac{1}{2} \times 432 \times 96 \times 344 = 71{,}332$ pounds. But this compressive stress measures the shearing stress S_h between the flange and the rib. This beam requires six $\frac{7}{8}$-inch bars for the reinforcement. We shall assume that the rib is to be 11 inches wide, and that four of the bars are placed in the bottom row, and two bars about 2 inches above them. The effect of this will be to deepen the beam slightly, since d measures the depth of the beam to the *center* of the reinforcement, and, as already computed numerically in Article 290, the center of gravity of this combination will be $\frac{8}{10}$ of an inch above the center of gravity of the lower row of bars. Substituting in Equation **40** the values $S_h = 71{,}332$, $b = 11$, and $L = 20$, we find, for the unit-value of z, 108 pounds per square inch. This shows that the assumed dimensions of the beam are satisfactory in this respect, since the true shearing stress permissible in concrete is higher than this.

But the beam must be tested also for its ability to withstand shear in vertical planes along the sides of the rib. Since the slab in this case is 5 inches thick and we can count on both surfaces to withstand the shear, we have a width of 10 inches to withstand the shear, as compared with the 11 inches on the underside of the slab. The unit-shear would therefore be $\frac{11}{10}$ of the unit-shear on the under side of the slab, and would equal 119 pounds per square inch. Even this would not be unsafe, but the danger of failure in this respect is usually guarded against by the fact that the slab almost invariably contains bars which are inserted to reinforce the slab, and which have such an area that they will effectively prevent any shearing in this way.

Testing Example 2 similarly, we may find the total compression C from Equation **32**, and that it equals $As = 16{,}000 \times 3.37 = 54{,}000$ pounds. The steel reinforcement is six $\frac{3}{4}$-inch bars, and by Table XVIII we find that if placed side by side, the beam must be 13.19 inches in width, or, in round numbers, 13¼ inches. Substituting these values in Equation **37**, we find, for the value of z, 45 pounds per square inch. Such a value is of course perfectly safe. The shear along the sides of the beam will be considerably greater, since the slab is only four inches thick, and twice the thickness is but 8 inches; therefore the maximum unit-shear along the sides will equal 45 times the ratio of 13.25 to 8, or 75 pounds per square inch. Even this would be perfectly safe, to say nothing of the additional shearing strength afforded by the slab bars.

295. **Shear in a T-Beam.** The shear here referred to is the shear of the beam as a whole on any vertical section. It does not refer to the shearing stresses between the slab and the rib.

The theoretical computation of the shear of a T-beam is a very complicated problem. Fortunately it is unnecessary to attempt to solve it exactly. The shearing resistance is certainly far greater in the case of a T-beam than in the case of a plain beam of the same width and total depth and loaded with the same total load. Therefore, if the shearing strength is sufficient, according to the rule, for a plain beam, it is certainly sufficient for the T-beam. In the first example of Article 291, the total load on the beam is 30,000 pounds. Therefore the maximum shear V at the end of the beam, is 15,000 pounds. In this particular case, $d - x = 12.25$. For this beam, $d = 13.75$ inches, and $b = 11$ inches. Substituting these values in Equation **31**, we have:

$$v = \frac{V}{b(d-x)} = \frac{15{,}000}{11 \times 12.25} = 111 \text{ pounds per square inch.}$$

Although this is probably a very safe stress for direct shearing, it is more than double the allowable direct tension due to the diagonal stresses; and therefore ample reinforcement must be provided. If only two of the ⅞-inch bars are turned at an angle of 45° at the end, these two bars will have an area of 1.54 square inches, and will have a working tensile strength (at the unit-stress of 16,000 pounds) of 24,640 pounds. This is more than the total vertical shear at the ends of the beam; and we may therefore consider that the beam is protected against this form of failure.

296. **Numerical Illustration of Slab, Beam, and Girder Construction.** Assume a floor construction as outlined in skeleton form in Fig. 107. The columns are spaced 16 feet by 20 feet. Girders which support the alternate rows of beams, connect the columns in

the 16-foot direction. The live load on the floor is 150 pounds per square foot. The concrete is to be a 1:2:4 mixture, with $r = 10$, and $c = 600$. Required the proper dimensions for the girders, beams, and slab.

The load on the girders may be computed in either one of two ways, both of which give the same results. We must consider that each *beam* supports an area of 8 feet by 20 feet. We may therefore consider that girder *d* supports the load of *b* (on a floor area 8 ft. by 20 ft.) as a concentrated load in the center. Or, we may consider that, ignoring the beams, the girder supports a uniformly distributed load on an area 16 ft. by 20 ft. The moment in either case is the same. Assume that we shall use a 1 per cent reinforcement in the slab. Then, from Table XV, with $r = 10$, and $p = .01$, we find that $k = .358$; then $x = .119\,d$, or $(d - x) = .881\,d$. As a trial, we estimate that a 5-inch slab (or $d = 4$) will carry the load. This will weigh 60 pounds per square foot, and make a total live and dead load of 210 pounds per square foot. A strip one foot wide and 8 feet long will carry a total load of 1,680 pounds, and its moment will be $\frac{1}{8} \times 1{,}680 \times 96 = 20{,}160$ inch-pounds. Using the first half of Equation **29**, we can substitute the known values, and say that:

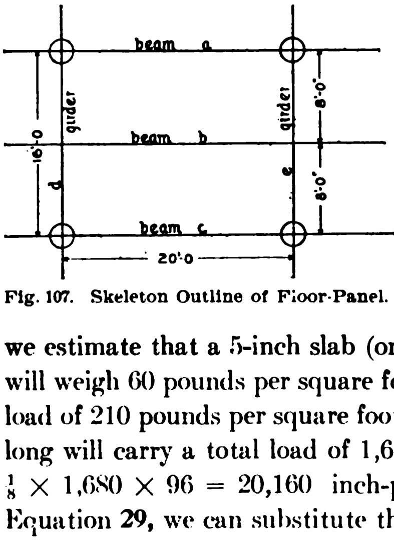


Fig. 107. Skeleton Outline of Floor-Panel.

$$20{,}160 = \frac{1}{2} \times 600 \times 12 \times .358\,d \times .881\,d$$

$$= 1{,}135\,d^2$$

$$d^2 = 17.76$$

$$d = 4.21$$

In this case the span of the slab is considered as the distance from center to center of the beams. This is evidently more exact than to use the net span (which equals eight feet, *less* the still unknown width of beam), since the true span is the distance between the *centers of pressure* on the two beams. It is probable that the true span (really indeterminable) will be somewhat less than 8 feet, which would probably justify using the round value of $d = 4$ inches, and the slab thickness as 5 inches, as first assumed. The area of the steel per *inch* of width of the slab = $pbd = .01 \times 1 \times 4.21 = .0421$ square inch. Using ½-inch round bars whose area equals .1963 square inch, the required spacing of the bars will be $.1963 \div .0421 = 4.66$ inches. Practically this would be called 4⅝ inches.

The load on a beam is that on an area of 8 feet by 20 feet, and equals $8 \times 20 \times 210 = 33{,}600$ pounds for live and dead load. As a rough trial value, we shall assume that the beam will be 12 inches wide and 15 inches deep below the slab, or a volume of $1 \times 1.25 \times 20$ cubic feet = 25 cubic feet, which will weigh 3,750 pounds. Adding this, we have 37,350 pounds as the total live and dead load carried by each beam. The load is uniformly distributed; and the moment:

$$M = \frac{1}{8} \times 37{,}350 \times 240 = 1{,}120{,}500 \text{ inch-pounds.}$$

We shall assume that the beam is to have a depth d to the reinforcement, of 22 inches, and shall utilize Equation 39 to obtain an approximate value for the area. Substituting the known quantities in Equation 39, we have:

$$1{,}120{,}500 = A \times 16{,}000 \times (22 - 1.67)$$
$$A = 3.44 \text{ square inches.}$$

For T-beams with very wide slabs and great depth of beam, the percentage of steel is always very small. In this case, $p = 3.44 \div (96 \times 22) = .00163$. Such a value is beyond the range of those given in Table XV, and therefore we must compute the value of k from Equation 27; and we find that $k = .165$; $kd = 3.63$, which shows that the neutral axis is within the slab; $x = \frac{1}{3}\,kd = 1.21$, and therefore $(d - x) = 20.79$. Substituting these values in the upper part of Equation 29 in order to find the value of c, we find that $c = 309$ pounds per square inch. Substituting the known values in the second half of Equation 29, in order to obtain a more precise value of s, we find that $s = 15{,}737$ pounds per square inch.

The required area (3.44 square inches) of the bars will be afforded by six ⅞-inch round bars ($6 \times .60 = 3.60$) with considerable to spare. From Table XVIII we find that six ⅞-inch bars (either square or round), if placed in one row, would require a beam 14.72 inches wide. This is undesirably wide, and so we shall use four bars in the lower row, and two above, and make the beam 11 inches wide. This will add nearly an inch to the depth, and the total depth will be

22 + 3, or 25 inches. The concrete below the slab is therefore 11 inches wide by 20 inches deep, instead of 12 inches wide by 15 inches deep, as assumed when computing the dead load. The section of 220 square inches will therefore weigh more than the suggested section of 180 square inches; but the difference in dead load weight is so small that it is unnecessary to alter the calculations, especially since the unit-stresses in the concrete and steel are both lower than the working limits. It should also be noted that the span of these beams was considered as 20 feet, which is the distance from center to center of the columns (or of the girders). This is certainly more nearly correct than to use the *net* span between the columns (or girders), which is yet unknown, since neither the columns nor the girders are yet designed. There is probably *some* margin of safety in using the span as 20 feet.

The load on one beam is computed above as 37,350 pounds. The load on the girder is therefore the equivalent of this load *concentrated* at the center, or of *double* the load (74,700 pounds) uniformly distributed. Assuming for a trial value that the girder will be 12 inches by 22 inches below the slab, its weight for sixteen feet will be 4,392, or say 4,400 pounds. This gives a total of 79,100 pounds as the equivalent total live and dead load uniformly distributed over the girder. Its moment in the center therefore equals $\frac{1}{8} \times 79{,}100 \times 192 = 1{,}898{,}400$ inch-pounds.

The width of the slab in this case is almost indefinite, being twenty feet, or forty-eight times the thickness of the slab. We shall therefore assume that the compression is confined to a width of fifteen times the slab thickness, or that $b' = 75$ inches. Assume for a trial value that $d = 25$ inches; then from Equation **39**, if $s = 16{,}000$, we find that $A = 5.08$ square inches. Then $p = .0027$; and, from Equation **27**, $k = .207$, and $kd = 5.175$. This shows that the neutral axis is below the slab, and that it belongs to Case 1, Article 286. Checking the computation of kd from Equation **34**, we compute $kd = 5.18$, which is probably the more correct value because computed more directly. The discrepancy is due to the dropping of decimals during the computations. From Equation **33**, we compute that $x = 1.72$; then $(d - x) = 23.28$. Substituting the value of the moment and of the dimensions in the upper part of Equation **35**, we compute c to be 420 pounds per square inch. Similarly, making substitutions in the lower part of Equation 35, using the more precise value of $(d - x)$ for the lever-arm of the steel, we find $s = 16{,}052$ pounds per square inch. The student should verify in detail all these computations.

The total required area of 5.08 square inches may be divided into, say, 8 round bars $\frac{7}{8}$ inch in diameter. These would have an area of 4.81 square inches. The discrepancy is about five per cent. These bars, placed in two rows, would require that the beam should be at least 10.78 inches wide. We shall call it 11 inches. The total depth of the beam will be three inches greater than d, or 28 inches. This means 23 inches below the slab, and the area of concrete below the slab is therefore $11 \times 23 = 253$ square inches, rather than $12 \times 22 = 264$ square inches, as assumed for trial.

Shear. The shearing stresses between the rib and slab of the girder are of special importance in this case. The quantity S_h of Article 293 equals the total compression in the concrete, which equals the total tension in the steel, which equals, in this case, $16{,}052 \times 5.08 = 81{,}544$ pounds. This equals $3\,bzl$, in which $b = 11$, $l = 16$ (feet), and z is to be determined.

$$z = 81{,}544 \div (3 \times 11 \times 16) = 154 \text{ pounds per square inch.}$$

This measures the maximum shearing stress under the slab, and is almost safe, even without the assistance furnished by the stirrups and the bars, which would come up diagonally through the ends of the beam (where this maximum shear occurs) nearly to the top of the slab. The vertical planes on each side of the rib have a combined width of 10 inches, and therefore the *unit-stress* is $\frac{11}{10} \times 154 = 169$ pounds per square inch. This is a case of true shear, and a 1:2:4 concrete should stand such a stress with a large factor of safety. But there are still other shearing stresses in these vertical planes. Considering a strip of the slab, say, one foot wide, which is reinforced by slab bars that are *parallel* to the girder, the elasticity of such a strip (if *disconnected* from the girder) would cause it to sag in the center. This must be prevented by the shearing strength of the concrete in the vertical plane along each edge of the girder rib. On account of the combined shearing stresses along these planes, it is usual to specify that when girders are parallel with the slab bars, bars shall be placed across the girder and through the top of the slab for the special purpose of resisting these shearing stresses. Some of the stresses are indefinite, and therefore no precise rules can be computed for the amount of the reinforcement. But since the amount required is evidently very small, no great percentage of accuracy is important. A recent specification on this point required ⅜-inch bars, 5 feet long, spaced 12 inches apart.

The shear of the girder, taken as a whole, should be computed as for simple beams, as already discussed in Article 295; and stirrups should be used, as described in Article 279.

Another special form of shear must be considered in this problem. Where the beams enter the girders, there is a *tendency* for the beams to tear their way out through the girder. The total load on the girder by the two beams on each side, is of course equal to the total load on one beam, and equals 37,350 pounds. Some of the reinforcing bars of the beam will be bent up diagonally so that they enter the girder near its top, and therefore the beam could not tear out without shearing through the girder from near its top or for a depth of, say, 22 inches (3 inches less than d). We therefore have $2 \times 22 \times 11 = 484$ square inches, the area to be sheared out. Dividing this into 37,350 gives 77 pounds per square inch. Although this is probably a safe shearing stress, many engineers would consider it advisable to use special V-shaped stirrups (see *a*, Fig. 108) to strengthen the beam against such stress. If the angle of these stirrups with the vertical is, say, 45°, then the stress in the bars on each side will be .707 of the total load, assuming that these bars were to take *all* the stress. This would mean that these bars would have a stress of about 26,406 pounds, and at 16,000 pounds per square inch would require a total area of 1.65 square inches. Three ¾-inch bars would therefore more than provide the necessary area, even assuming that these stirrups took the entire load, and disregarding the stirrups such as would ordinarily be placed in the beam, and also disregarding the shearing strength of the concrete. If, therefore, these stirrups are made of

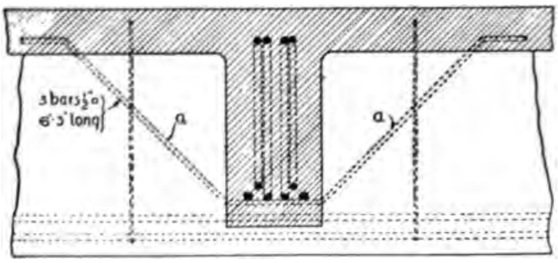


Fig. 108. Detail of Reinforcement at Junction of Beam and Girder.

½-inch bars instead of ¾-inch bars, the shearing stresses in the concrete due to the beam will be amply provided for. A complete detailed drawing will show all of the bars required for a panel between four of these columns. The student should study this drawing (see Fig. 109) in connection with the foregoing demonstrations of the dimensions of the bars and of the concrete.

FOOTINGS

297. Simple Footings. When a definite load, such as a weight carried by a column, is to be supported on a subsoil whose bearing power has been estimated at some definite figure, the required area of the footing becomes a perfectly definite quantity, regardless of the method of construction of the footing. But with the area of the footing once determined, it is possible to effect considerable economy in the construction of the footing, by the use of reinforced concrete. An ordinary footing of masonry is usually made in a pyramidal form, although the sides will be stepped off instead of being made sloping. It may be approximately stated that the depth of the footing below the base of the column, when ordinary masonry is used, must be

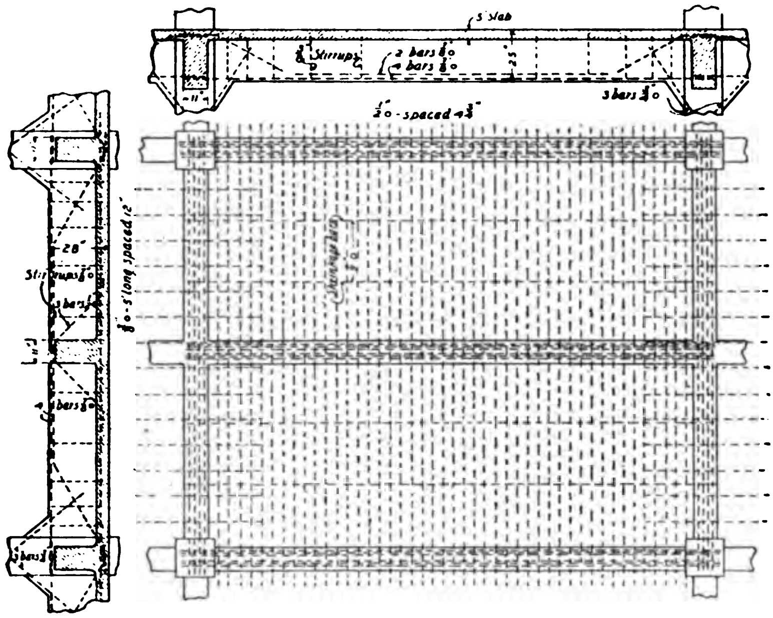


Fig. 109. Detail of Floor-Panel.

practically equal to the width of the footing. The offsets in the masonry cannot ordinarily be made any greater than the heights of the various steps. Such a plan requires an excessive amount of masonry.

A footing of reinforced concrete consists essentially of a slab, which is placed no deeper in the ground than is necessary to obtain a proper pressure from the subsoil. In the simplest case, the column is placed in the middle of the footing, and thus acts as a concentrated load in the middle of the plate (Fig. 110). The mechanics of such a problem are somewhat similar to those of a slab supported on four sides and carrying a concentrated load in the center, with the very important exception, that the resistance, instead of being applied merely at the edges of the slab, is uniformly distributed over the entire surface. Since the column has a considerable area, and the slab merely overlaps the column on all sides, the common method is to consider the overlapping on each side to be an inverted cantilever carrying a uniformly distributed load, which is in this case an upward pressure. The maximum moment evidently occurs immediately below each vertical face of the column. At the extreme outer edge of the slab the moment is evidently zero, and the thickness of the slab may therefore be reduced considerably at the outer edge. The depth of the slab, and the amount of reinforcement, which is of course placed near the bottom, can be determined according to the usual rules for obtaining a moment. This can best be illustrated numerically.

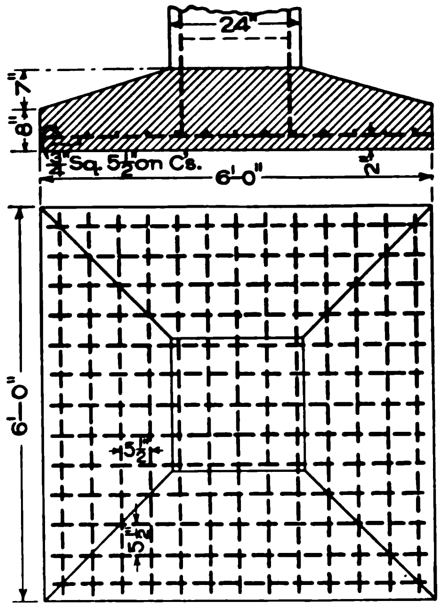


Fig. 110. Simple Footing of Reinforced Concrete.

Example. Assume that a load of 252,000 pounds is to be carried by a column, on a soil which consists of hard, firm gravel. Such soil will ordinarily safely carry a load of 7,000 pounds per square foot. On this basis, the area of the footing must be 36 square feet, and therefore a footing 6 feet square will answer the purpose. A concrete column 24 inches square will safely carry such a loading. Placing such a column in the middle of a footing will leave an offset 2 feet broad outside each face of the column. We may consider a section of the footing made by passing a vertical plane through one face of the column. This leaves a block of the footing 6 feet long and 2 feet wide, on which there is an upward pressure of 12 × 7,000 = 84,000 pounds. The center of pressure is 12 inches from the section, and the moment is therefore 12 × 84,000 = 1,008,000 inch-pounds. Multiplying this by 4, we have 4,032,000 inch-pounds as the ultimate moment. Applying Equation **21**, we place this equal to $397\ bd^2$, in which $b = 72$ inches. Solving this for d, we have $d = 11.9$ inches. A total thickness of 15 inches would therefore answer the purpose. The amount of steel required per inch of width $= .0084\ d = .0084 \times 11.9 = .100$ square inch of steel per inch of width. Therefore ¾-inch bars spaced 5.6 inches apart will serve the purpose. A similar reinforcing of bars should be placed perpendicularly to these bars.

The above very simple solution would be theoretically accurate in the case of an offset 2 feet wide for the footing of a wall of indefinite length, assuming that the upward pressure was 7,000 pounds per square foot. The development of such a moment uniformly along the section of our square footing, implies a resistance to bending near the outer edges of the slab which will not actually be obtained. The moment will certainly be greater under the edges of the column. On the other hand, we have used bars in both directions. The bars passing under the column in each direction are just such as are required to withstand the moment produced by the pressure on that part of the footing directly in front of each face of the column. It may be considered that the other bars have their function in tying the two systems into one plate whose several parts mutually support one another. If further justification of such a method is needed, it may be said that experience has shown that it practically fulfils its purpose.

A more effective method of reinforcing a simple footing is shown in Fig. 111. Two sets of the reinforcing bars are at *a-a* and *b-b*, and are placed only under the column. To develop the strength of the corners of the footings, bars are placed diagonally across the footing, as at *c-c* and *d-d*. In designing this footing, the projections of the footing beyond the column are treated as free cantilever beams, or by the method discussed above. The maximum shear occurs near the center; and therefore, if it is necessary to take care of this shear by means of reinforcement, it should be provided by using stirrups.

Example. Assume that a load of 300,000 pounds is to be carried by a column 28 inches square, on a soil that will safely carry a load of 6,000 pounds per square foot. What should be the dimensions of the footing and the size and spacing of the reinforcing bars? The bars are to be placed diagonally as well as directly across the footing, as illustrated in Fig. 111. Also investigate the shear.

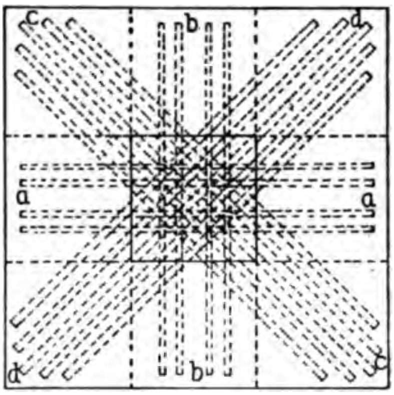


Fig. 111. Reinforcement for Footing.

Solution. The load of 300,000 pounds will evidently require an area of 50 square feet. The sides of the square footing will evidently be 7.07 feet, or, say, 85 inches; and the offset on each side of the 28-inch column is 28.5 inches. The area of each cantilever wing which is straight out from the column is 28.5 × 28 = 798 square inches = 5.54 square feet. The load is therefore 5.54 × 6,000 = 33,250 pounds. Its lever-arm is one-half of 28.5 inches, or 14.25 inches. The moment is therefore 473,812 inch-pounds. Adopting the straight-line formula $M_c = 80bd^2$, on the basis that $p = .0086$, we may write the equation:

$$473{,}812 = 80 \times 28 \times d^2,$$

from which,

$$\begin{aligned} d^2 &= 211; \\ d &= 14.5; \\ A = pbd &= .0086 \times 28 \times 14.5 \\ &= 3.59 \text{ square inches.} \end{aligned}$$

This area of metal may be furnished by eight ¾-inch round bars, and therefore there should be eight ¾-inch round bars spaced about 3.5 inches apart under the column in *both* directions.

The mechanics of the reinforcement of the corner sections, which are each 28.5 inches square, is exceedingly complicated in its precise theory. The following approximation to it is probably sufficiently precise. The area of each corner section is the square of 28.5 inches, or 812.25 square inches. At 6,000 pounds per square foot, the pressure on such a section will be 33,844 pounds, and the center of gravity of this section is of course at the center of the square, which is 14.25 × 1.414 = 20.15 inches from the corner of the column. A bar immediately under this diagonal line would have a lever-arm of 20.15 inches. A bar parallel to it would have the same lever-arm from the middle of the bar to the point were it passes under the column. Therefore, if we consider that this entire pressure of 33,844 pounds has an average lever-arm of 20.15 inches, we have a moment of 681,957 inch-pounds. Using, as before, the moment equation $M_c = 80bd^2$, we may transpose this equation to read $b = M_c \div 80d^2$. Then,

$$\begin{aligned} A = pbd &= p\frac{M}{80d^2}d = p\frac{M}{80d} \\ &= .0086 \times \frac{681{,}957}{80 \times 14.5} \\ &= 5.06 \text{ square inches.} \end{aligned}$$

This area of steel will be furnished by five 1⅛-inch round bars. The diagonal reinforcement will therefore consist of five 1⅛-inch round bars running diagonally in *both* directions. These bars should be spaced about 4 inches apart. Those that are precisely under the diagonal lines of the square should be about 9 feet 8 inches long; those parallel to them will each be 8 inches shorter than the next bar.

Shear. The total load of this column is 300,000 pounds. The shear in the footing is of course a maximum immediately under the edges of the column. The perimeter of the column is four times 28 inches, or 112 inches. The thickness of the footing is something greater than the value found above for d (14.5 inches), and we shall therefore make it, say, 18 inches. This will mean that the surface area which would need to be punched out if the column were to shear its way through the footing would be 18 × 112 inches, or 2,016 square inches. Since the area of the column is approximately one-ninth of the area of the footing, the shearing force is about eight-ninths of the total load on a column, or it is eight-ninths of 300,000 pounds, which is 266,667 pounds. Dividing this by 2,016, we have about 130 pounds per square inch as the shearing force on the concrete of the footing, ignoring the assistance from the 26 bars in the footing. There is therefore no occasion to provide for shear in such a footing. The intensity of the shear decreases from the maximum value just given, to zero at the edges of the footing.

298. **Continuous Beams.** Continuous beams are sometimes used to save the expense of underpinning an adjacent foundation or wall. These footings are designed as simple beams, but the steel is placed in the top of the beams.

Example. Assume that the columns on one side of a building are to be supported by a continuous footing; that the columns are 22 inches square, spaced 12 feet on center; and that they support a load of 195,000 pounds each. If the soil will safely support 6,000 pounds per square foot, the area required for a footing will be 195,000 ÷ 6,000 = 32.5 square feet. Since the columns are spaced 12 feet apart, the width of footing will be 32.5 ÷ 12 = 2.71 feet, or 2 feet 9 inches. To find the depth and amount of reinforcement necessary for this footing, it is designed as a simple inverted beam supported at both ends (the columns), and loaded with an upward pressure of 6,000 pounds per square foot on a beam 2 feet 9 inches wide. In computing the moment of this beam, the continuous-beam principle may be utilized on all except the end spans, and thus reduce the moment and therefore the required dimensions of the beam. Many engineers ignore this principle, since it merely increases the factor of safety to do so.

299. **Beam Footing.** When a simple footing supports a single column, the center of pressure of the column must pass vertically through the center of gravity of the footing, or there will be dangerous transverse stresses in the column, as discussed later. But it is sometimes necessary to support a column on the edge of a property when it is not permissible to extend the foundations beyond the property line. In such a case, a simple footing is impracticable. The method of such a solution is indicated in Fig. 112, without numerical computation. The nearest interior column (or even a column on the opposite side of the building, if the building be not too wide) is selected, and a combined footing is constructed under both columns. The weight on both columns is computed. If the weights are equal, the center of gravity is half-way between them; if unequal, the center of gravity is on the line joining their centers, and at a distance from

them such that (see Fig. 112) $x:y::W_2:W_1$. In this case, evidently W_2 is the greater weight. The area *a b c d* must fulfil two conditions:

(1) The area must equal the total loading ($W_1 + W_2$), divided by the allowable loading per square foot; and,

(2) The center of gravity must be located at *O*.

An analytical solution of the relative and absolute values of *a b* and *c d* which will fulfil the two conditions, is very difficult, and fortunately is practically unnecessary. If x and y are equal, *a b c d* is a rectangle. If W_2 is greater than 2 W_1, then y will be less than $\frac{1}{2}x$; and even a triangle with the vertex under the column W_1 would not fulfil the condition. In fact, if W_1 is very small compared with W_2, it might be impracticable to obtain an area sufficiently large to sustain the weight. The proper area can be determined by a few trials, with sufficient accuracy for the purpose.

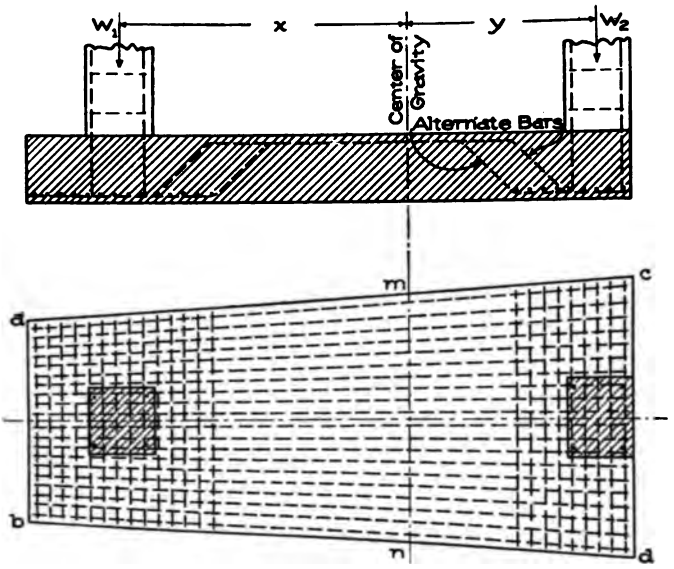


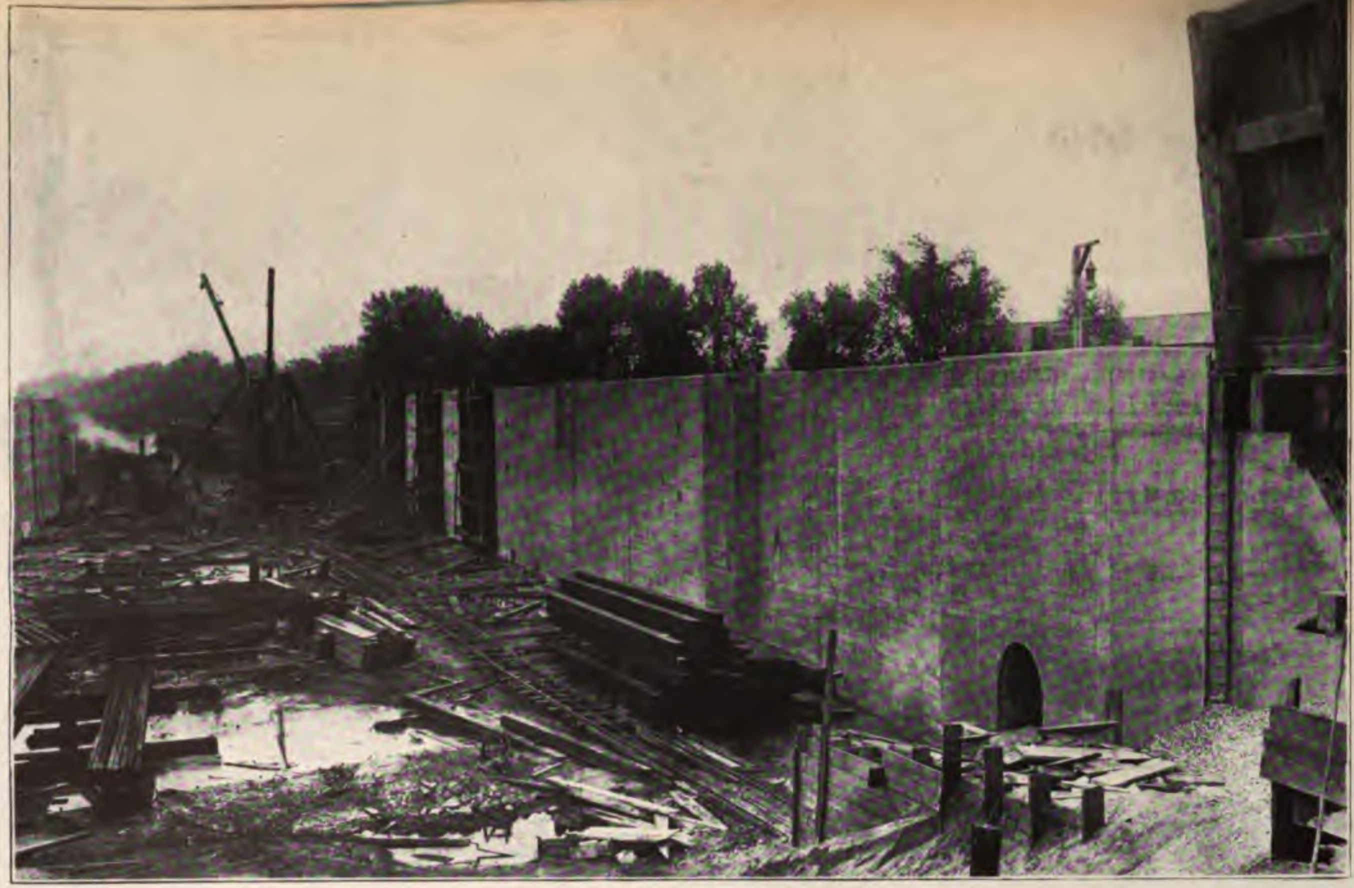
Fig. 112. Combined Footing for Two Columns, One on Edge of Property.

The footing must be considered as an inverted beam at the section *m n*, where the moment $= W_2 y - \frac{1}{2} W_1 y$. The width is *m n*; and the required depth and the area of the steel must be computed by the usual methods. The bars will here be in the top of the footing, but will be bent down to the bottom under the columns, as shown in Fig. 112. The cross-bars under each column will be designed, as in the case of the simple footing, to distribute the wei[illegible] each column across the width of the footing, and to transfer the [illegible] to the longitudinal bars.

RETAINING WALLS

300. **Essential Principles.** The economy of a retaining wall of reinforced concrete lies in the fact that by adopting a skeleton form of construction and utilizing the tensional and transverse strength which may be obtained from reinforced concrete, a wall may be built, of which the volume of concrete is, in some cases, not more than one-third the volume of a retaining wall of plain concrete which would answer the same purpose. Although the cost of reinforced concrete per cubic foot will be somewhat greater than that of plain concrete, it sometimes happens that such walls can be constructed for one-half the cost of plain concrete walls. The general outline of a reinforced-concrete retaining wall is similar to the letter L, the base of which is a base-plate made as wide as (and generally a little wider than) the width usually considered necessary for a plain concrete wall. As a general rule, the width of the base should be about one-half the height. The face of the wall is made of a comparatively thin plate whose thickness is governed by certain principles, as explained later. At intervals of 10 feet, more or less, the base-plate and the face are connected by *buttresses*. These buttresses are very strongly fastened by tie-bars to both the base-plate and the face-plate. The stress in the buttresses is almost exclusively tension. The pressure of the earth tends to force the face-plate outward; and therefore the face-plate must be designed on the basis of a vertical slab subjected to transverse stresses which are maximum at the bottom and which reduce to zero at the top.

If the wall is "surcharged" (which means that the earth at the top of the wall is not level, but runs back at a slope), then the face-plate will have transverse stresses even at the top. The base-plate is held down by the pressure of the superimposed earth. The buttresses must transmit the bursting pressure on the face of the wall backward and downward to the base-plate. The base-plate must therefore be designed by the same method as a horizontal slab carrying a load equal and opposite to the upward pull in each buttress. If the base-plate extends in front of the face of the wall, thus forming



IMPROVEMENT OF THE MISSISSIPPI RIVER

Construction of lock at Moline, Illinois. View of south wall, looking east from west end.

an extended toe, as is frequently done with considerable economy and advantage, even that toe must be designed to withstand transverse bending at the wall line, and also shearing at that point. The application of these principles can best be understood by an illustration.

301. *Numerical Example.* Assume that it is required to design a retaining wall to withstand an ordinary earthwork pressure of 20 feet, the earth being level on top. We are at once confronted with the determination of the actual lateral pressure of the earthwork. Unfortunately, this is an exceedingly uncertain quantity, depending upon the nature of the soil, upon its angle of repose, and particularly upon its condition whether wet or dry. The *angle of repose* is the largest angle with the horizontal at which the material will stand without sliding down. A moment's consideration will show that this angle depends very largely on the condition of the material, whether wet or dry, etc. On this account any great refinement in these calculations is utterly useless.

Assuming that the back face of the wall is vertical, or practically so; that the upper surface of the earth is horizontal; and that the angle of repose of the material is 30°, the total pressure of the wall equals $\frac{1}{6}\, w\, h^2$, in which h is the total height of the wall, and w is the weight per unit-volume of the earth. If the angle of repose is steeper than this, the pressure will be less.' If the angle of repose is less than this, the fraction $\frac{1}{6}$ will be larger, but the unit-weight of the material will *probably* be smaller. Assuming the weight at the somewhat excessive figure of 96 pounds per cubic foot, we can then say, as an ordinary rule, that the total pressure of the earth on a vertical strip of the wall one foot wide will equal $16\, h^2$, in which h is the height of the wall in feet. The average pressure, therefore, equals $16\, h$; and the maximum pressure at a depth of h feet equals $32\, h$. Applying this figure to our numerical example, we have a total pressure on a vertical strip one foot wide, of $16 \times 20^2 = 6{,}400$ pounds. The pressure at a depth of 20 feet $= 32 \times 20 = 640$ pounds.

It is usual to compute the thickness and reinforcement of a strip one foot wide running horizontally between two buttresses. Practically the strip at the bottom is very strongly reinforced by the baseplate, which runs at right angles to it; but if we design a strip at the bottom of the wall without allowing for its support from the base-plate, and then design all the strips toward the top of the wall in the same proportion, the upper strips will have their proper design, while the lower strip merely has an excess of strength. We shall assume, in this case, that the buttresses are spaced 15 feet center to center. Then the load on a horizontal strip of face-plate 12 inches high, 15 feet long, and 19 feet 6 inches from the top, will be 15 × 19.5 × 32, or 9,360 pounds. Multiplying this by 4, we have an ultimate load of 37,440 pounds. The span in inches equals 180. Then,

$$M_o = \frac{37,440 \times 180}{8} = 842,400 \text{ inch-pounds.}$$

Placing this equal to 397 bd^2, in which $b = 12$ inches, we find that $d^2 = 176.8$, and $d = 13.3$ inches. At one-half the height of the wall, the moment will equal one-half of the above, and the required thickness d would be 9.4 inches. The actual thickness at the bottom, including that required outside of the reinforcement, would therefore make the thickness of the wall about 16 inches at the bottom. At one-half the height, the thickness must be about 12 inches. Using a uniform taper, this would mean a thickness of 8 inches at the top.

The reinforcement at the bottom would equal .0084 × 13.3 = .112 square inch of metal per inch of height. Such reinforcement could be obtained by using ¾-inch bars spaced 5 inches apart. The reinforcement at the center of the height would be .0084 × 9.4 = .079 square inch per inch of width. This could be obtained by using ⅝-inch bars about 5 inches apart, or by using ¾-inch bars about 7 inches apart. The selection and spacing of bars can thus be made for the entire height. While there is no method of making a definite calculation for the steel required in a vertical direction, it may be advisable to use ½-inch bars spaced about 18 inches apart.

302. **Base-Plate.** We shall assume that the base-plate has a width of one-half the height of the wall, or is 10 feet wide. If the inner face of the face-plate is 2 feet 6 inches from the toe, the width of the base-plate sustaining the earth pressure is 7 feet 6 inches. The actual pressure on the base-plate is that due to the total weight of the earth. The upward pull on the buttresses is less than this, and is measured by the moment of the horizontal pressure tending to tip the wall over. To resist this overturning tendency, there must be a downward pressure on the plate whose moment equals the moment of the

couple tending to turn the wall over. The pressure on the wall on a vertical strip one foot wide, as found above, is 6,400 pounds, which has a lever-arm, about the center of the base of the face-plate, of 6 feet 8 inches. The vertical pressure to resist this will be applied at the center of the 7-foot 6-inch base, or 4 feet 5 inches from the center of the face-plate. The total necessary pressure will therefore be $\frac{6{,}400 \times 6.67}{4.42}$, or 9,653 pounds. This means an average pressure of 1,287 pounds per square foot. Making a similar calculation for this base-plate to that previously made for the face-plate, we find that the thickness $d = 19.1$ inches. This shows that our base-plate should have a total thickness of about 22 inches.

The amount of steel *per inch of width* of the slab equals .0084 × 19.1 = .160 square inch. This can be provided by ⅞-inch bars spaced 4¾ inches apart, or by 1-inch bars spaced 6¼ inches apart. This reinforcement will be uniform across the total width of the base-plate.

303. **Buttresses.** The total pressure on a vertical strip one foot wide is 6,400 pounds. For a panel of 15 feet, this equals 96,000 pounds; and its moment about the base of the wall equals 96,000 × 80 inches = 7,680,000 inch-pounds. If the tie-bars in the buttresses are placed about 3 inches from the face of the buttresses, their distance from the center of the base of the face-wall will be about 89 inches. Therefore the tension in the bars in each buttress will equal $\frac{7{,}680{,}000}{89}$ = 86,292 pounds.

Since the earth pressures considered above are actual pressures, we must here consider working stresses in the metal. Allowing 15,000 pounds' tension in the steel, it will require 5.75 square inches of steel for the tie-bar of each buttress. Six 1-inch square bars will more than furnish this area. Even these bars need not all be extended to the top of the buttress, since the tension is gradually being transferred to the face-plate.

The width of the buttress is not very definitely fixed. It must have enough volume to contain the bars properly, without crowding them. In this case, for the six 1-inch bars, we shall make the width 12 inches. At the base of the buttresses, these bars should be bent around bars running through the base-plate, so that the lower part of the buttress will be very thoroughly anchored into the base-plate. It is also necessary to tie the buttress to the face-plate. The amount of this tension is definitely calculated for each foot of height, from the total pressure on the face-plate in each panel for that particular foot of height. At a depth of 19.5 feet, we found a bursting pressure of 624 pounds per square foot, or 9,360 pounds on the 15-foot panel. This would therefore be the required bond between the buttress and the face-plate at a depth of 19.5 feet. With a working tension of 15,000 pounds per square inch, such a tension would be furnished by .624 square inch of metal. This equals .05 square inch of metal for each inch of height, and ½-inch bars spaced 5 inches apart will furnish this tension. The amount of this tension varies from the above, to zero at the top of the wall. This tension is usually provided by small bars, such as ½-inch bars, which are bent at a right angle so as to hook over the horizontal bars in the face-plate and run backward to the back of the buttress.

In the design described above, the extension of the toe beyond the face of the wall is so short that there is no danger that the toe will be broken off on account of either shearing or transverse stress. It is usually good policy to place some transverse bars in the base-plate which are perpendicular to the face of the wall, and to have them extend nearly to the point of the toe. No definite calculation can be made of the required number of these bars, unless they are required to withstand transverse bending of the toe.

If there is any danger that the subsoil is liable to settle, and thus produce irregular stresses on the base-plate, a large reinforcement in this direction may prove necessary. It is good policy to place at least ½-inch bars every 12 inches through the base-plate, for the prevention of cracks; and this amount should be increased as the uncertainty in the stress in the base-plate increases. Although there are no definite stresses in the top of the wall, it is usual to make the thickness of the face-plate at least 6 inches at the top, and also to place a finishing cornice on top of the wall, somewhat as is shown in Fig. 113.

When the subsoil is very unreliable, it is even possible that there might be a tendency for the front and back of the base-plate to sink, and to break the base-plate by tension of the top. This can be resisted by bars in the upper part of the base-plate which are perpendicular to the wall.

304. L-Shaped Retaining Walls. Retaining walls of very moderate height may be constructed in L-shaped sections without buttresses, by thickening the walls at the base, and by using sufficient reinforcement to resist the transverse stresses, which, of course, have their maximum value at the base of the wall (Fig. 114). From the standpoint of cubic yards of concrete and pounds of steel, such a wall is not as economical as the buttressed wall, but the forms are very much more simple and are less expensive. A low wall is always made much thicker than mere theoretical computation would call for, and in such a case the additional thickening for the L design might be little or nothing. For high walls—twenty feet or more—the economy utterly disappears. The mechanics of this form of wall is quite different from the form with buttresses. In the case of a buttressed wall, the vertical plate between the buttresses is merely designed to resist the bursting pressure on a slab which has the buttresses as abutments. When there are no abutments, the pressure on each unit vertical strip of the wall must be computed; and the strength

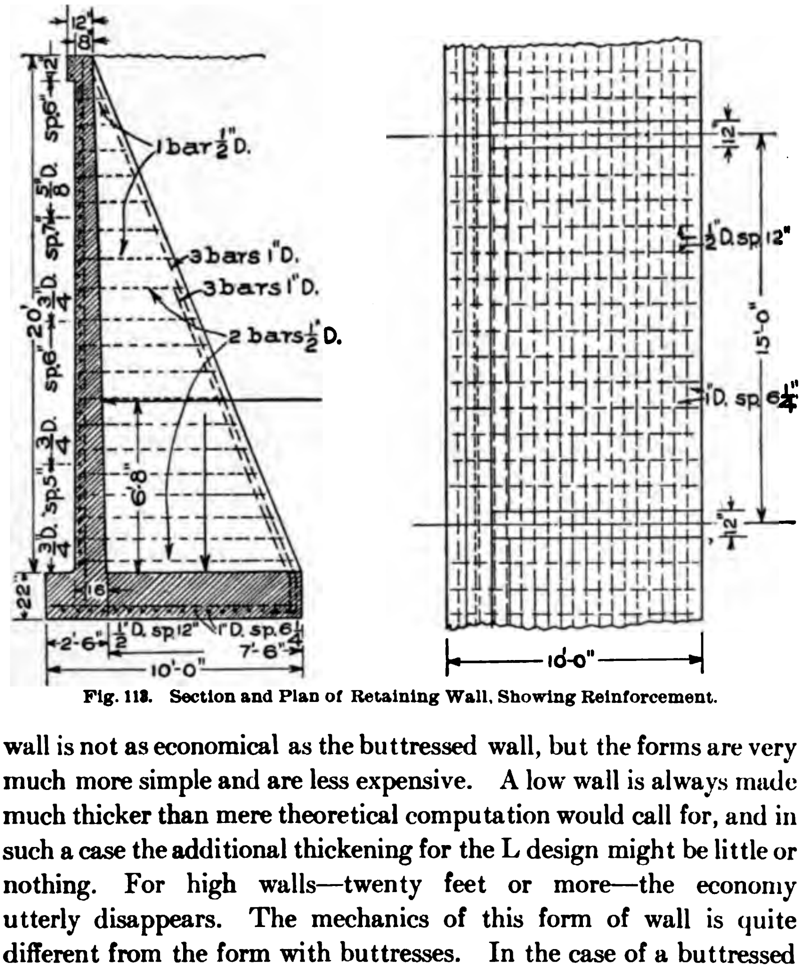


Fig. 113. Section and Plan of Retaining Wall, Showing Reinforcement.

at every section (vertically) must be computed on the basis of a cantilever acted on by horizontal forces. This practically means tha the moment increases from zero at the top of the wall to a maximu at the base just above the base-plate. Of course the mechanics the wall taken as a whole, in its pressure on the subsoil, is identic with that of the other form of retaining wall.

WIND BRACING

305. **General Principles.** The practical applications of t principles of reinforced concrete which have already been discuss have been almost exclusively th required for sustaining vertical loa but a structure consisting simply beams, girders, slabs, and colum *may* fall down, like a house of car unless it is provided with late bracing to withstand wind press and any lateral forces tending turn it over. The necessary pro ion for such stresses is usually m by placing *brackets* in the an between posts and girders, as been illustrated in Fig. 102. Th brackets are reinforced with which will resist any tensile st on the brackets. The compres strength of concrete may be relie to resist a tendency to crush brackets by compression. Us such brackets will occur in pai each end of a beam supported on columns. If we consider that any given moment is to be div equally between two brackets, then, if we are to have a working sion of 15,000 pounds per square inch in the steel, and a wor compression of 500 pounds per square inch in the concrete, the of the concrete must be 30 times the area of the steel. But since outer face of the concrete will have practically twice the compres of the concrete at the angle of the beam and column, and since maximum of 500 pounds per square inch must not be exceeded

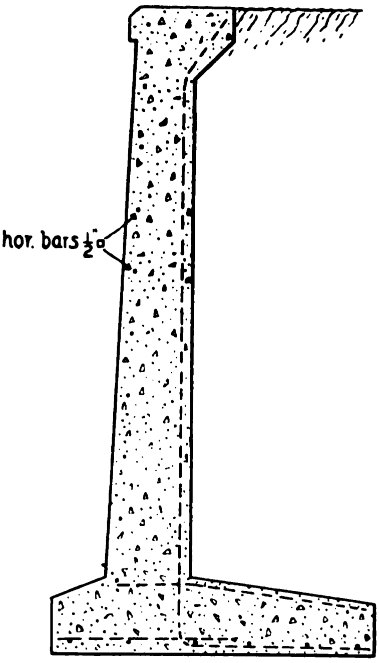


Fig. 114. L-Shaped Retaining Wall.

must have twice that area of concrete; or, in other words, the area of the concrete from the point of the angle down to the face must be 60 times the area of the steel.

Although these brackets are frequently put in without any definite design, it is possible to make some sort of computation, especially when a building is directly exposed to wind pressure, by computing the moment of the wind pressure. For example, if a building is 100 feet long and 50 feet high, and is subjected to a wind pressure of 30 pounds per square foot, the total wind pressure will be 50 × 100 × 30 = 150,000 pounds. Considering the center of pressure as applied at half the height, this would give a moment about the base of the building, of 150,000 × 25 = 3,750,000 foot-pounds = 45,000,000 inch-pounds. If this 100-foot building had eight lines of columns with a pair of brackets on each line, and was four stories high, there would be 64 such brackets to resist wind pressure. Each bracket would therefore be required to resist $\frac{1}{64}$ of 45,000,000 inch-pounds, or about 700,000 inch-pounds. We shall assume that the bracket will have a depth of 25 inches, from the intersection of the center lines of the column and the beam to the steel near the face of the bracket. Then, since each bracket must withstand a moment of 700,000 inch-pounds, the stress in the steel will be 700,000 ÷ 25 = 28,000 pounds. If the actual stress in the steel is 15,000 pounds per square inch, this would require 1.87 square inches of steel, which would be more than supplied by four $\frac{3}{4}$ inch square bars. If these brackets were 12 inches wide and 25 inches deep, the area of concrete is 300 square inches, which is 160 times the area of the steel. There is, therefore, an ample amount of concrete to withstand compression, on the part of those brackets which are subject to compression rather than tension. It is probable that the above calculation is excessive on the side of safety, since it is quite improbable that such a broad area would ever be subject to a pressure of 30 pounds per square foot over the whole area. The method of calculation also ignores the fact that the monolithic character of a reinforced-concrete structure furnishes a very considerable resistance at the junction of columns and girders, and that they should not by any means be considered as if they were pin-connected structures, which would require that the whole of the lateral stiffening should be supplied by these brackets. Nevertheless these brackets must be designed according to some such method.

VERTICAL WALLS

306. **Curtain Walls.** Vertical walls which are not intended to carry any weight, are sometimes made of reinforced concrete. They are then called *curtain walls*, and are designed merely to fill in the panels between the posts and girders which form the skeleton frame of the building. When these walls are interior walls, there is no definite stress which can be assigned to them, except by making assumptions that may be more or less unwarranted. When such walls are used for exterior walls of buildings, they must be designed to withstand wind pressure. This wind pressure will usually be exerted as a pressure from the outside tending to force the wall inward; but if the wind is in the contrary direction, it may cause a lower atmospheric pressure on the outside, while the higher pressure of the air within the building will tend to force the wall outward. It is improbable, however, that such a pressure would ever be as great as that tending to force the wall inward. Such walls may be designed as slabs carrying a uniformly distributed load, and supported on all four sides. If the panels are approximately square, they should have bars in both directions, and should be designed by the same method as "slabs reinforced in both directions," as has previously been explained. If the vertical posts are much closer together than the height of the floor, as sometimes occurs, the principal reinforcing bars should be horizontal, and the walls should be designed as slabs having a span equal to the distance between the posts. Some small bars spaced about 2 feet apart should be placed vertically to prevent shrinkage. The pressure of the wind corresponding to the loading of the slab, is usually considered to be 30 pounds per square foot, although the actual wind pressure will very largely depend on local conditions, such as the protection which the building receives from surrounding buildings. A pressure of thirty pounds per square foot is usually sufficient; and a slab designed on this basis will usually be so thin, perhaps only 4 inches, that it is not desirable to make it any thinner. Since designing such walls is such an obvious application of the equations and problems already solved in detail, no numerical illustration will here be given.

CULVERTS

307. **Box Culverts.** The permanency of concrete, and particularly reinforced concrete, has caused its adoption in the construction of culverts of all dimensions, from a cross-sectional area of a very few square feet, to that of an arch which might be more properly classified under the more common name *masonry arch.* The smaller sizes can be constructed more easily, and with less expense for the forms, by giving them a rectangular cross-section. The question of foundations is solved most easily by making a concrete bottom, as well as side walls and top. The structure then becomes literally a *box.* Its design consists in the determination of the external pressure exerted by the earth, and of the required thickness of the concrete to withstand the pressure on the flat sides considered as slabs. The most uncertain part of the computation lies in the determination of the actual pressure of the earth. Under the heading "Retaining Walls," this uncertainty was discussed.

One very simple method is to assume that the earth pressure is equivalent to that of a liquid having a unit-weight equal to that of the weight of a cubic foot of the earth, which is nearly 100 pounds. Under almost any circumstances, these figures would be sufficiently large, and perhaps very excessive. Calculations on such a basis are therefore certainly safe. If the pressure is computed on this basis, and a factor of safety of 2 is used, it is equivalent to an actual pressure of only one-half the amount (which is more probable), having a factor of 4. If the depth of the earth is quite large compared with the dimensions of the culvert, we may consider that the upward pressure on the bottom, as well as the lateral pressure on the sides, is practically the same as the downward pressure on the top. If the bottom of the culvert is laid on rock, or on soil which is practically unyielding, there will be no necessity of considering that there is any upward pressure on the bottom slab tending to burst that slab upward. The softer the soil, the greater will be the tendency to transverse bending in the bottom slab.

Since the design of rectangular box culverts is purely an application of the equations for transverse bending, after the external pressures have been determined, no numerical example will here be given. These structures are not only reinforced with bars, considering the sides as slabs, but should also have bars placed across the corners, which will withstand a tendency of the section to collapse in case the pressure on opposite sides is unequal. They must also be reinforced with bars running longitudinally with the culvert. As in the other cases of longitudinal reinforcement, no definite design can be made for its amount. A typical cross-section for such a culvert is shown in Fig. 115. The longitudinal bars are indicated in this figure. They are used to prevent cracks owing to expansion or contraction, and also to resist any tendency to rupture which might be caused by a settling or washing-out of the subsoil for any considerable distance under the length of the culvert.

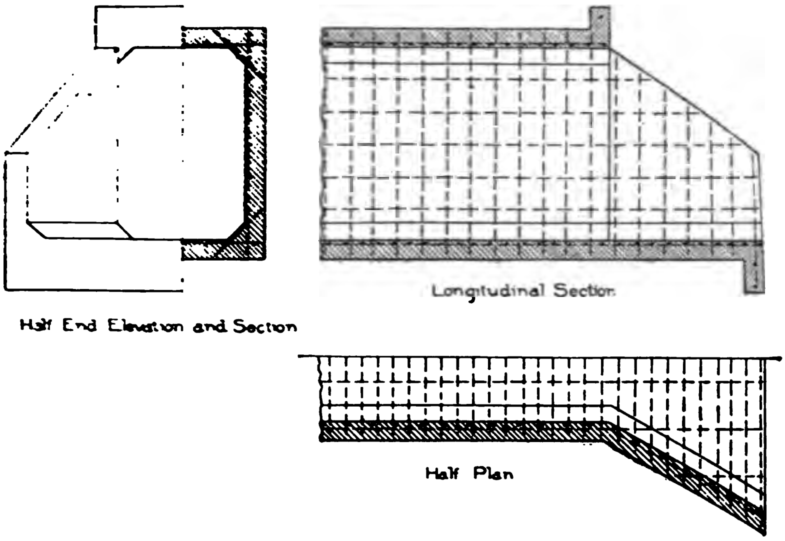


Fig. 115. Rectangular Box Culvert.

308. **Arch Culverts.** The general subject of arches, and especially the application of reinforced concrete to arch construction, are taken up in Part V, and therefore will not be further discussed here.

COLUMNS

309. **Methods of Reinforcement.** The laws of mechanics, as well as experimental testing on full-sized columns of various structural materials, show that very short columns, or even those whose length is ten times their smallest diameter, will fail by crushing or shearing

of the material. If the columns are very long, say twenty or more times their smallest diameter, they will probably fail by bending, which will produce an actual tension on the convex side of the column. The line of division between long and short columns is practically very uncertain, owing to the fact that the center line of pressure of a column is frequently more or less eccentric because of irregularity of the bearing surface at top or bottom. Such an eccentric action will cause buckling of the column, even when its length is not very great. On this account, it is always wise (especially for long columns) to place reinforcing bars within the column. The reinforcing bars consist of longitudinal bars (usually four, and sometimes more with the larger columns), and bands of small bars spaced from 6 to 18 inches apart vertically, which bind together the longitudinal bars. The longitudinal bars are used for the purpose of providing the necessary transverse strength to prevent buckling of the column. As it is practically impossible to develop a satisfactory theory on which to compute the required tensional strength in the convex side of a column of given length, without making assumptions which are themselves of doubtful accuracy, no exact rules for the sizes of the longitudinal bars in a column will be given. The bars ordinarily used vary from ½ inch square to 1 inch square; and the number is usually four, unless the column is very large (400 square inches or larger) or is rectangular rather than square. It has been claimed by many, that longitudinal bars in a column may actually be a source of danger, since the buckling of the bars outward may tend to disintegrate the column. This buckling can be avoided, and the bars made mutually self-supporting, by means of the bands which are placed around the column. These bands are usually ¼-inch or ⅜-inch round or square bars. The specifications of the Prussian Public Works for 1904 require that these horizontal bars shall be spaced a distance not more than 30 times their diameter, which would be 7½ inches for ¼-inch bars, and 11¼ inches for ⅜-inch bars. The bands in the column are likewise useful to resist the bursting tendency of the column, especially when it is short. They will also reinforce the column against the tendency to shear, which is the method by which failure usually takes place. The angle between this plane of rupture and a plane perpendicular to the line of stress, is stated to be 60°. If, therefore, the bands are placed at a distance apart equal to the smallest diameter of the column, any probable plane of rupture will intersect one of the bands, even if the angle of rupture is somewhat smaller than 60°.

The unit working pressure permissible in concrete columns is usually computed at from 350 to 500 pounds per square inch. The ultimate compression for transverse stresses for 1:3:5 concrete has been taken at 2,000 pounds per square inch. With a factor of 4, this gives a working pressure of 500 pounds per square inch; but the ultimate stress in a column of plain concrete is generally less than 2,000 pounds per square inch. Tests of a large number of 12 by 12-inch plain concrete columns showed an ultimate compressive strength of approximately 1,000 pounds per square inch; but such columns generally begin to fail by the development of longitudinal cracks. These would be largely prevented by the use of lateral reinforcement or bands. Therefore the use of 500 pounds per square inch as a working stress for columns which are properly reinforced, may be considered justifiable although not conservative.

310. **Design of Columns.** It may be demonstrated by theoretical mechanics, that if a load is jointly supported by two kinds of material with dissimilar elasticities, the proportion of the loading borne by each will be in a ratio depending on their relative areas and moduli of elasticity. The formula for this may be developed as follows:

C = Total unit-compression upon concrete and steel in pounds per square inch = Total load divided by the combined area of the concrete and the steel;
c = Unit-compression in the concrete, in pounds per square inch;
s = Unit-compression in the steel, in pounds per square inch;
p = Ratio of area of steel to total area of column;
$r = \frac{E_s}{E_c}$ = Ratio of the moduli of elasticity;
ϵ_s = Deformation per unit of length in the steel;
ϵ = Deformation per unit of length in the concrete;
A_s = Area of steel;
A_c = Area of concrete.

The total compressive force in the concrete $= A_c \times c$; and that in the steel $= A_s \times s$.

The sum of these compressions = the total compression; and therefore,

$$C(A_c + A_s) = A_c c + A_s s.$$

The actual linear compression of the concrete equals that of the steel; therefore,

$$\frac{c}{E_c} = \frac{s}{E_s}.$$

From this equation, since $r = \frac{E_s}{E_c}$, we may write the equation $rc = s$. Solving the above equation for C, we obtain:

$$C = \frac{A_c\, c + A_s\, s}{A_c + A_s}.$$

Substituting the value of $s = rc$, we have:

$$C = c\left(\frac{A_c + A_s\, r}{A_c + A_s}\right) = c\left(\frac{A_s + A_c - A_s + A_s\, r}{A_c + A_s}\right).$$

If p = the ratio of cross-section of steel to the *total* cross-section of the column, we have:

$$p = \frac{A_s}{A_c + A_s}.$$

Substituting this value of $\frac{A_s}{A_c + A_s}$ in the above equation, we may write:

$$C = c\,(1 - p + pr).$$

Solving this equation for p, we obtain:

$$p = \frac{C - c}{c\,(r - 1)} \quad \ldots\ldots\ldots\ldots\ldots (41)$$

Example 1. A column is designed to carry a load of 160,000 pounds. If the column is made 18 inches square, and the load per square inch to be carried by the concrete is limited to 400 pounds, what must be the ratio of the steel, and how much steel would be required?

Answer. A column 18 inches square has an area of 324 square inches. Dividing 160,000 by 324, we have 494 pounds per square inch as the total unit-compression upon the concrete and the steel, which is C in the above formula. Assume that the concrete is 1:3:5 concrete, and that the ratio of the moduli of elasticity (r) is therefore 12. Substituting these values in Equation 41, we have:

$$p = \frac{494 - 400}{400\,(12 - 1)} = .0214.$$

Multiplying this ratio by the total area of the column, 324 square inches, we have 6.93 square inches of steel required in the column. This would very nearly be provided by four bars 1¼ inches square. Four round bars 1½ inches in diameter would give an excess in area.

Either solution would be amply safe under the circumstances, provided the column was properly reinforced with bands.

Example 2. A column 16 inches square is subjected to a load of 115,000 pounds, and is reinforced by four ⅞-inch square bars besides the bands. What is the actual compressive stress in the concrete per square inch?

Answer. Dividing the total stress (115,000) by the area (256), we have the combined unit-stress C = 449 pounds per square inch. By inverting one of the equations above, we can write:

$$c = \frac{C}{1 - p + r\,p}.$$

In the above case, the four ⅞-inch bars have an area of 3.06 square inches; and therefore,

$$p = \frac{3.06}{256} = .012;\ r = 12.$$

Substituting these values in the above equation, we may write:

$$c = \frac{449}{1 - .012 + (.012 \times 12)} = \frac{449}{1.132} = 397 \text{ pounds per square inch.}$$

The net area of the concrete in the above problem is 252.94 square inches. Multiplying this by 397, we have the total load carried by the concrete, which is 100,117 pounds. Subtracting this from 115,000 pounds, the total load, we have 14,883 pounds as the compressive stress carried by the steel. Dividing this by 3.06, the area of the steel, we have 4,864 pounds as the unit compressive stress in the steel. This is practically twelve times the unit-compression in the concrete, which is an illustration of the fact that if the compression is shared by the two materials in the ratio of their moduli of elasticity, the unit-stresses in the materials will be in the same ratio. This unit-stress in the steel is about one-third of the working stress which may properly be placed on the steel. It shows that we cannot economically use the steel in order to reduce the area of the concrete, and that the chief object in using steel in the columns is in order to protect the columns against buckling, and also to increase their strength by the use of bands.

It sometimes happens that in a building designed to be structurally of reinforced concrete, the column loads in the columns of the lower story may be so very great that concrete columns of sufficient size would take up more space than it is desirable to spare for such a purpose. For example, it might be required to support a load of 320,000 pounds on a column 18 inches square. If the concrete (1:3:5) is limited to a compressive stress of 400 pounds per square inch, we may solve for the area of steel required, precisely as was done in example 1. We should find that the required percentage of steel was 13.4 per cent, and that the required area of the steel was therefore 43.3 square inches. But such an area of steel could carry the entire load of 320,000 pounds without the aid of the concrete, and would have a compressive unit-stress of only 7,400 pounds. In such a case, it would be more economical to design a steel column to carry the entire load, and then to surround the column with sufficient concrete to fireproof it thoroughly. Since the stress in the steel and the concrete are divided in proportion to their relative moduli of elasticity, which is usually about 10 or 12, we cannot develop a working stress of, say, 15,000 pounds per square inch in the steel, without at the same time developing a compressive stress of 1,200 to 1,500 pounds in the concrete, which is objectionably high as a working stress.

311. **Hooped Columns.** It has been found that the strength of a column is very greatly increased and even multiplied by surrounding the column by numerous hoops or bands or by a spiral of steel. The basic principle of this strength can best be appreciated by considering a section of stovepipe filled with sand and acting as a column. The sand alone, considered as a column, would not be able to maintain its form, much less to support a load, especially if it was dry. But when it is confined in the pipe, the columnar strength is very considerable. Concrete not only has great crushing strength, even when plain, but can also be greatly strengthened against failure by the tensile strength of bands which confine it. The theory of the amount of this added resistance is very complex, and will not here be given. The general conclusions, in which experimental results support the theory, are as follows:

1. The deformation of a hooped column is practically the same as that of a plain concrete column of equal size for loads up to the maximum for a plain column.
2. Further loading of a hooped column still further increases the shortening and swelling of the column, the bands stretching out, but without causing any apparent failure of the column.
3. Ultimate failure occurs when the bands break or, having passed their elastic limit, stretch excessively.

Hooped columns may thus be trusted to carry a far greater unit-load than plain columns, or even columns with longitudinal rods and a few bands. There is one characteristic that is especially useful for a column which is at all liable to be loaded with a greater load than its nominal loading. A hooped column will shorten and swell very perceptibly before it is in danger of sudden failure, and will thus give ample warning of an overload.

Considère has developed an empirical formula based on actual tests, for the strength of hooped columns, as follows:

$$\text{Ultimate strength} = c'A + 2.4s'pA \quad \ldots\ldots (42)$$

in which,

c' = Ultimate strength of the concrete;
s' = *Elastic limit* of the steel;
p = Ratio of area of the steel to the whole area;
A = Whole area of the column.

This formula is applicable only for reinforcement of mild steel. Applying this formula to a hooped column tested to destruction by Professor Talbot, in which the ultimate strength (c') of similar concrete was 1,380 pounds per square inch, the elastic limit of the steel (s') was 48,000 pounds per square inch; the ratio of reinforcement (p) was .0212; and the area (A) was 104 square inches; and substituting these quantities in Equation 42, we have, for the computed ultimate strength, 409,900 pounds. The actual ultimate by Talbot's test was 351,000 pounds, or about 86 per cent.

Talbot has suggested the following formulæ for the ultimate strength of hooped columns *per square inch*:

$$\text{Ultimate strength} = 1{,}600 + 65{,}000\,p \text{ (for mild steel)} \quad \ldots (43)$$

$$\text{"} \quad \text{"} \quad = 1{,}600 + 100{,}000\,p \text{ (for high steel)} \quad \ldots (44)$$

In these formulæ, p applies only to the area of concrete within the hooping; and this is unquestionably the correct principle, as the concrete outside of the hooping should be considered merely as fire protection and ignored in the numerical calculations, just as the concrete below the reinforcing steel of a beam is ignored in calculating the strength of the beam. The ratio of the area of the steel is computed by computing the area of an equivalent thin cylinder of steel which would contain as much steel as that actually used in the bands or spirals. For example, suppose that the spiral reinforcement con-

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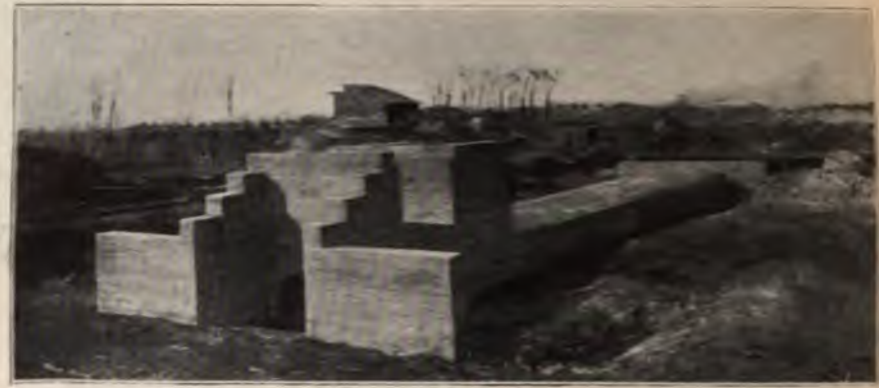
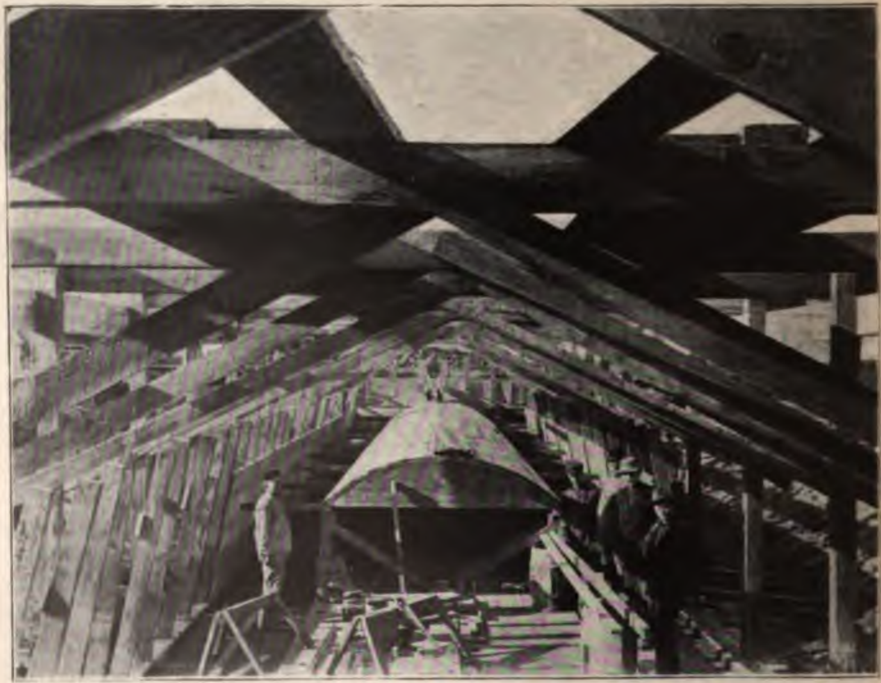


Reinforced-Concrete Arch Culvert, Completed.



Forms for Construction of Arched Culvert Shown Above.

VIEWS OF CONSTRUCTION ON LINE OF ILLINOIS AND MISSISSIPPI CANAL sisted of a ½-inch round rod, the spiral having a pitch of 3 inches. A ½-inch round rod has an area of .196 square inch. That area for 3 inches in height would be the equivalent of a solid band .0653 inch thick. If the spiral had a diameter of, say, 11 inches, its circumference would be 34.56 inches, and the area of metal in a horizontal section would be 34.56 × .0653 = 2.257 square inches. The area of the concrete within the spiral is 95.0 square inches. The value of p is therefore 2.257 ÷ 95.0 = .0237. If the ½-inch bar were made of high-carbon steel, the *ultimate* strength per square inch of the column would be 1,600 + (100,000 × .0237) = 1,600 + 2,370 = 3,970. The unit-strength is considerably more than doubled. The ultimate strength of the whole column is therefore 95 × 3,970 = 377,150 pounds. Such a column could be safely loaded with about 94,300 pounds, provided its length was not so great that there was danger of buckling. In such a case, the unit-stress should be reduced according to the usual ratios for long columns, or the column should be liberally reinforced with longitudinal rods, which would increase its transverse strength.

312. Effect of Eccentric Loading of Columns. It is well known that if a load on a column is eccentric, its strength is considerably less than when the resultant line of pressure passes through the axis of the column. The theoretical demonstration of the amount of this eccentricity depends on assumptions which may or may not be found in practice. The following formula is given without proof or demonstration, in Taylor and Thompson's treatise on Concrete:

Let e = Eccentricity of load;
b = Breadth of column;
f = Average unit-pressure;
f' = Total unit-pressure of outer fibre nearest to line of vertical pressure

Then,

$$f' = f\left(1 + \frac{6e}{b}\right) \qquad (45)$$

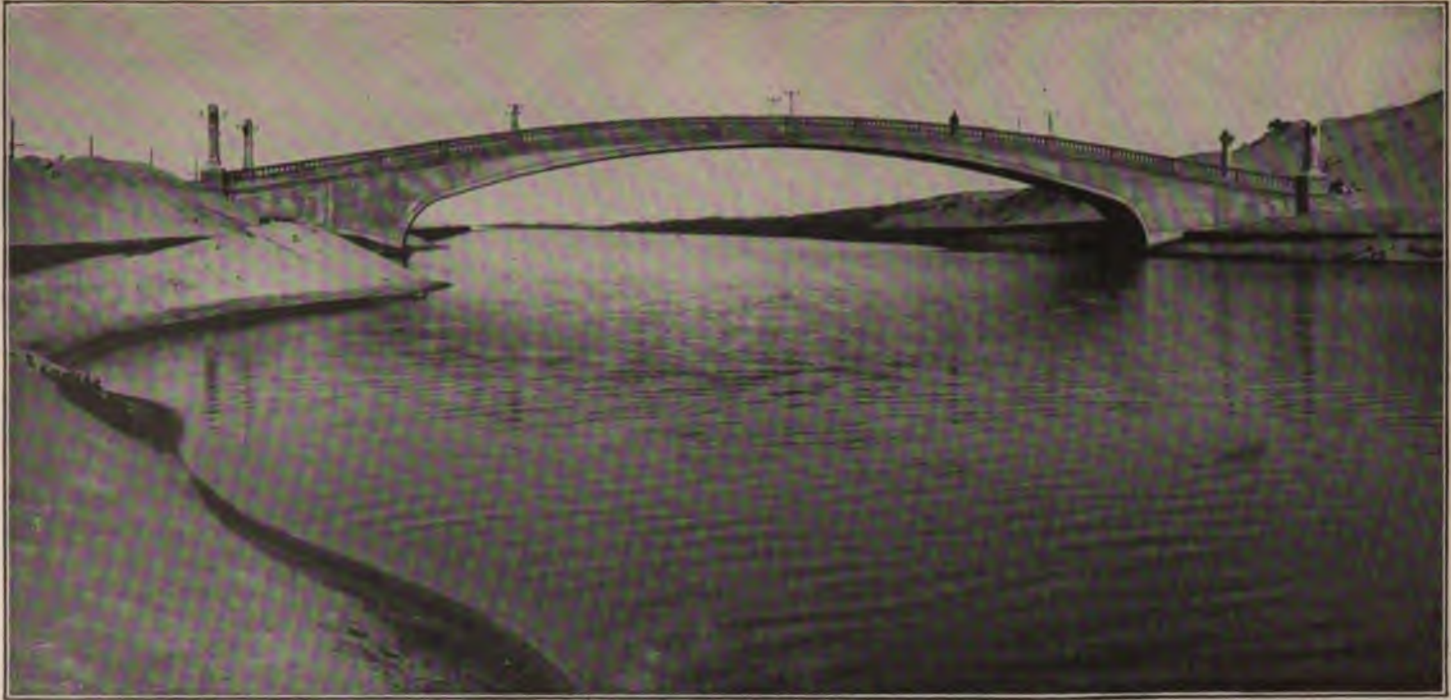
As an illustration of this formula, if the eccentricity on a 12-inch column were 2 inches, we should have $b = 12$, and $e = 2$. Substituting these values in Equation 45, we should have $f' = 2f$, which means that the maximum pressure would equal twice the average pressure. In the extreme case, where the line of pressure came to the outside of the column, or when $e = \frac{1}{2}b$, we should have that the maximum pressure on the edge of the column would equal four times the average pressure.

Any refinements in such a calculation, however, are frequently overshadowed by the uncertainty of the actual location of the center of pressure. A column which supports two equally loaded beams on each side, is probably loaded more symmetrically than a column which supports merely the end of a beam on one side of it. The best that can be done is arbitrarily to lower the unit-stress on a column which is probably loaded somewhat eccentrically.

TANKS

313. **Design.** The extreme durability of reinforced-concrete tanks, and their immunity from deterioration by rust, which so quickly destroys steel tanks, have resulted in the construction of a large and increasing number of tanks in reinforced concrete. Such tanks must be designed to withstand the bursting pressure of the water. If they are very high compared with their diameter, it is even possible that failure might result from excessive wind pressure.

The method of designing one of these tanks may best be considered from an example. Suppose that it is required to design a reinforced-concrete tank with a capacity of 50,000 gallons, which shall have an inside diameter of 18 feet. At 7.48 gallons per cubic foot, a capacity of 50,000 gallons will require 6,684 cubic feet. If the inside diameter of the tank is to be 18 feet, then the 18-foot circle will contain an area of 254.5 square feet. The depth of the water in the tank will therefore be 26.26 feet. The lowest foot of the tank will therefore be subjected to a bursting pressure due to 25.76 vertical feet of water. Since the water pressure per square foot increases $62\frac{1}{2}$ pounds for each foot of depth, we shall have a total pressure of 1,610 pounds per square foot on the lowest foot of the tank. Since the diameter is 18 feet, the bursting pressure it must resist on *each* side is one-half of $18 \times 1{,}610 = \frac{1}{2} \times 28{,}980 = 14{,}490$ pounds. If we allow a working stress of 15,000 pounds per square inch, this will require .966 square inch of metal in the lower foot. Since the bursting pressure is strictly proportional to the depth of the water, we need only divide this number proportionally to the depth to obtain the bursting pressure at other depths. For example, the ring one foot high, at one-half the depth of the tank, should have .483 square inch



BRIDGE OF REINFORCED CONCRETE, AT PLAYA DEL REY, NEAR LOS ANGELES, CAL.

Extreme Length, 205 feet 8 inches; Span, 146 feet; Width, 19 feet; Spring, 18 feet; Height above water, 20 feet.

of metal; and that at one-third of the depth, should have .322 square inch of metal. The actual bars required for the lowest foot may be figured as follows: .966 square inch per foot equals .0805 square inch per inch; ¾-inch square bars, having an area .5625 square inch, will furnish the required strength when spaced 7 inches apart. At one-half the height, the required metal per linear inch of height is half of the above, or .040. This *could* be provided by using ¾-inch bars spaced 14 inches apart; but this is not so good a distribution of metal as to use ⅝-inch square bars having an area of .39 square inch, and to space the bars nearly 10 inches apart. It would give a still better distribution of metal, to use ½-inch bars spaced 6 inches apart at this point, although the ½-inch bars are a little more expensive per pound, and, if they are spaced very closely, will add slightly to the cost of placing the steel. The size and spacing of bars for other points in the height can be similarly determined.

A circle 18 feet in diameter has a circumference of somewhat over 56 feet. Assuming as a preliminary figure that the tank is to be 10 inches thick at the bottom, the mean diameter of the base ring would be 18.83 feet, which would give a circumference of over 59 feet. Allowing a lap of 3 feet on the bars, this would require that the bars should be about 62 feet long. Although it is possible to have bars rolled of this length, they are very difficult to handle, and require to be transported on the railroads on *two* flat cars. It is therefore preferable to use bars of slightly more than half this length, and to make two joints in each band.

The bands which are used for ordinary wooden tanks are usually fastened at the ends by screw-bolts. Some such method is necessary for the bands of concrete tanks, provided the bands are made of plain bars. Deformed bars have a great advantage in such work, owing to the fact that, if the bars are overlapped from 18 inches to 3 feet, according to their size, and are then wired together, it will require a greater force than the strength of the bar to pull the joints apart after they are once thoroughly incased in the concrete and the concrete has hardened.

314. **Test for Overturning.** Since the computed depth of the water is over 26 feet, we must calculate that the tank will be, say, 28 feet high. Its outer diameter will be approximately 20 feet. The total area exposed to the surface of the wind, will be 560 square feet.

We may assume that the wind has an average pressure of 50 pounds per square foot; but owing to the circular form of the tank, we shall assume that its effective pressure is only one-half of this; and therefore we may figure that the total overturning pressure of the wind equals 560 × 25 = 14,000 pounds. If this is considered to be applied at a point 14 feet above the ground, we have an overturning moment of 196,000 foot-pounds, or 2,352,000 inch-pounds.

Although it is not strictly accurate to consider the moment of inertia of this circular section of the tank as it would be done if it were a strictly homogeneous material, since the neutral axis, instead of being at the center of the section, will be nearer to the compression side of the section, our simplest method of making such a calculation is to assume that the simple theory applies, and then to use a generous factor of safety. The effect of shifting the neutral axis from the center toward the compression side, will be to increase the unit-compression on the concrete, and reduce the unit-tension in the steel; but, as will be seen, it is generally necessary to make the concrete so thick that its unit compressive stress is at a very safe figure, while the reduction of the unit-tension in the steel is merely on the side of safety.

Applying the usual theory, we have, for the moment of inertia of a ring section, .049 $(d_1^4 - d^4)$. Let us assume as a preliminary figure, that the wall of the tank is 10 inches thick at the bottom. Its outside diameter is therefore 18 feet + twice 10 inches, or 236 inches. The moment of inertia I = .049 $(236^4 - 216^4)$ = 45,337,842 biquadratic inches. Calling c the unit-compression, we have, as the ultimate moment due to wind pressure:

$$M = \frac{c\ I}{\frac{1}{2}\ d_1} = \frac{c \times 45.337.842}{\frac{1}{2}\ d_1} = 2,352,000 \text{ inch-pounds},$$

in which $\frac{1}{2}\ d_1$ = 118 inches.

Solving the above equation for c, we have c equals a fraction less than 6 pounds per square inch. This pressure is so utterly insignificant, that, even if we double or treble it to allow for the shifting of the neutral axis from the center, and also double or treble the allowance made for wind pressure, although the pressure chosen is usually considered ample, we shall still find that there is practically no danger that the tank will fail owing to a crushing of the concrete due to wind pressure.

The above method of computation has its value in estimating the amount of steel required for vertical reinforcement. On the basis of 6 pounds per square inch, a sector with an average width of 1 inch and a diametral thickness of 10 inches would sustain a compression of about 60 pounds. Since we have been figuring working stresses, we shall figure a working tension of, say, 16,000 pounds per square inch in the steel. This tension would therefore require $\frac{60}{16,000} =$ .0037 square inch of metal per inch of width. Even if ¼-inch bars were used for the vertical reinforcement, they would need to be spaced only about 17 inches apart. This, however, is on the basis that the neutral axis is at the center of the section, which is known to be inaccurate.

A theoretical demonstration of the position of the neutral axis for such a section, is so exceedingly complicated that it will not be considered here. The theoretical amount of steel required is always less than that computed by the above approximate method; but the necessity for preventing cracks, which would cause leakage, would demand more vertical reinforcement than would be required by wind pressure alone.

315. **Practical Details of the Above Design.** It was assumed as an approximate figure, that the thickness of the concrete side wall at the base of the tank should be 10 inches. The calculations have shown that, so far as wind pressure is concerned, such a thickness is very much greater than is required for this purpose; but it will not do to reduce the thickness in accordance with the apparent requirements for wind pressure. Although the thickness at the bottom might be reduced below 10 inches, it probably would not be wise to do so. It may, however, be tapered slightly towards the top, so that at the top the thickness will not be greater than 6 inches, or perhaps even 5 inches. The vertical bars in the lower part of the side wall must be bent so as to run into the base slab of the tank. This will bind the side wall to the bottom. The necessity for reinforcement in the bottom of the tank depends very largely upon the nature of the foundation, and also to some extent on the necessity for providing against temperature cracks, as has been discussed in a previous section. Even if the tank is placed on a firm and absolutely unyielding foundation, some reinforcement should be used in the bottom, in

order to prevent cracks which might produce leakage. These bars should run from a point near the center, and be bent upward at least 2 or 3 feet into the vertical wall. Sometimes a gridiron of bars running in both directions is used for this purpose. This method is really preferable to the radial method. The methods of making tanks water-tight have already been discussed.

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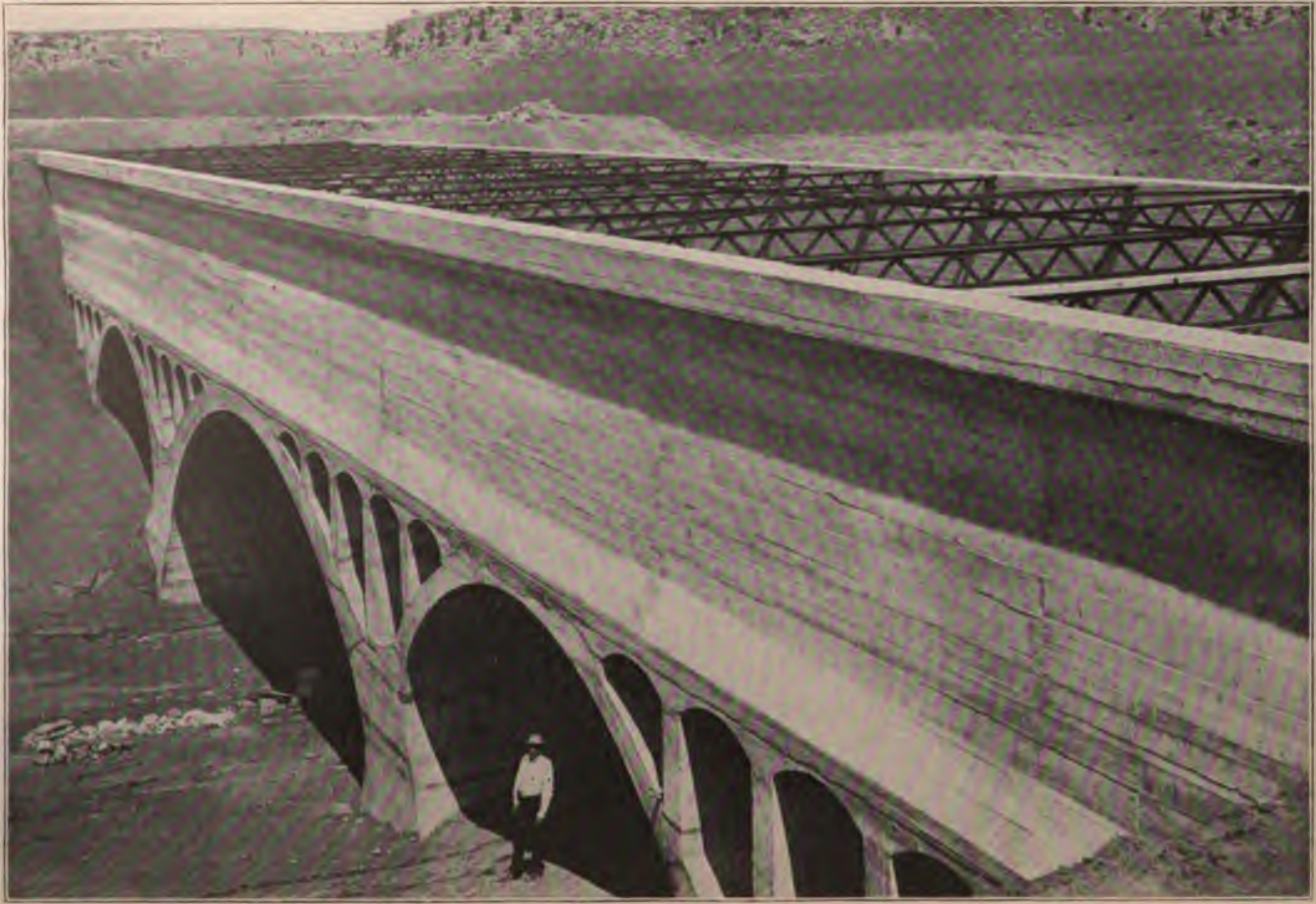

TOP AND SIDE VIEW OF SPRING CANYON FLUME ON INTERSTATE CANAL, NORTH PLATTE IRRIGATION PROJECT, NEBRASKA

Built of steel and reinforced concrete.

Photo by Reclamation Service.

MASONRY AND REINFORCED CONCRETE

PART IV.

FINISHING SURFACES OF CONCRETE

316. Imperfections. To give a satisfactory finish to exposed surfaces of concrete is a rather difficult problem. Usually, when the forms are taken down, the surface of the concrete shows the joints, knots, and grain of the wood. It has more the appearance of a piece of rough carpentry work than that of finished masonry. Or, failure to tamp or flat-spade the surfaces next to the forms, will result in rough places or *stone pockets*. Lack of homogeneity in the concrete will cause a variation in the surface texture of the concrete. Variation of color, or discoloration, is one of the most common imperfections. Old concrete adhering to the forms will leave pits in the surface; or the pulling-off of the concrete in spots, as a result of it adhering to the forms when they are removed, will cause a roughness.

To guard against these imperfections, the forms must be well constructed of dressed lumber, and the pores should be well filled with soap or paraffine. The concrete should be thoroughly mixed, and, when placed, care should be taken to compact the concrete thoroughly, next to the forms. The variation in color is usually due to the leaching-out of lime, which is deposited in the form of an efflorescence on the surface; or to the use of different cements in adjacent parts of the same work. The latter case usually can be avoided by using the same brand of cement on the entire work, and the former will be treated under the heading of *Efflorescence* (Article 329).

317. Plastering. Plastering is not usually successful, although there are cases where a mixture of equal parts of cement and sand has apparently been successful; and when finished rough, it did not show any cracks. It is generally considered impossible to apply mortar in thin layers to a concrete surface, and make it adhere for any

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length of time. When the plastering begins to scale off, it looks worse than the unfinished surface. This paragraph is intended more as a warning against this manner of finishing concrete surfaces than as a description of it as an approved method of finish.

318. **Mortar Facing.** The following method has been adopted by the New York Central Railroad for giving a good finish to exposed concrete surfaces:

The forms of 2-inch tongued-and-grooved pine were coated with soft soap, all openings in the joints of the forms being filled with hard soap. The concrete was then deposited, and, as it progressed, was drawn back from the face with a square-pointed shovel, and 1:2 mortar poured in along the forms. When the forms were removed, and while the concrete was green, the surface was rubbed, with a circular motion, with pieces of white firebrick, or brick composed of one part cement and one part sand. The surface was then dampened and painted with a 1:1 grout, rubbed in, and finished with a wooden float, leaving a smooth and hard surface when dry.

The following method of placing mortar facing has been found very satisfactory, and has been adopted very extensively in the last few years: A sheet-iron plate 6 or 8 inches wide and about 5 or 6 feet long, has riveted across it on one side angles of $\frac{3}{4}$-inch size, or such other size as may be necessary to give the desired thickness of mortar facing, these angles being spaced about two feet apart (Fig 116). In operation, the ribs of the angles are placed against the forms; and the space between the plate and forms is filled with mortar, which is mixed in small batches, and thoroughly tamped. The concrete back-filling is then placed; the mould is withdrawn; and the facing and back-filling are rammed together. The mortar facing is mixed in the proportion of one part cement, to 1, 2, or 3 parts sand; usually a 1:2 mixture is employed, mixed wet and in small batches as used. As mortar facing shows the roughness of the forms more readily than

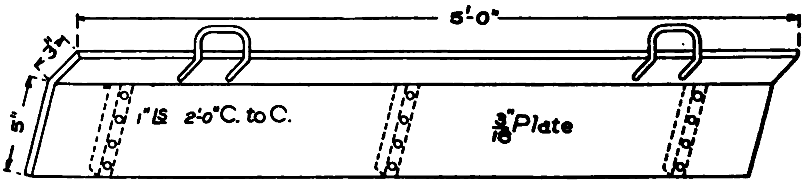


Fig. 116. Mould for Mortar Facing.

concrete does, care is required in constructing, to secure a smooth finish. When the forms are removed, the face may be treated either in the manner already described, or according to the following method taken from the "Proceedings" of the American Railway Engineering and Maintenance of Way Association:

> "After the forms are removed, any small cavities or openings in the concrete shall be filled with mortar, if necessary. Any ridges due to cracks or joints in the lumber shall be rubbed down; the entire face shall be washed with a thin grout of the consistency of whitewash, mixed in the proportion of 1 part cement to 2 parts of sand. The wash shall be applied with a brush."

319. **Masonry Facing.** Concrete surfaces may be finished to represent ashlar masonry. The process is similar to stone-dressing; and any of the forms of finish employed for cut stone can be used for concrete. Very often, when the surface is finished to represent ashlar masonry, vertical and horizontal three-sided pieces of wood are fastened to the forms to make V-shaped depressions in the concrete, as shown in Fig. 117.

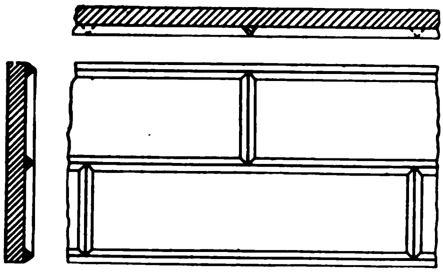


Fig. 117. Masonry Facing of Concrete.

320. **Stone or Brick Facing.** A facing of stone or brick is frequently used for reinforced concrete, and is a very satisfactory solution of the problem of finish. The same care is required with a stone or brick facing as if the entire structure were stone or brick. The Ingalls Building at Cincinnati, Ohio, 16 stories, is veneered on the outside with marble to a height of three stories, and with brick and terra-cotta above the third story. Exclusive of the facing, the wall is 8 inches thick.

In constructing the Harvard University Stadium, care was taken, after the concrete was placed in the forms, to force the stones back from the face and permit the mortar to cover every stone. When the forms were removed, the surface was picked with a tool as shown in Fig. 118. A pneumatic tool has also been adopted for this purpose.

The number of square feet to be picked per day, depends on the hardness of the concrete. If the picking is performed by hand, it is done by a common laborer; and he is expected to cover, on an average, about 50 square feet per day of ten hours. With a pneumatic tool, a man would cover from 400 to 500 square feet per day.

321. **Granolithic Finish.** Several concrete bridges in Philadelphia have been finished according to the following specifications; and their appearance is very satisfactory:

> "Granolithic surfacing, where required, shall be composed of 1 part cement, 2 parts coarse sand or gravel, and 2 parts granolithic grit, made into a stiff mortar. Granolithic grit shall be granite or trap rock, crushed to pass a ¼-inch sieve, and screened of dust. For vertical surfaces, the mixture shall be deposited against the face forms to a minimum thickness of 1 inch, by skilled workmen, as the placing of the concrete proceeds; and it thus forms a part of the body of the work. Care must be taken to prevent the occurrence of air-spaces or voids in the surface. The face shall be removed as soon as the concrete has sufficiently hardened; and any voids that may appear shall be filled with the mixture. The surface shall then be immediately washed with water until the grit is exposed and rinsed clean, and shall be protected from the sun and kept moist for three days. For bridge-seat courses and other horizontal surfaces, the granolithic mixture shall be deposited on the concrete to a thickness of at least 1½ inches, immediately after the concrete has been tamped and before it has set, and shall be troweled to an even surface, and, after it has set sufficiently hard, shall be washed until the grit is exposed."

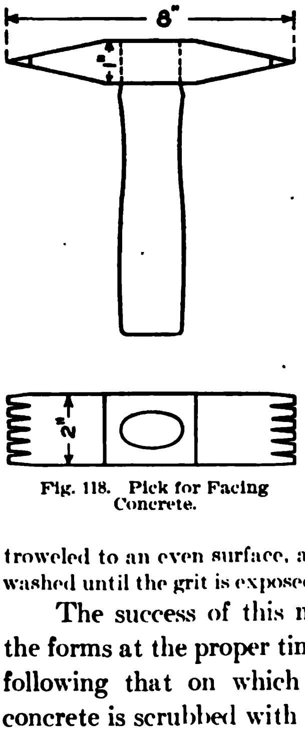
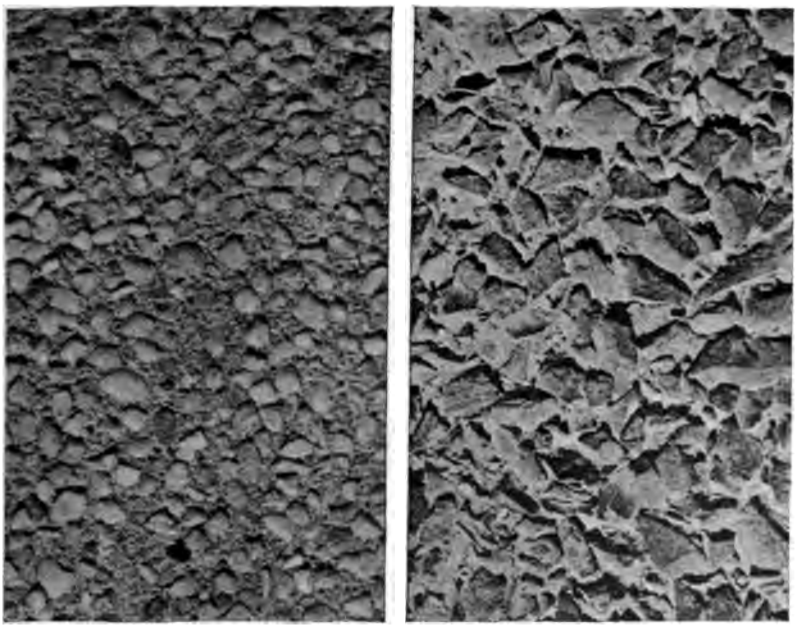


Fig. 118. Pick for Facing Concrete.

The success of this method depends greatly on the removal of the forms at the proper time. In general the washing is done the day following that on which the concrete was deposited. The fresh concrete is scrubbed with an ordinary scrubbing-brush, removing the film, the impressions of the forms, and exposing the sand and stone of the concrete. If this is done at the right time—that is, when the material is at the proper degree of hardness—merely a few rubs of an ordinary house scrubbing-brush, with a free flow of water to cut and to rinse clean, constitutes all the work and apparatus required. The cost of scrubbing is small if done at the right time. A laborer will wash 100 square feet in an hour; but if that same area is permitted to get hard, it may require two men a day with wire brushes to secure the desired results. The practicability of removing the forms at the proper time for such treatment, depends upon the character of the structure and the conditions under which the work must be done. This method is applicable to vertical walls, but it would not be applicable to the soffit of an arch. (See Fig. 119.)

322. The Acid Treatment. This treatment consists in washing the surface of the concrete with diluted acid, then with an alkaline solution. The diluted acid is applied first, to remove the cement and expose the sand and stone; the alkaline solution is then applied to remove all of the free acid; and finally the surface is washed with clear water. The treatment may be applied at any time after the forms are removed. It is simple and effective. Limestone cannot be used in the concrete for any surfaces that are to have this treatment, as the limestone would be affected by the acid. This process has been used very successfully. It is said to be patented.



Aggregate $\frac{3}{16}$-Inch White Pebbles. Aggregate $\frac{3}{4}$-Inch Screened Stone.

Fig. 119. Quimby's Finish on Concrete Surfaces. Reproduced at actual size.

323. Dry Mortar Finish. The dry mortar method consists of a dry, rich mixture, with finely crushed stone. The concrete is usually composed of 1 part cement, 3 parts sand, and 3 parts crushed stone known at the $\frac{1}{4}$-inch size, and mixed dry so that no mortar will flush to the surface when well rammed in the forms. When placed, the concrete is not spaded next to the forms; and being dry, there is no smooth mortar surface, but there should be an even-grained, rough surface. With the dry mixture, the imprint of joints of the forms is hardly noticed, and the grain of the wood is not seen at all. This style of finish has been extensively used in the South Park system of Chicago, and there has been no efflorescence apparent on the surface, which is explained by "the dryness of the mix and the porosity of the surface."

324. **Cast Slab Veneer.** Cast concrete slab veneer can be made of any desired thickness or size. It is set in place like stone veneer, with the remainder of the concrete forming the backing. It is usually cast in wooden moulds, face down. A layer of mortar, 1 part cement, 1 part sand, and 2 or 3 parts fine stone or coarse sand, is placed in the mould to a depth of about 1 inch, and then the mould is filled up with a 1:2:4 concrete. Any steel reinforcement that is desired may be placed in the concrete. Usually, cast concrete slab veneer is cheaper than concrete facing cast in place, and a better surface finish is secured.

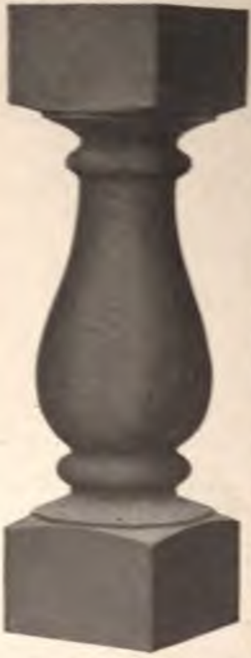
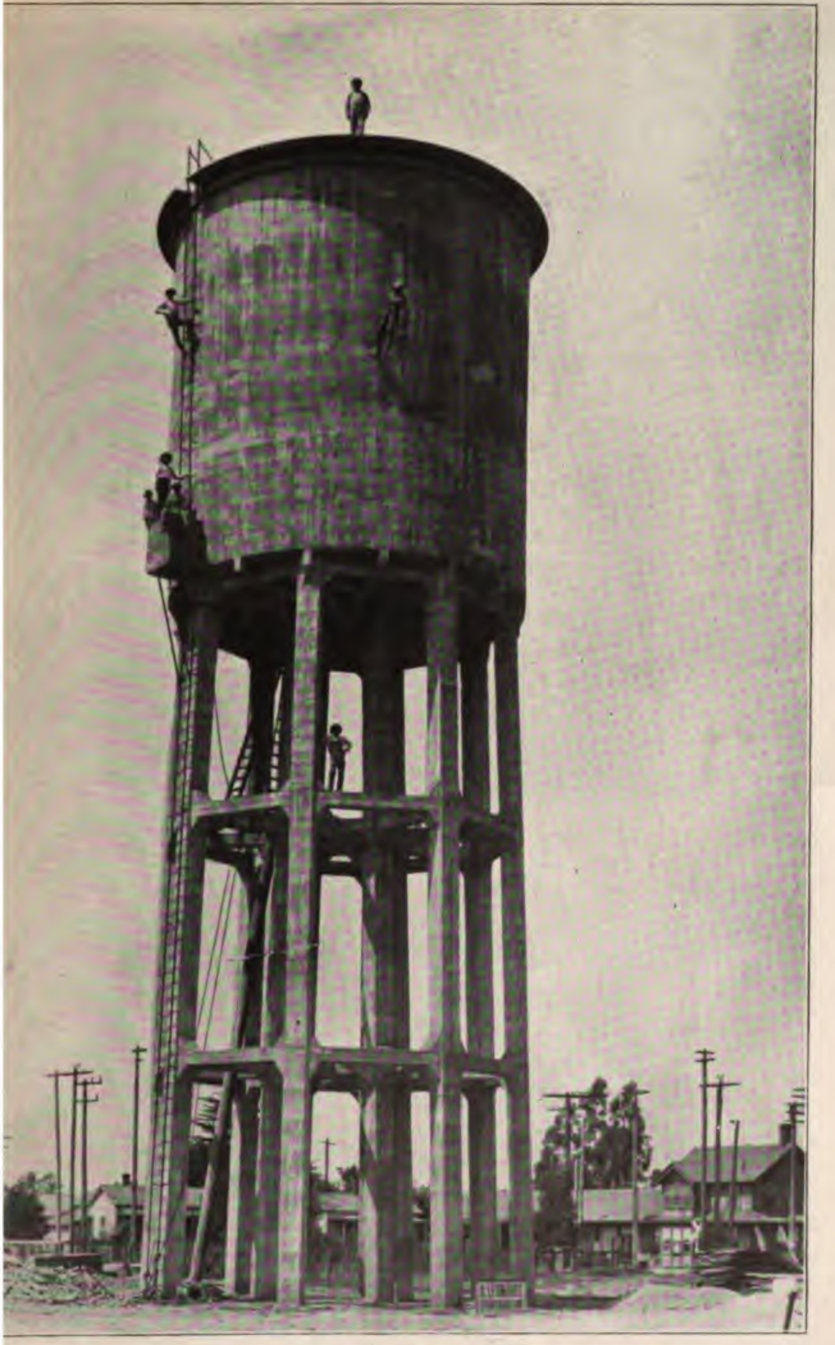


Fig. 120. Balustrade.

325. **Mouldings and Ornamental Shapes.** Concrete is now in demand in ornamental shapes for buildings and bridges. They may be either constructed in place, or moulded in sections and placed the same as cut stone. Plain cornices or panels are usually constructed in place, and complicated moulding or balusters (Fig 120) are usually made in sections and erected in separate pieces.

The moulds may be constructed of wood, metal, or plaster of Paris, or moulded in sand. The operation of casting concrete in sand is similar to that of casting iron. The pattern is made of wood the exact size required. It is then moulded in flasks exactly as done in casting iron. The ingredients for concrete consist of cement and sand or fine crushed stone; the mixture, with a consistence about



REINFORCED-CONCRETE WATER TANK, ANAHEIM, CALIFORNIA

Total height, 113 ft.; supporting frame, 75 ft. high; tank, 38 ft. high, 32 ft. in diameter, with 5 in. thick at bottom, tapering to 3 in. thick at top. Capacity, 200,000 gallons. Cost, 00, or 75 per cent of lowest estimate on a steel tank and tower of like dimensions.

that of cream, is poured into the mould with the aid of a funnel and a T-pipe. Generally the casting is left in the sand for three or four days, and, after being taken out of the sand, should harden in the air a week or ten days before being placed. Balusters are very often made in this manner.

326. **Colors for Concrete Finish.** Coloring matter has not been used very extensively in concrete work, except in ornamental work. It has not been very definitely determined what coloring matters are detrimental to concrete. Lampblack (boneblack) has been used more extensively than any other coloring matter. It gives different shades of gray, depending on the amount used. Common lampblack and Venetian red should not be used, as they are apt to run or fade. Dry mineral colors, mixed in proportions of two to ten per cent of the cement, give shades approaching the color used. Red lead should never be used; even one per cent is injurious to the concrete. Variations in the color of cement and character of the sand used will affect the results obtained in using coloring matter.

COLORED MORTARS

Colors Given to Portland Cement Mortars Containing Two Parts River Sand to One Part Cement

Dry Material Used	Weight of Dry Coloring Matter to 100 Lbs. of Cement				Cost of Coloring Matter per Pound
	½ Pound	1 Pound	2 Pounds	4 Pounds	
Lampblack	Light Slate	Light Gray	Blue Gray	Dark Blue Slate	15 cents
Prussian Blue	Light Green Slate	Light Blue Slate	Blue Slate	Bright Blue Slate	50 "
Ultramarine Blue	....	Light Blue Slate	Blue Slate	Bright Blue Slate	20 "
Yellow Ocher	Light Green	....	....	Light Buff	3 "
Burnt Umber	Light Pinkish Slate	Pinkish Slate	Dull Lavender Pink	Chocolate	10 "
Venetian Red	Slate, Pink Tinge	Bright Pinkish Slate	Light Dull Pink	Dull Pink	2½ "
Red Iron Ore	Pinkish Slate	Dull Pink	Terra-Cotta	Light Brick Red	2½ "

The above table is taken from Sabin's "Cement and Concrete."

327. **Painting Concrete Surface.** Special paints are made for painting concrete surfaces. Ordinary paints, as a general rule, are not

satisfactory. Before the paint is applied, the surface of the wall should be washed with dilute sulphuric acid, 1 part acid to 100 parts water.

328. **Finish for Floors.** Floors in manufacturing buildings are often finished with a 1-inch coat of cement and sand, which is usually mixed in the proportions of 1 part cement to 1 part sand, or 1 part cement to 2 parts sand. This finishing coat must be put on before the concrete base sets, or it will break up and shell off, unless it is made very thick, 1½ to 2 inches. A more satisfactory method of finishing such floors is to put 2 inches of cinder concrete on the concrete base, and then put the finishing coat on the cinder concrete. The finish coat and cinder concrete bond together, making a thickness of three inches. The cinder concrete may consist of a mixture of 1 part cement, 2 parts sand, and 6 parts cinders, and may be put down at any time; that is, this method of finishing a floor can be used as satisfactorily on an old concrete floor as on one just constructed.

In office buildings, and generally in factory buildings, a wooden floor is laid over the concrete. Wooden stringers are first laid on the concrete, about 1½ to 2 feet apart. The stringers are 2 inches thick and 3 inches wide on top, with sloping edges. The space between the stringers is filled with cinder concrete, as shown in Fig. 121, usually mixed 1:4:8. When the concrete has set, the flooring is nailed to the stringers.

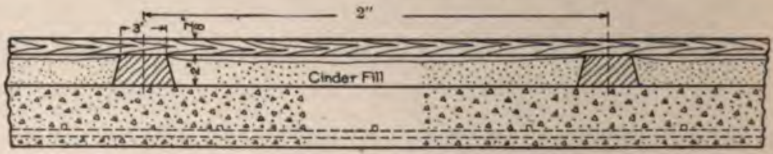


Fig. 121. Cinder Fill between Stringers.

329. **Efflorescence.** The white deposit found on the surface of concrete, brick, and stone masonry is called *efflorescence*. It is caused by the leaching of certain lime compounds, which are deposited on the surface by the evaporation of the water. This is believed to be due primarily to the variation in the amount of water used in mixing the mortar. An excess of water will cause a segregation of the coarse and fine materials, resulting in a difference of color. In a very wet mixture, more lime will be set free from the cement and

brought to the surface. When great care is used as to the amount of water, and care is taken to prevent the separation of the stone from the mortar when deposited, the concrete will present a fairly uniform color when the forms are removed. There is greater danger of the efflorescence at joints than at any other point, unless special care is taken. If the work is to be continued within 24 hours, and care is taken to scrape and remove the *laitance,* and then, before the next layer is deposited, the scraped surface is coated with a thin cement mortar, the joint should be impervious to moisture, and no trouble with efflorescence should be experienced.

A very successful method of removing efflorescence from a concrete surface, consists in applying a wash of diluted hydrochloric acid. The wash consists of 1 part acid to 5 parts water, and is applied with scrubbing brushes. Water is kept constantly played on the work, by means of a hose, to prevent the penetration of the acid. The cleaning is very satisfactory, and for plain surfaces costs about 20 cents per square yard.

330. **Laitance.** Laitance is whitish, spongy material that is washed out of the concrete when it is deposited in water. Before settling on the concrete, it gives the water a milky appearance. It is a semi-fluid mass, composed of a very fine, flocculent matter in the cement; generally contains hydrate of lime; stays in a semi-fluid state for a long time; and acquires very little hardness at its best. Laitance interferes with the bonding of the layers of concrete, and should always be thoroughly cleaned from the surface before another layer of concrete is placed.

MACHINERY FOR CONCRETE WORK

331. **Concrete Plant.** No general rule can be given for laying out a plant for concrete work. Every job is generally a problem by itself, and usually requires a careful analysis to secure the most economical results. Since it is much easier and cheaper to handle the cement, sand, and stone before they are mixed, the mixing should be done as near the point of installation as possible. All facilities for handling these materials, charging the mixer, and distributing the concrete after it is mixed, must be secured and maintained. The charging and distributing are often done by wheelbarrows or carts; and economy of operation depends largely upon system and regu-

larity of operation. Simple cycles of operations, the maintenance of proper runways, together with clock-like regularity, are necessary for economy. To shorten the distance of wheeling the concrete, it is very often found, on large buildings, that it is more economical to have two medium-sized plants located some distance apart, than to have one large plant. In city work, where it is usually impossible to locate the hoist outside of the building, it is constructed in the elevator shaft or light well. In purchasing a new plant, care must be exercised in selecting machinery that will not only be satisfactory for the first job, but that will fulfil the general needs of the purchaser on other work. All parts of the plant, as well as all parts of any one machine, should be easily duplicated from stock, so that there will not be any great delay from any breakdown or worn-out parts.

The design of a plant for handling the material and concrete, and the selection of a mixer, depend upon local conditions, the amount of concrete to be mixed per day, and the total amount required on the contract. It is very evident that on large jobs it pays to invest a large sum in machinery to reduce the number of men and horses; but if not over 50 cubic yards are to be deposited per day, the cost of the machinery is a big item, and hand labor is generally cheaper. The interest on the plant must be charged against the number of cubic yards of concrete; that is, the interest on the plant for a year must be charged to the number of cubic yards of concrete laid in a year. The depreciation of the plant is found by taking the cost of the entire plant when new, and then appraising it after the contract is finished, and dividing the difference by the total cubic yards of concrete laid. This will give the depreciation per cubic yard of concrete manufactured.

332. **Concrete Mixers.** The best concrete mixer is the one that turns out the maximum of thoroughly mixed concrete at the minimum of cost for power, interest, and maintenance. The type of mixer with a complicated motion gives better and quicker results than one with a simpler motion. There are two general classes of concrete mixers—*continuous* mixers and *batch* mixers. A *continuous* mixer is one into which the materials are fed constantly, and from which the concrete is discharged constantly. *Batch* mixers are constructed to receive the cement with its proportionate amount of sand and stone, all at one charge; and, when mixed, it is discharged in a

mass. No very distinct line can be drawn between these two classes, for many of these mixers are adapted to either continuous or batch mixing. Generally, batch mixers are preferred, as it is a very difficult matter to feed the mixers uniformly unless the materials are mechanically measured.

Continuous mixers usually consist of a long screw or pug mill, that pushes the materials along a drum until they are discharged in a continuous stream of concrete. Where the mixers are fed with automatic measuring devices, the concrete is not regular, as there is no reciprocating motion of the materials. In a paper recently read before the Association of American Portland Cement Manufacturers, S. B. Newberry says:

> "For the preparation of concrete for blocks in which thorough mixing and use of an exact and uniform proportion of water are necessary, continuous mixing machines are unsuitable; and batch mixers, in which a measured batch of the material is mixed the required time, and then discharged, are the only type which will be found effective."

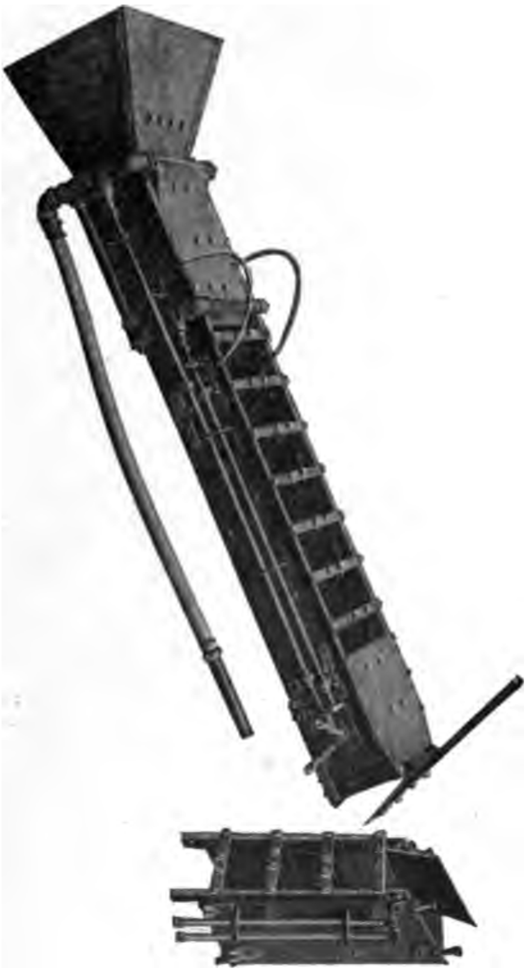


Fig. 122. Portable Gravity Mixer.

There are three general types of concrete mixers: *gravity* mixers, *rotary* mixers, and *paddle* mixers.

Gravity mixers are the oldest type of concrete mixers. They require no power, the materials being mixed by striking obstructions which throw them together in their descent through the machine.

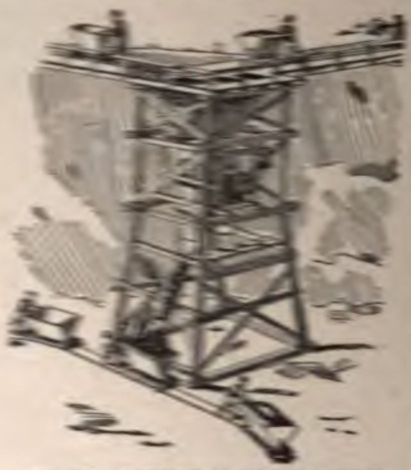


Fig. 123. Operation of Portable Gravity Mixer

Their construction is very simple. Fig. 122 illustrates a portable gravity mixer. This mixer, as will be seen from the figure, is a steel trough or chute in which are contained mixing members consisting of pins or blades. The mixer is portable, and requires no skilled labor to operate it. There is nothing to get out of order or cause delays. It is adapted for both large and small jobs. In the former case, it is usually fed by measure, and by this method will produce concrete as fast as the materials can be fed to their respective bins and the mixed concrete can be taken from the discharge end of the

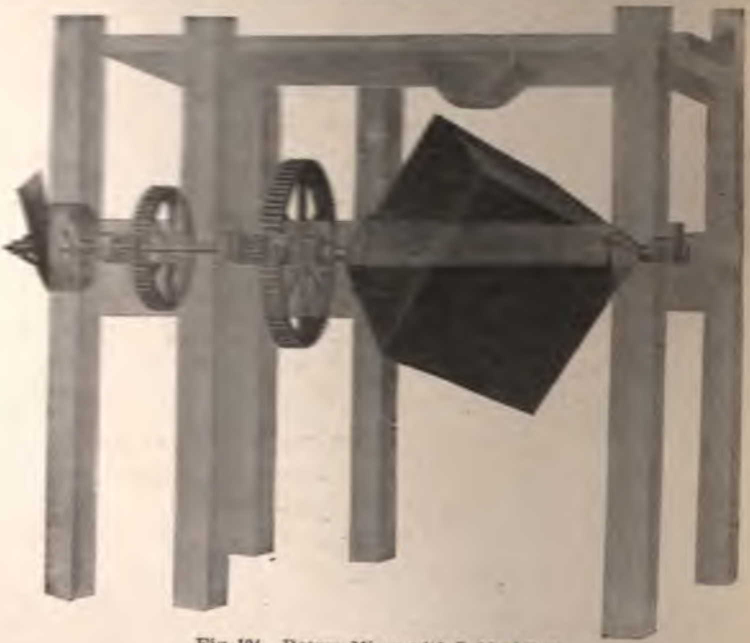


Fig. 124. Rotary Mixer with Cubical Box.

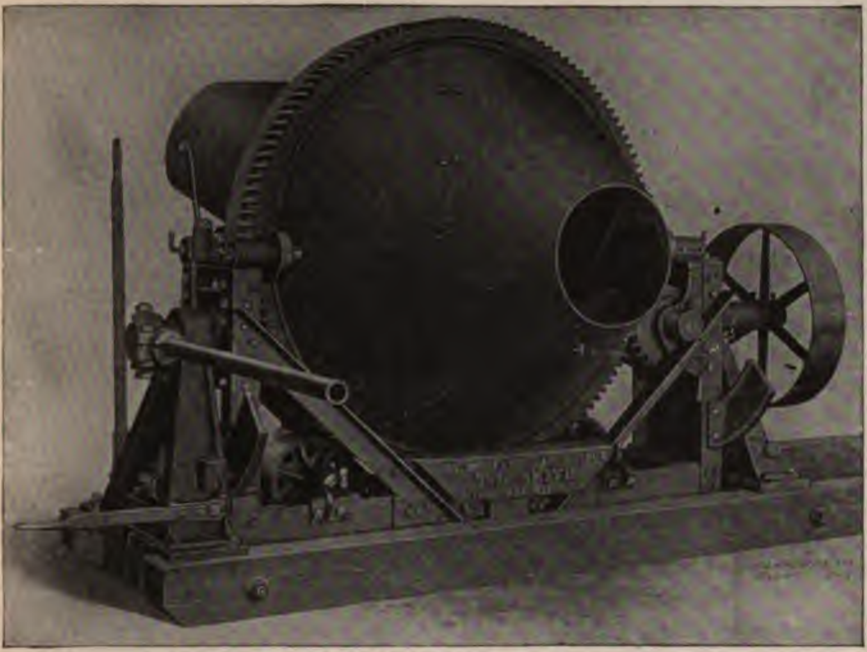


Fig. 125. Rotary Mixer Mounted on Frame.

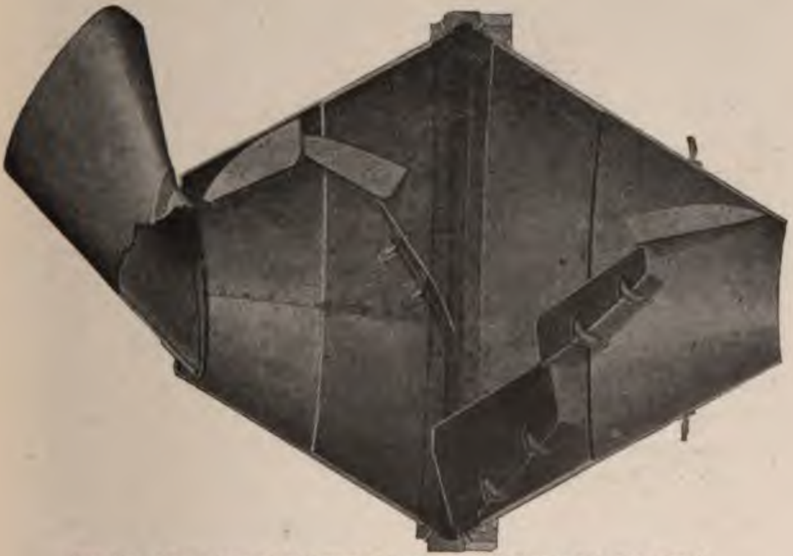


Fig. 126. Cross-Section of Drum of Rotary Mixer (front half cut away), Showing Blades and Lining.

mixer. On very small jobs, the best way to operate is to measure the batch in layers of stone, sand, and cement respectively, and feed to the mixer by men with shovels.

There are two spray pipes placed on the mixer: for feeding by hand, one spray only would be used; the other spray is intended for use only when operating with the measure and feeder, and a large amount of water is required. These sprays are operated by handles which control two gate-valves and regulate the quantity of water flowing from the spray pipes.

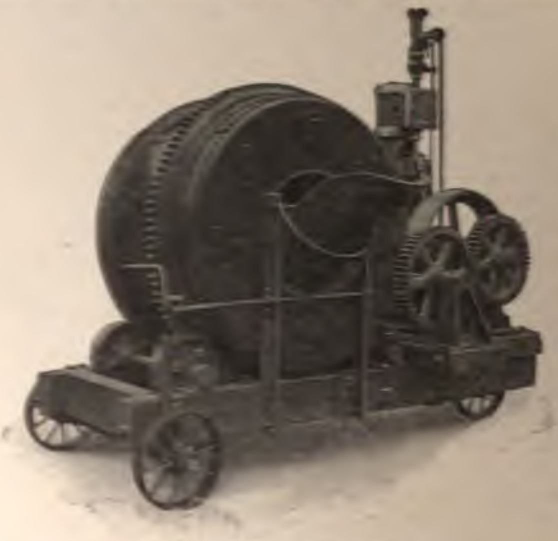


Fig. 122. Ransome Batch Mixer.

These mixers are made in two styles, sectional and non-sectional. The sectional can be made either 4, 6, or 8 feet long. The non-sectional are in one length of 6, 8, or 10 feet. Both are constructed of $\frac{1}{8}$-inch steel. To operate this mixer, the materials must be raised to a platform, as shown in Fig. 123.

Rotary mixers, Fig. 124, generally consist of a cubical box made of steel and mounted on a wooden frame. This steel box is supported by a hollow shaft through two diagonally opposite corners, and the water is supplied through openings in the hollow shaft. Materials are dropped in at the side of the mixer, through a hinged door. The machine is then revolved several times, usually about 15 times; the door is opened; and the concrete is dumped out into carts or cars. There are no paddles or blades of any kind inside the box to assist in the mixing. This mixer is not expensive itself, but the erection of the frame and the hoisting of the stone and sand often render it less economical than some of the more expensive devices.

Rotating mixers which contain reflectors or blades, Fig. 125, are usually mounted on a suitable frame by the manufacturers. The rotating of the drum tumbles the material, and it is thrown against the mixing blades, which cut it and throw it from side to side. Many of these machines can be filled and dumped while running, either by tilting or by their chutes. Fig. 125 illustrates the Smith mixer, and Fig. 126 gives a sectional view of the drum, and shows the arrangement of the blades. This mixer is furnished on skids with

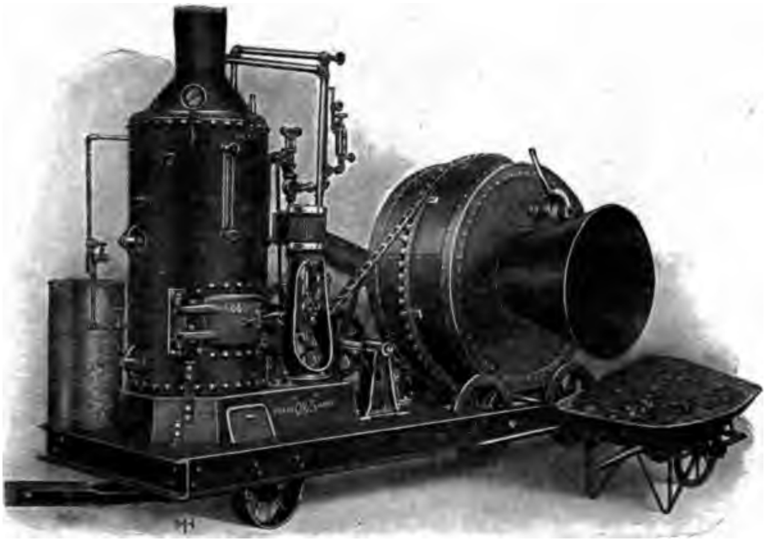


Fig. 128. McKelvey Batch Mixer.

driving pulley. The concrete is discharged by tilting the drum, which is done by power.

Fig. 127 represents a Ransome mixer, which is a batch mixer. The concrete is discharged after it is mixed, without tilting the body of the mixer. It revolves continuously even while the concrete is being discharged. Riveted to the inside of the drum are a number of steel scoops or blades. These scoops pick up the material in the bottom of the mixer, and, as the mixer revolves, carry the material upward until it slides out of the scoops, which therefore assist in mixing the materials.

Fig. 128 represents a McKelvey batch mixer. In this mixer, the lever on the drum operates the discharge. The drum is fed and discharged while in motion, and does not change its direction or its position in either feeding or discharging. The inside of the drum is provided with blades to assist in the mixing of the concrete.

Paddle mixers may be either continuous or of the batch type. Mixing paddles, on two shafts, revolve in opposite directions, and the concrete falls through a trap door in the bottom of the machine. In the continuous type the materials should be put in at the upper end so as to be partially mixed dry. The water is supplied near the middle of the mixer. Fig. 129 represents a type of the paddle mixer.

333. **Automatic Measures for Concrete Materials.** Mechanical measuring machines for concrete materials have not been very extensively developed. One difficulty is that they require the constant attention of an attendant, unless the materials are perfectly uniform. If the machine is adjusted for sand with a certain percentage of moisture, and then is suddenly supplied with sand having greater or less moisture, the adjustment must be changed or the mixture will not be uniform. If the attendant does not watch the condition of the materials very closely, the proportions of the ingredients will vary greatly from what they should.

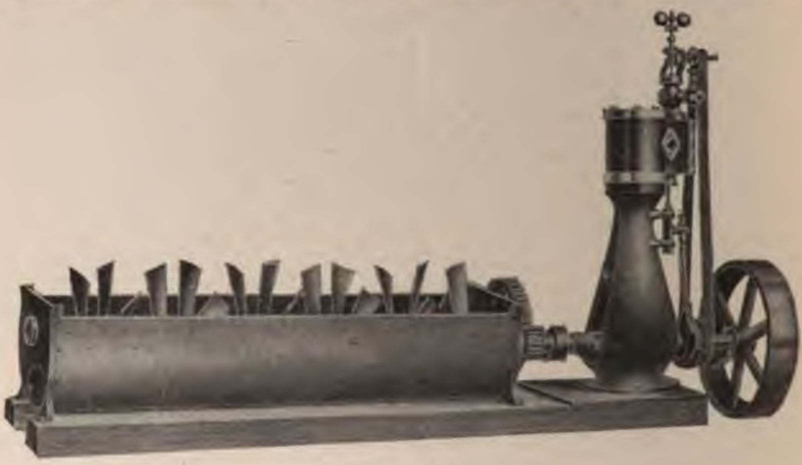


Fig. 129. Paddle Mixer.

The *Trump* measuring device, shown in Fig. 130, consists of a horizontal revolving table on which rests the material to be measured, and a stationary knife set above the table and pivoted on a vertical shaft outside the circumference. The knife can be adjusted to extend a proper distance into the material, and to peel off, at each revolution of the table, a certain amount, which falls into the chute. The material peeled off is replaced from the supply contained in a bottomless storage cylinder somewhat smaller in diameter than the table and revolving with it. The depth of the cut of the knife is adjusted by swinging the knife around on its pivot so that it extends a greater or less distance into the material. The swing is controlled by a screw attached to an arm cast as part of the knife. A micrometer scale, with pointer, indicates the position of the knife. When it is desired to measure off and mix three materials, the machines are made with three tables set one above the other and mounted on the same spindle so that they revolve together. Each table has its own storage cylinder above it, the cylinders being placed one within the other, as shown in Fig. 131.

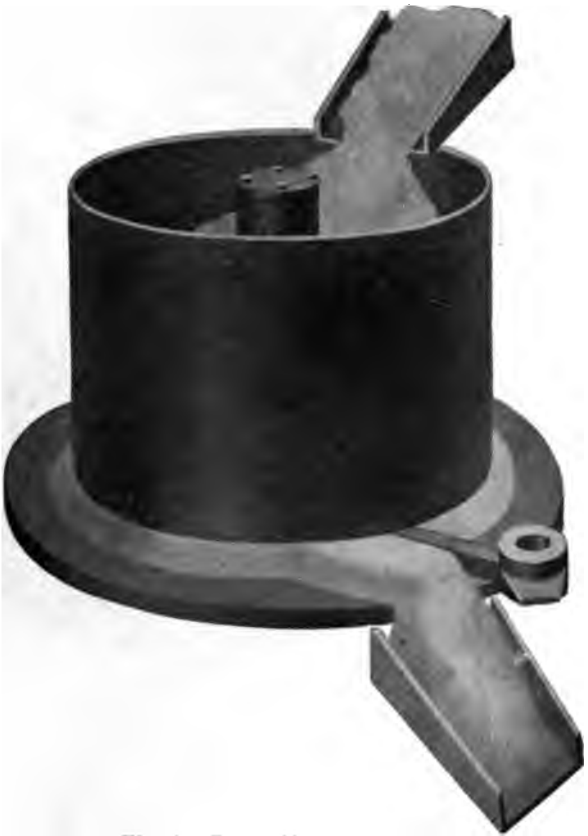


Fig. 130. Trump Measuring Device.

334. **Source of Power.** In each case the source of power for operating the mixer, conveyors, hoists, derricks, or cableways must be considered. If it is possible to run the machinery by electricity, it is generally economical to do so. But this will depend a great deal upon the local price of electricity. When all the machinery can be supplied with steam from one centrally located boiler, this arrangement will be found perhaps more efficient.

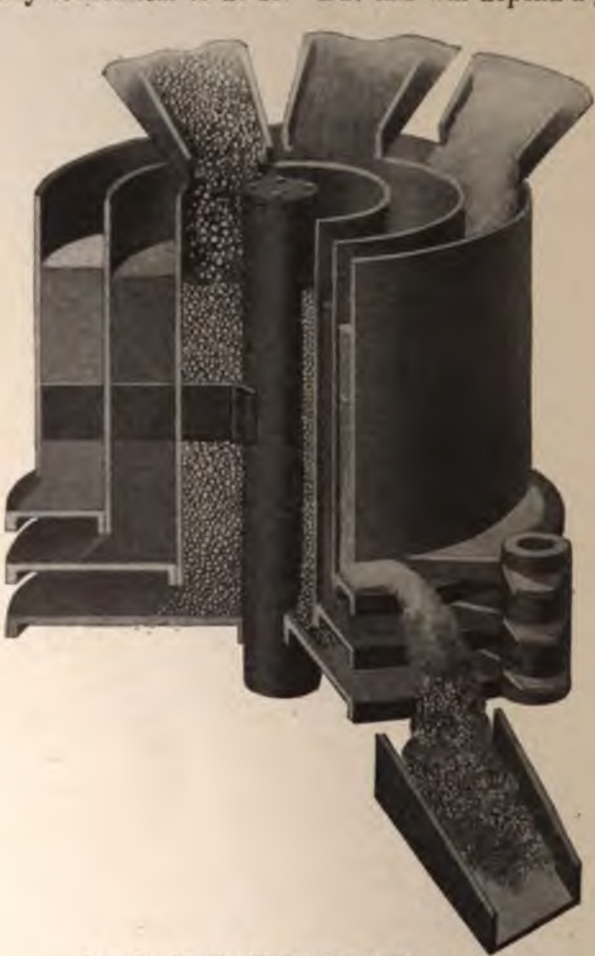


Fig. 131. Interior View of Trump Concrete Mixer.

In the construction of some reinforced-concrete buildings, a part of the machinery was operated by steam and part by electricity. In constructing the Ingalls Building, Cincinnati, the machinery was operated by a gas engine, electric motor, and a steam engine. The mixer was generally run by a motor; but by shifting the belt, it could be run by the gas engine. The hoisting was done by a 20-horsepower Lidgerwood engine. This engine was also connected up to a boom derrick, to hoist lumber and steel. The practice of operating the machinery of one plant by power from different sources, is to be questioned; but the practice of operating the mixer by steam and the hoist by electricity seems to be very common in the construction of buildings. A contractor, before purchasing machinery for concrete work, should carefully investigate the different sources of power for operating the machinery, not forgetting to consider the local conditions as well as general conditions.

335. **Power for Mixing Concrete.** A vertical steam engine is generally used to operate the mixer. The smaller sizes of engines and mixers are mounted on the same frame; but on account of the weight, it is necessary to mount the larger sizes on separate frames. Fig. 132 shows a Ransome disc crank vertical engine, and Table XIX is taken from a Ransome catalogue on concrete machinery. These engines are well-built, heavy in construction, and will stand hard work and high speed.

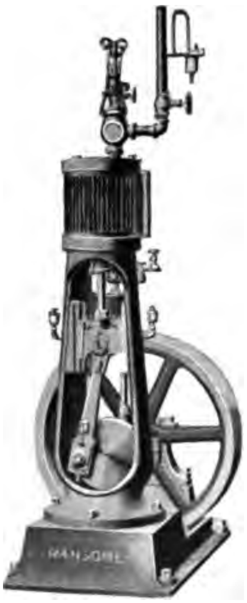


Fig. 132. Ransome Disc Crank Vertical Engine.

336. **Gasoline Engines.** Gasoline engines are used to some extent to operate concrete mixers. Their use so far has been limited chiefly to portable plants such as are used for street work. The fuel for the gasoline engine is much easier moved from place to place than the fuel for a steam engine. Another advantage that the gasoline engine has over the steam engine is that it does not require the constant attention of an engineer.

There are two types of engines—the *horizontal* and the *vertical*. The vertical engines occupy much less floor space for a given horsepower than the horizontal. While each type has its advantages and disadvantages, there does not really appear to be any very great advantage fo one type over the other. Both types of engines are

TABLE XX

Tables of Sizes of the Lambert Hoisting Engines

Horse-Power Usually Rated	Dimensions of Cylinders		Dimensions of Drums		Weight Hoisted, Single Line	Suitable Weight for Pile-Driving Hammer for Quick Work
	Diameter (Inches)	Stroke (Inches)	Diameter (Inches)	Length between Flanges (Inches)		
10	5¼	8	12	16	2,500	1,600
14	6¼	8	12	16	3,000	2,000
20	7	10	14	18	5,000	3,000
25	7½	10	14	24	6,500	4,000
30	8¼	10	14	24	8,000	5,000
35	9	10	14	24	9,000	5,000
40	9½	10	16	23	10,000	6,000

maximum only a part of the time, even though the hoist may be operated practically continuously. From an economical point of view, these conditions give the electric motor-driven hoist special advantages, in that the electric hoist should always be ready, but using power only when in actual operation and then only in proportion to the load handled. The ease with which a motor is moved, and the simplicity of the connection to the service supply, requiring only two wires to be connected, are also in favor of the electric motor.

Fig. 136 shows a motor made by the Westinghouse Electric & Manufacturing Company, which is designed for the operation of cranes, hoists, or for intermittent service in which heavy starting torques and a wide speed variation are required. The frames are enclosed to guard against dirt and moisture, but are so designed that the working parts may be exposed for inspection or adjustment without dismantling. These motors are series-wound, and are designed for operating on direct-current circuits. The motor frames are of cast steel, nearly square in section and very compact. The frame is built in two parts, and so divided that the upper half of the field can be removed without disturbing the gear or shaft, making it easy to take out a pole-piece and field-coils, or to remove the armature. Fig. 137 shows the controller for this type of motor. These controllers, when used for crane service, may be placed directly in the crane cage and operated by hand, or mounted on the resistance frames outside the cage, and operated by bell cranks and levers, so that the

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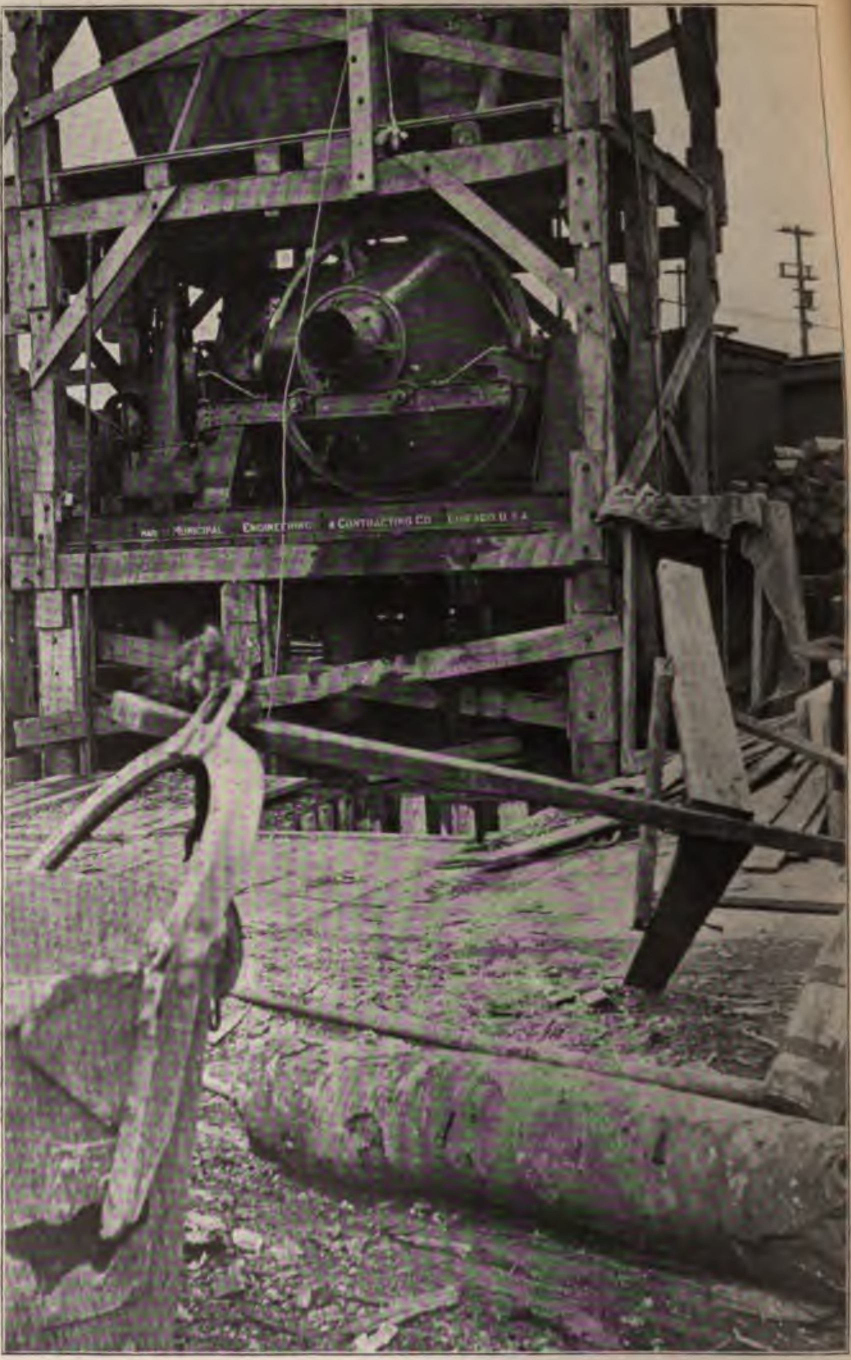


CONCRETE MIXER USED IN CONSTRUCTION OF TUNNEL UNDER DETROIT RIVER

Note the large bin above the mixer, for cement and gravel; also the shaft down which the concrete is passed through a chute.

attendant may stand closer to the operating handles and away from the contacts and resistance.

Polyphase induction motors are being used to some extent for general hoisting and derrick work. These motors may be of the two-phase or three-phase type; but the latter are slightly more efficient. These motors are provided with resistances in the rotor circuit, and with external contacts for varying the same. Two capacities of resistance can be furnished: (*a*) Intermittent service, zero to full load; and (*b*) Intermittent service, zero to half-speed; and continuous service, half-speed to full speed. The controllers are of the drum type, similar to those used on street-cars.

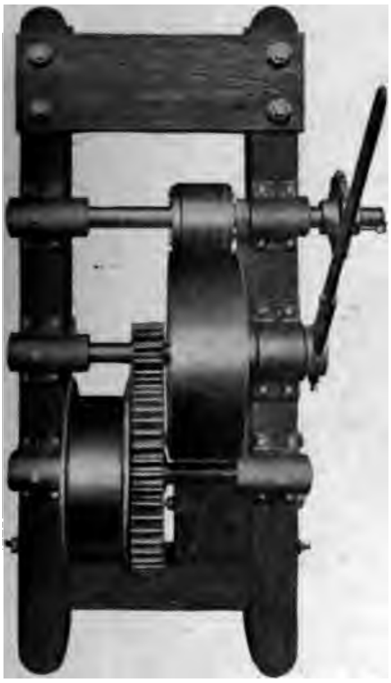


Fig. 135. Ransome Friction Crab Hoist.

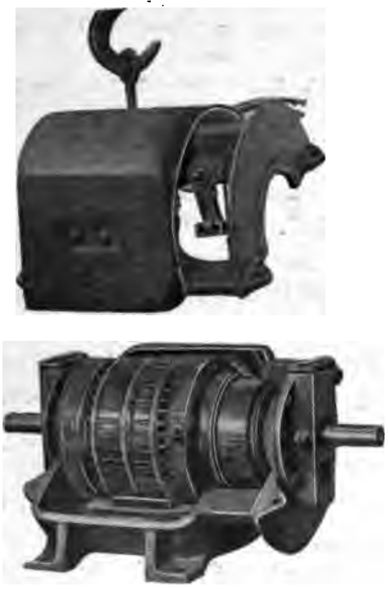


Fig. 136. Motor with Fields Parted. For operation of cranes, hoists, etc.

340. Hoisting Lumber and Steel. In constructing large reinforced-concrete buildings, usually a separate hoist is used to hoist the steel and lumber for the forms. This hoist may be equipped with either an electric motor or an engine, depending upon the general

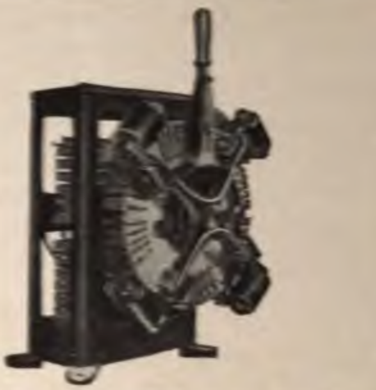


Fig. 137. Regulating and Reversing Controller for Motor of Fig. 136.

arrangement of the plant. These hoists are usually of the single-drum type.

341. **Hoisting Buckets.** In building construction, concrete is usually hoisted in automatic dumping buckets. The bucket is designed to slide up and down a light framework of timber, as shown in Fig. 138, and to dump automatically when it reaches the proper place to dump. The dumping of the buckets is accomplished by the bucket pitching forward at the point where the front guide in the hoisting tower is cut off. The bucket rights itself again automatically as soon as it begins to descend. These buckets are often used for hoisting sand and stone as well as concrete. The capacity of these buckets varies from 10 cubic feet to 40 cubic feet. Fig. 139 shows a Ransome bucket which has been satisfactorily used for this purpose.

342. **Charging Mixers.** The mixers are usually charged by means of wheelbarrows, although other means are

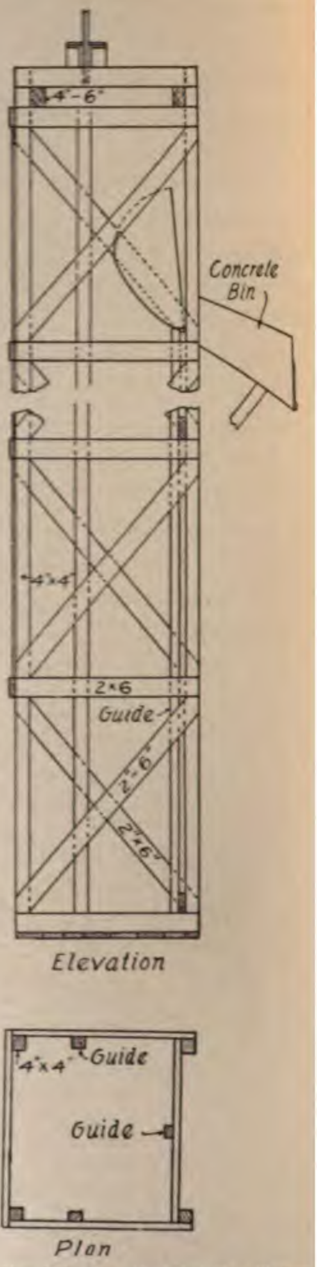


Fig. 138. Detail of Hoisting Tower, with Automatic Dumping Bucket.

sometimes used. Fig. 140 shows the type of wheelbarrow generally used for this work. The capacity varies from 2 cubic feet to 4 cubic feet, the latter size being generally used, as with good runways, a man can handle four cubic feet of stone or sand in a well-constructed wheelbarrow.

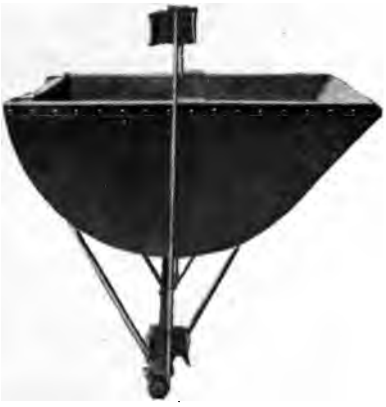


Fig. 139. "Ransome" Concrete Hoist-Bucket.

In ordinary massive concrete construction, as foundations, piers, etc., where it is not necessary to hoist the concrete after it is mixed, the mixer is usually elevated so that the concrete can be discharged directly into wheelbarrows, carts, cars, or a chute from which the wheelbarrows or carts are filled. It is much better to discharge the concrete into a receiving chute than to discharge it directly into the conveyor. The chute can be emptied while the mixer is being charged and rotated; while, if the concrete is discharged directly into wheelbarrows or carts, there must be sufficient wheelbarrows or carts *waiting* to receive the discharge, or the man charging the mixer will be idle while the mixer is being discharged. A greater objection is that if the man in charge of the mixer finds that the charging men or conveying men are waiting, he is very apt to discharge the concrete before it is thoroughly mixed, in an effort to keep all the men busy. A platform is built at the elevation of the top of the hopper, through which the materials are fed to the mixer, Fig. 141. This is a rather

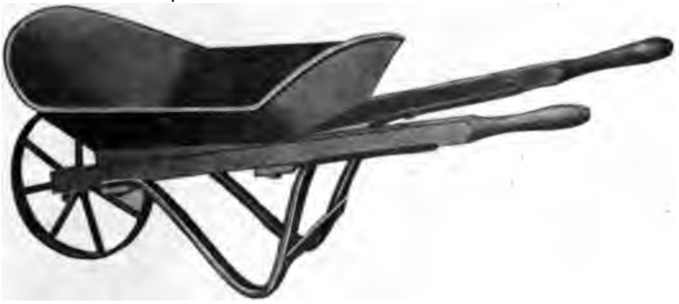


Fig. 140. "Sterling" Contractor's Wheelbarrow.

expensive operation for mixing concrete, and should always be avoided when possible.

Fig. 142 shows a charging elevator devised by the McKelvey Machinery Company. The bucket is raised and lowered by the same engine by which the concrete is mixed, and operated by the same man. The capacity of the charging bucket is the same as that of the mixer.

In Fig. 143 is shown an automatic loading bucket which has been devised by the Koehring Machine Company for charging the mixers made by them. The bucket is operated by a friction clutch, and is provided with an automatic stop. In using either make of these charging buckets, it is necessary to use wheelbarrows to charge the buckets, unless the materials are close to the mixer.

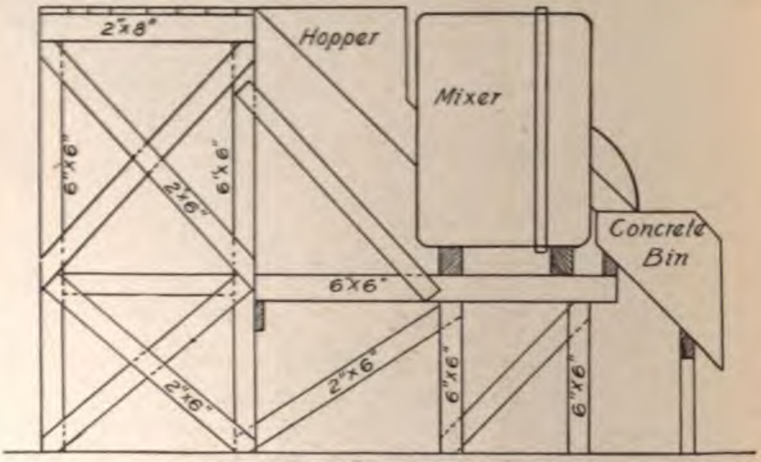


Fig. 141. Concrete Mixer Erected.

343. **Transporting Concrete.** Concrete is usually transported by wheelbarrows, carts, cars, or derricks, although other means are frequently used. It is essential, in handling or transporting concrete, that care be taken to prevent the separation of the stone from the mortar. With a wet mixture, there is not so much danger of the stone separating. Owing to the difference in the time of setting of Portland cement and Natural cement, the former can be conveyed much farther and with less danger of the initial setting taking place before the concrete is deposited. When concrete is mixed by hand, wheelbarrows are generally used to transport the concrete; and they

are very often used also for transporting machine-mixed concrete. The wheelbarrows used are of the same type as shown in Fig. 140. About two cubic feet of wet concrete is the average load for a man to handle in a wheelbarrow.

Fig. 144 shows a cart of the Koehring make, for transporting concrete. The capacity of these carts is six cubic feet. One man can push or pull these carts over a plank runway. The runway consists of two planks, each 8 to 10 inches wide, fastened together with 1-inch by 6-inch cross-pieces, and made in sections so that they can be easily handled by two men.

When it is necessary to convey concrete a longer distance than it is economical to do so by wheelbarrows or carts, a dumping car run on a track is often used. Fig. 145 shows a steel car for this purpose. The capacity of these cars is from 10 cubic feet to 40 cubic feet, and the track gauge is from 18 inches to 36 inches. Both end and side dumping cars are made.

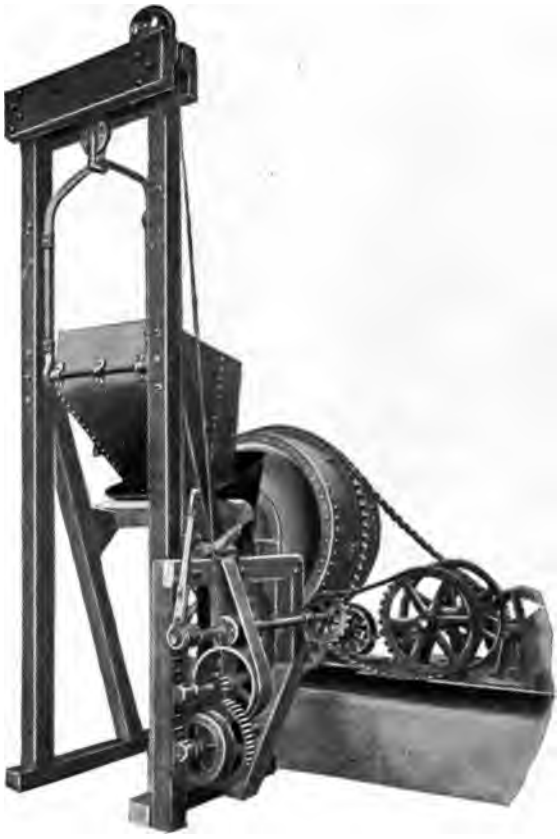


Fig. 142. "McKelvey" Charging Bucket Discharging Batch into Mixer Operated by Same Power.

If a large amount of concrete is to be deposited near where it is mixed, derricks are frequently used to convey the concrete. A com-

[illegible] is easily made by using flat cars with [illegible].

344. **Boilers.** Upright tubular boilers are generally used to supply steam for concrete mixers and hoists operated by steam engines, when they are isolated. For the smaller sizes of mixers, the boilers are on the same frame as the engine and mixer. Fig. 128 shows a McKelvey mixer, engine, and boiler mounted on the same frame. In a similar manner the boiler is often fastened to the same frame as the hoisting engine. This arrangement cannot be used for the larger sizes of mixers and hoists, as they are too heavy to be handled conveniently.

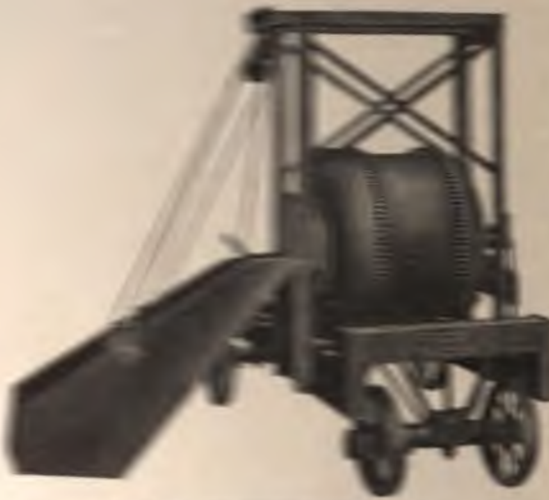


Fig. 143. Automatic Loading Bucket.

When it is possible, the mixer and hoists should be supplied with steam from one centrally located boiler. A portable boiler is then generally used.

345. **Woodworking Plant.** A portable woodworking plant can very often be used to advantage in shaping the lumber for the forms

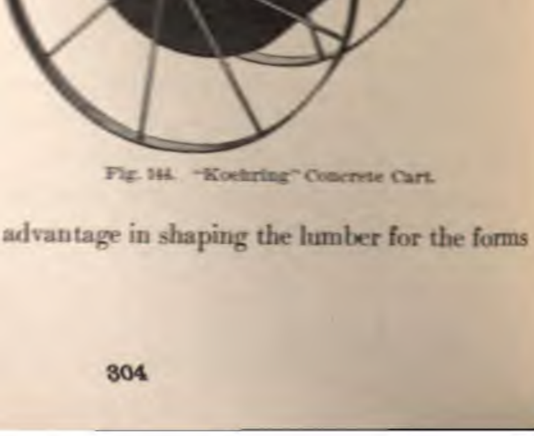


Fig. 144. "Koehring" Concrete Cart.

when a large building is to be erected. The plant can be set near the site of the building to be erected, and the woodworking done there. The machinery for such a plant should consist of a planer adapted for surfacing lumber on three sides, a rip saw, a crosscut circular saw; and in some cases a band saw can be used to advantage. Usually, the difference in cost between surfaced and unsurfaced lumber is so small that the lumber could not be surfaced in a plant of this kind, for the difference in cost; but perhaps it would be more uniform in thickness. In such a plant the rip saw and the crosscut saw would be found to be the most useful; and if reasonable care is taken, this machinery will soon pay for itself. It is often difficult to get work done at a planing mill when it is wanted; and if a contractor has his own woodworking machinery, he will be independent of any planing mill. A plant of this kind can be operated by a steam or gasoline engine or an electric motor.

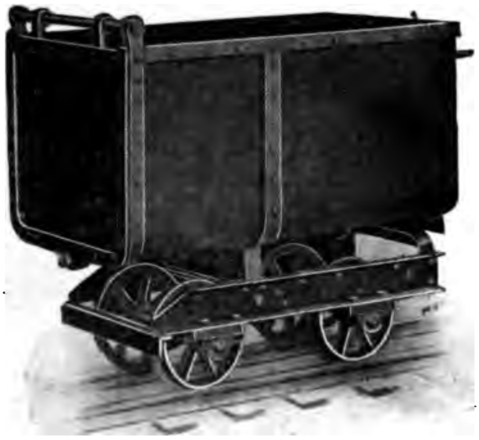


Fig. 145. Steel Body End Dump Car.

346. **Plant for Ten-Story Building.** The plant used by Cramp & Company in constructing a reinforced-concrete building for the Boyertown Burial Casket Company, Philadelphia, will be described, to show the arrangement of the plant rather than the make of the machinery used. The building is 80 feet by 120 feet, and is ten stories high; also, there is a mezzanine floor between the first and second floors. This building is structurally of reinforced concrete, except that the interior columns in the lower floors were constructed of angles and plates and fireproofed with concrete. The power plant for the building is to be located at a level of about seven feet below the basement floor. The hoisting shaft is built in the elevator shaft located in the rear of the building. The hoisting tower is constructed of four 4 by 4-inch corner-posts, and well braced with 2 by 6-inch

plank. Two guides are placed on opposite sides; also one on the front, Fig. 146. The front guide was made in lengths equal to the height of different floors of the building. Fig. 146 shows the location of all the machinery, all of which is of the Ransome make. The concrete was discharged directly from the mixer into the bucket, which rested at the bottom of the elevator shaft. At the elevation where it was desirable to dump the concrete, the front slide was taken out, and the concrete was dumped automatically by the bucket tipping forward. The bucket rights itself as soon as it begins to descend.

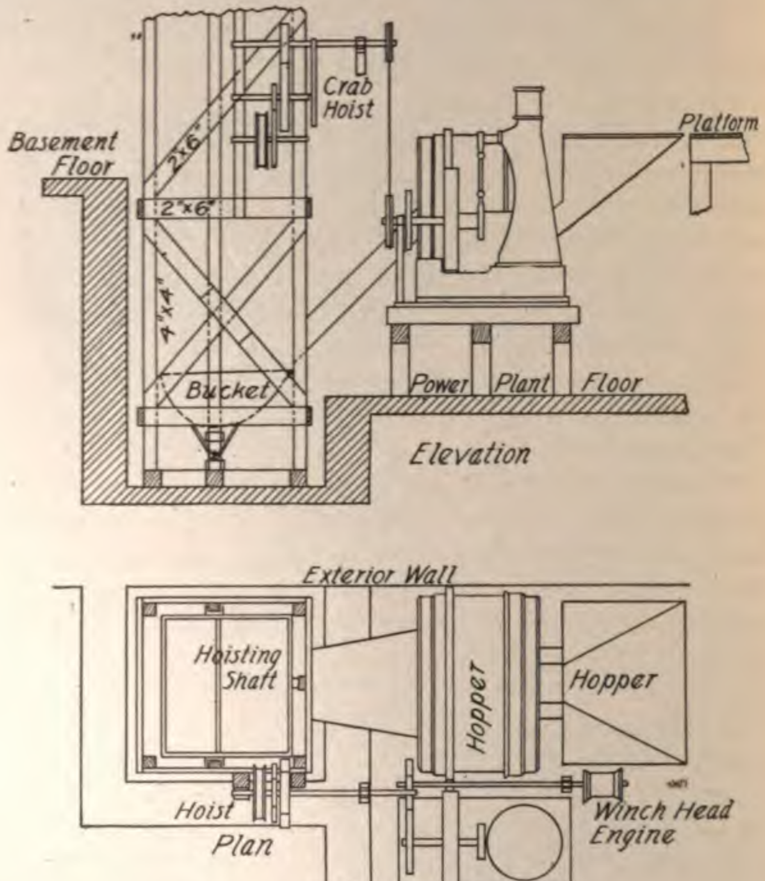


Fig. 146. Concrete Plant for Ten-Story Building.

The capacity of the mixer and hoisting bucket per batch, was 20 cubic feet. A 9 by 9-inch, 20-horse-power vertical engine was used to mix and hoist the concrete, steel, structural steel for columns, and lumber for the forms. A 30-horse-power boiler was used to supply the steam, which was located several feet from the engine, and is not shown in the plan view of the plant. A Ransome friction crab hoist was used to hoist the concrete, and was connected to the engine by a sprocket-wheel and chain. The steel and lumber were hoisted by means of a rope, wrapped three or four times around a winch-head which was on the same shaft as the mixer. The rope extended vertically up from the pulley, through a small hole in the floors, to a small pulley at the height required to hoist the lumber or steel; and then it extended horizontally to another pulley at the place where the material was to be hoisted. The rope descends over the pulley to the ground. A man was stationed at the engine to operate the rope. There were two rope-haulages operated from the pulley on the engine shaft, one being used at a time. On being given the signal, the operator wrapped the rope around the winch-head three or four times, kept it in place, and took care of the rope that ran off the pulley as material was being hoisted.

Wheelbarrows were used in charging the mixer, and hand-carts were used in distributing the concrete. The runways were made by securely fastening two 2 by 10-inch planks together in sections of 12 feet to 16 feet, which were handled by two men. By keeping the runway in good condition, two men were generally able to distribute the concrete, except on the lower floors, and when it was to be transported the full length of the building. The capacity of the carts was 6 cubic feet each. Concrete for the ninth floor was hoisted and placed at the rate of 15 cubic yards per hour.

347. **Plant for the Locust Realty Company Building.** The plant used for constructing a five-story reinforced-concrete building, 117 feet by 200 feet, for the Locust Realty Company, by Moore & Company, Inc., is a good example of a centrally located plant. Near the center of the building is an elevator shaft, in which was constructed the framework for hoisting the concrete. Fig. 147 shows the arrangement of the plant, which is located in the basement and near the center of the building. The mixer is located so that the concrete can be dumped directly into the hoisting bucket. The chute for receiving the materials being about 18 inches above the basement floor,

it was therefore necessary to wheel the materials up an incline. An excavation was made below the level of the basement floor for the hoisting bucket. The mixing was done by a steam engine located on the same frame as the mixer. The concrete was hoisted by a hoisting engine which was located about twenty feet from the shaft. A small hoisting engine was also used for hoisting the steel and lumber used for forms; as this engine was located some distance from the rest of the plant, it is not shown in Fig. 147. The three engines are supplied with steam from a portable boiler which is located as shown in the figure. The efficiency of this plant was shown in the mixing and hoisting of the concrete for the second floor, when 240 cubic yards were mixed and hoisted in 16 hours, or at an average rate of one cubic yard in four minutes.

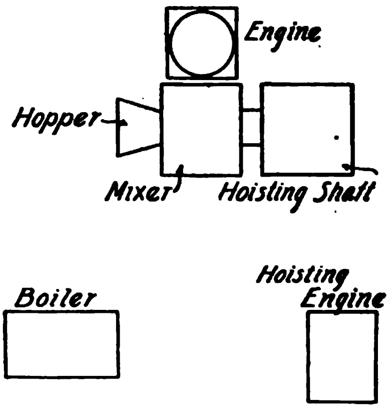


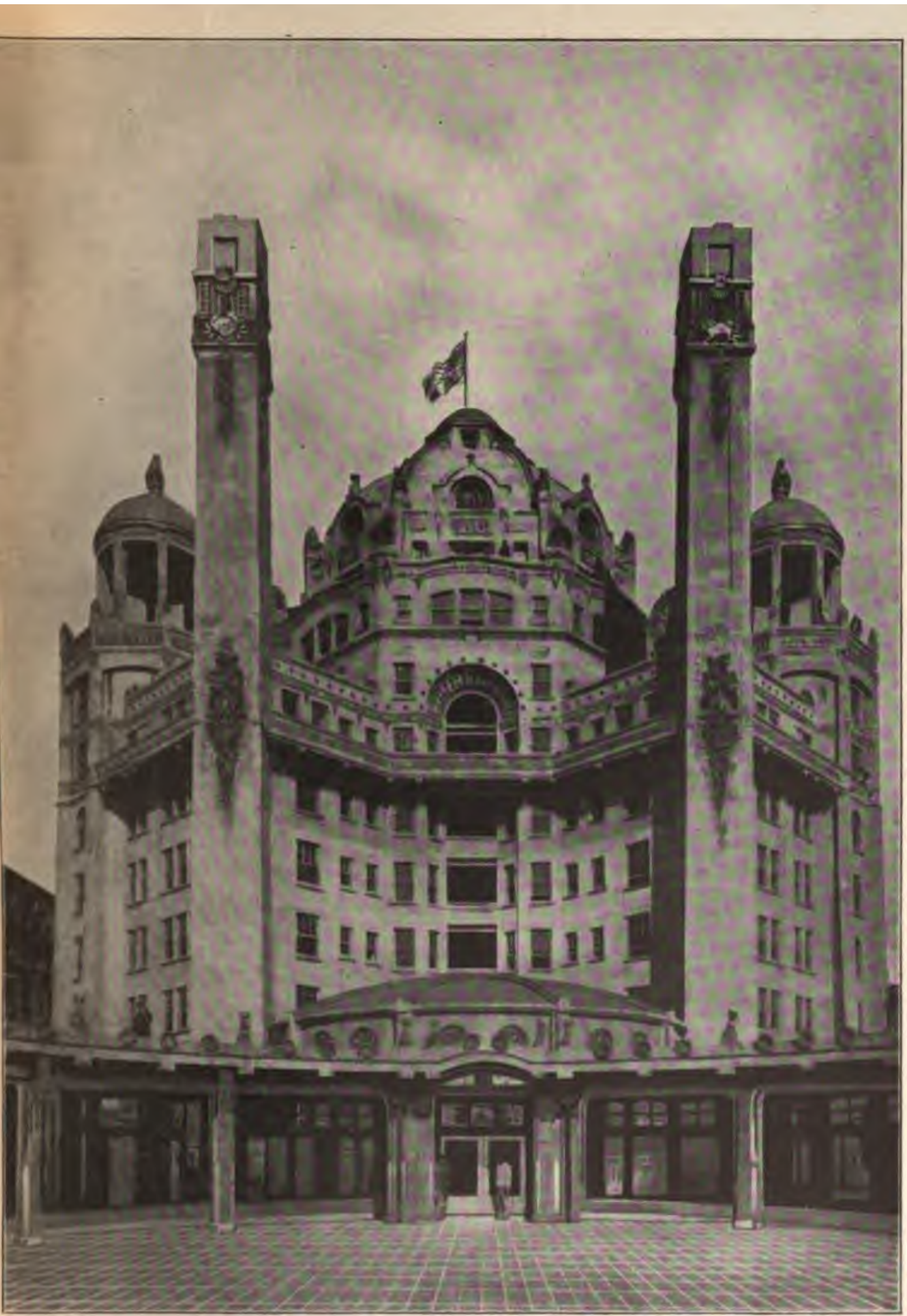
Fig. 147. Concrete Plant for Locust Realty Company Building.

All materials were delivered at the front of the building; and therefore it was necessary to transport the cement, sand, and stone about 100 feet to the mixer. This was done by means of wheelbarrows especially designed and made for Moore & Company, Inc., the capacity being four cubic feet. The concrete was a 1:2:4 mix, and was mixed in batches of 14 cubic feet. The materials for a batch, therefore, consisted of 2 bags of cement, 1 wheelbarrow of sand, and 2 wheelbarrows of stone.

The lumber for the forms was 1¼-inch plank, except the support and braces. Details of the forms will be given and discussed under the heading of "Forms."

348. **Concrete Plant for Street Work.** A self-propelling mixing and spreading machine has been found very desirable for laying concrete base for street pavements. Fig. 148 illustrates a plant of this kind, which has been devised by the Municipal Engineering & Contracting Company. One of these machines was very successfully used in Buffalo, N. Y., in 1907.

The mixer is of the improved cube type, mounted on a heavy



HOTEL MARLBOROUGH-BLENHEIM, ATLANTIC CITY, N. J.

This Magnificent Structure, Built Throughout of Reinforced Concrete is Especially Noteworthy as the Most Elaborate Application which has Yet been Made of this New Material of Construction. It is Located Near the Famous "Boardwalk," Overlooking the Ocean.

truck frame. The concrete is discharged into a specially designed bucket, which receives the whole batch and travels to the rear on a truck which is about 25 feet long. The head of the truck is supported by guys, and also by a pair of small wheels near the middle of the truck, which rest on the graded surface of the street. The truck or boom is pivoted at the end connected to the main truck, and has a horizontal swing of about 170 degrees, so that a street 50 feet wide is covered. An inclined track is also constructed, on which a bucket is operated for elevating and charging the mixer. The bucket is loaded while resting on the ground, with the proper ingredients for a batch, from the materials that have been distributed in piles along the street. The bucket is then pulled up the incline, and the contents dumped into the mixer. An automatic water-measuring supply tank

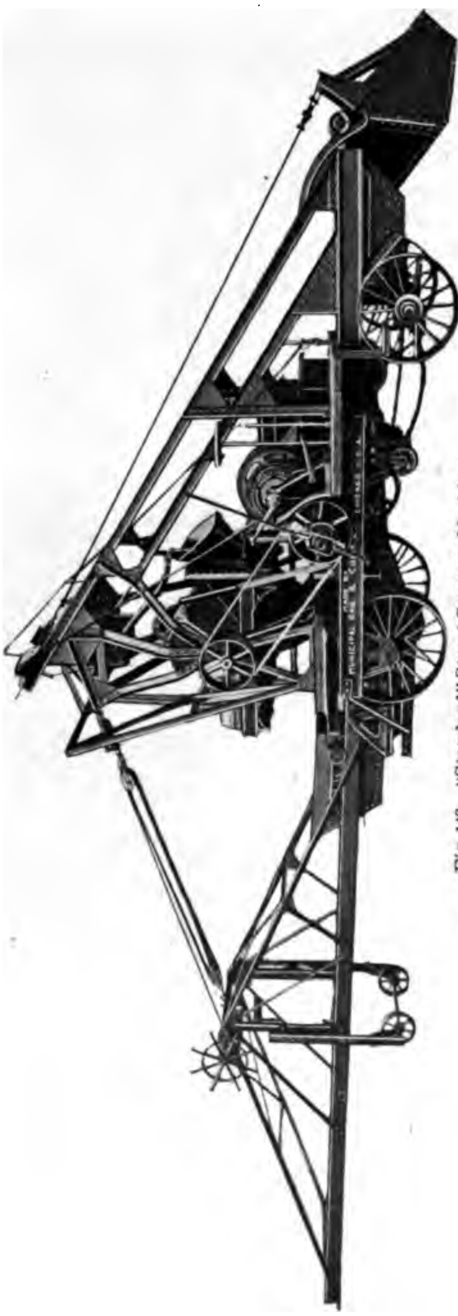


Fig. 148. "Standard" Street Concrete Machine.
Municipal Engineering & Contracting Company, Chicago, Ill.

mounted on the upper part of the frame insures a uniform amount of water for each batch mixed. The power for hoisting, mixing, and distributing the concrete, and propelling the machine, was furnished by a 16-horse-power gasoline engine of the automobile type. The machine can be moved backward as well as forward, and is supplied with complete steering gear.

The capacity per charge of the mixer and charging and distributing buckets used at Buffalo, was 11 cubic feet. The crew consisted of 16 men and a foreman, and they mixed and laid from 110 to 120 cubic yards per hour. Their best record was 1,000 cubic yards in an eight-hour day. The thickness of the concrete base laid was 6 inches.

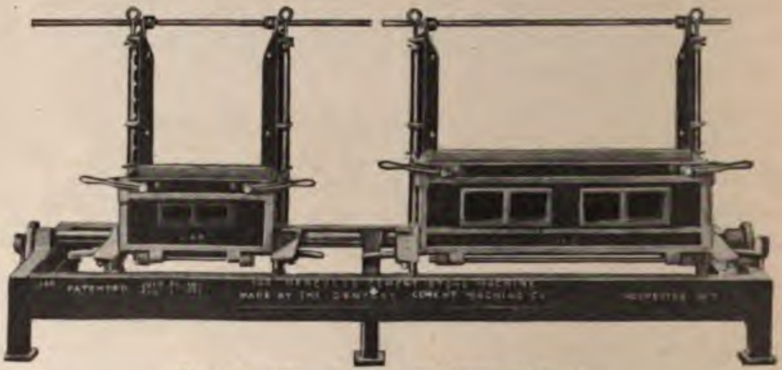


Fig. 149. "Hercules" Cement Stone Machine.

349. **Concrete Block Machines.** There are two general types of hollow concrete block machines on the market—those with a *vertical* face and those with a *horizontal* face. In making blocks with the vertical-faced machine, the face of the block is in a vertical position when moulded, and is simply lifted from the machine on its base-plate. The horizontal-faced (or *face-down*) block is made with the face down, the face-plate forming the bottom of the mould. The cores are withdrawn horizontally, or the mould is turned over and the core is taken out vertically; the block is then ready for removal. The principal difference in the two types of machine, is that, if it is desired to put a special facing on the block, it is more convenient to do it with a horizontal-faced machine. With the vertical-faced machine, the special facing is put on by the use of a parting plate. When the part-

ing plate is removed, the two mixtures of concrete are bonded together by tamping the coarser material into the facing mixture.

Fig. 149 shows a *Hercules* machine. The foundation parts can be attached for making any length of block up to 6 feet. The illustration shows two moulds of different lengths attached. These machines are constructed of iron and steel, except that the pallets (the plates on which the blocks are taken from the machine) may be either wood or steel. This type of machine is the horizontal or *face-down* machine.

Another machine of the face-down type is shown in Fig. 150. This machine, the *Ideal*, is simple in construction and operation; they are portable, which makes them convenient to operate. In making blocks with this machine, the cores are removed by means of a lever, while the block is in the position in which it was made. The mould and block are then turned over, and the face- and end-plates are released, and the block removed on the pallet.

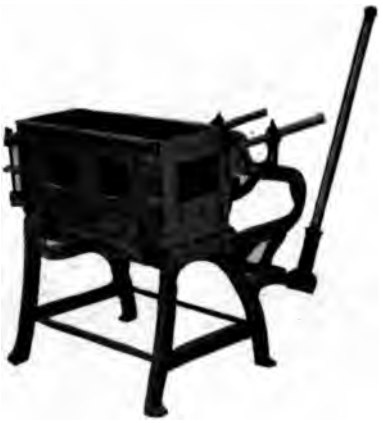


Fig. 150. "Ideal" Concrete Block Machine.

In Fig. 151 are shown a group of the various forms which may be made. The figure also illustrates the facility with which concrete may be utilized for ornamental as well as structural purposes.

Cement Brick Machines. Fig. 152 shows a machine for making cement brick. Ten bricks, $2\frac{3}{8}$ by $3\frac{7}{8}$ by 8 inches, are made at one operation. By using a machine in which the bricks are made on the side, a wetter mixture of concrete can be used than if they are made on the edge. The concrete usually consists of a mixture of 1 part Portland Cement and 4 parts sand. The curing of these bricks is the same as that for concrete blocks. In making these bricks, a number of wooden pallets are required, as the brick should not be removed from the pallet until the concrete has set.

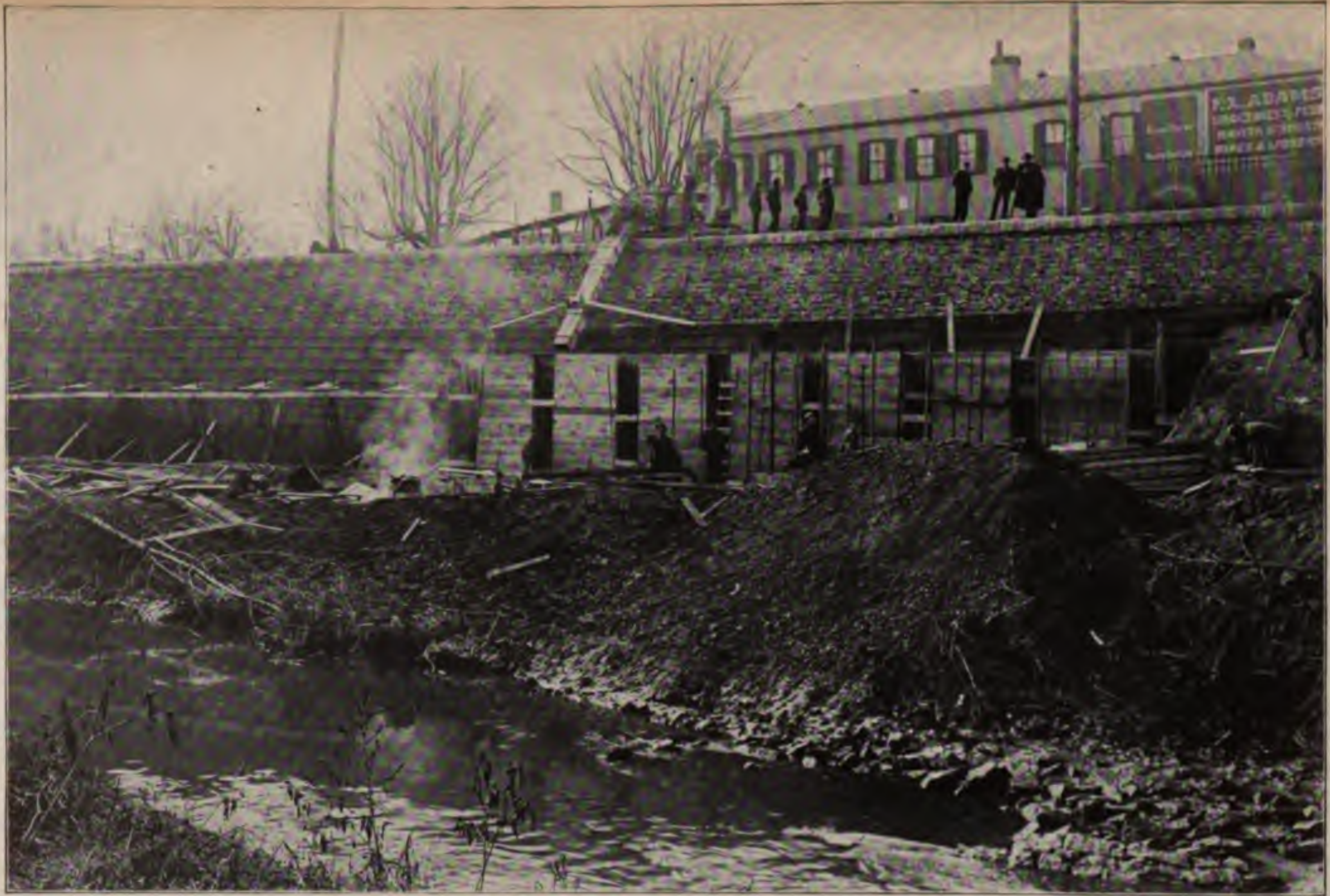
350. **Sand Washing.** It becomes necessary sometimes to wash dirty sand when clean sand can be secured only at a high cost, while

the dirty sand can be easily obtained. If only a small quantity is to be washed, it may be done with a hose. A trough should be built about 8 feet wide and 15 feet long, the bottom having a slope of about 19 inches in its entire length. The side should be about 8 inches high at the lower end, and increase gradually to a height of about 36 inches at the upper end. In the lower end of the trough, should be a gate about 6 inches high, sliding in guides so that it can be easily removed. The sand is placed in the upper end of the trough, and a stream of water is played on it. The sand and water flow down the trough, and the dirt passes over the gate with the overflow water. With a trough of the above dimensions, and a stream of water from a $\frac{3}{4}$-inch hose, three cubic yards of sand should be washed in an hour.

Fig. 151. Group of Blocks Made on "Hercules" Machines.

Concrete mixers are often used for washing sand. The sand is dumped into the mixer in the usual manner, and the water is turned on. When the mixer is filled with water so that it overflows at the discharge end, the mixer is started. By revolving the mixer, the water is able to separate the dirt from the sand, and it is carried off by the overflow of water. When the water runs clear, the washing is completed, and the sand is dumped in the usual way.

When large quantities of sand are required to be washed, special machinery for that purpose should be employed.



STOPPING THE SLIDING OF A RETAINING WALL

West side of Beargrass Creek, Louisville, Ky. The stone-filled timber crib and wooden piles supporting the old wall proving insufficient, a retaining wall of reinforced concrete was built a few feet in front of the sliding wall. The forms for its construction, with the reinforcing bars, are here shown.

Courtesy of J. P. Claybrook, Chief Engineer, Dept. of Public Works, Louisville, Ky.

FORMS

351. General Requirements. In actual construction work, the cost of forms is a large item of expense and offers the best field for the exercise of ingenuity. For economical work, the design should consist of a repetition of identical units; and the forms should be so devised that it will require a minimum of nailing to hold them, and of labor to make and handle them. Forms are constructed of the cheaper grades of lumber. To secure a smooth surface, the planks are planed on the side on which the concrete will be placed. Green lumber is preferable to dry, as it is less affected by wet concrete. If the surface of the planks that is placed next to the concrete is well oiled, the planks can be taken down much easier, and, if they are kept from the sun, can be used several times.

Crude oil is an excellent and cheap material for greasing forms, and can be applied with a whitewash brush. The oil should be applied every time the forms are used. The object is to fill the pores of the wood, rather than to cover it with a film of grease. Thin soft-soap, or a paste made from soap and water, is also sometimes used.

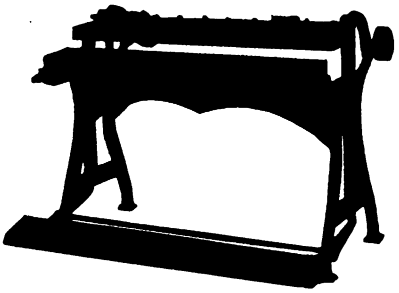


Fig. 152. "Century" Cement Brick Machine.

In constructing a factory building of two or three stories, usually the same set of forms are used for the different floors; but when the building is more than four stories high, two or more sets of forms are specified, so as always to have one set of forms ready to move.

The forms should be so tight as to prevent the water and thin mortar from running through, and thus carrying off the cement. This is accomplished by means of tongued-and-grooved or beveled-edge boards (Fig. 153); but it is often possible to use square lumber if it is thoroughly wet so as to swell it before the concrete is placed. The beveled-edge boards are often preferred to tongued-and-grooved boards, as the edges tend to crush as the boards swell, and beveling prevents buckling.

Lumber for forms may be made of 1-inch, 1½-inch, or 2-inch plank. The spacing of studs depends in part upon the thickness of concrete to be supported, and upon the thickness of the boards on which the concrete is placed. The size of the studding depends upon the height of the wall and the amount of bracing used. Except in very heavy or high walls, 2 by 4-inch or 2 by 6-inch studs are used. For ordinary floors with 1-inch plank, the supports should be placed about 2 feet apart; with 1½-inch plank, about 3 feet apart; and with 2-inch plank, 4 feet apart.

The length of time required for concrete to set depends upon the weather, the consistency of the concrete, and the strain which is to come on it. In good drying weather, and for very light work, it is often possible to remove the forms in 12 to 24 hours after placing the concrete, if there is no load placed on it. The setting of concrete is greatly retarded by cold or wet weather. Forms for concrete arches and beams must be left in place longer than in wall work, because of the tendency to fail by rupture across the arch or beam. In small, circular arches, like sewers, the forms may be removed in 18 to 24 hours if the concrete is mixed dry; but if wet concrete is used, in 24 to 48 hours. Forms for large arch culverts and arch bridges are seldom taken down in less than 14 days; and it is often specified that they must not be struck for 28 days after placing the last concrete. In ordinary floor construction consisting of slabs, girders, and beams, the forms are usually left in place at least a week.

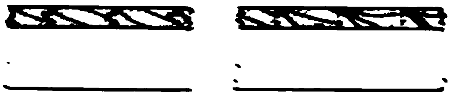


Fig. [illegible]. Corrugated [illegible] Bar

[illegible] Bar

[illegible]. **Forms for Columns.** In constructing columns, the forms should be erected complete the full height of the columns; and concrete is deposited from the top. The concrete must be mixed very wet, and it must be rammed very thoroughly at the bottom, and care must be taken not to displace the steel. Sometimes the forms are constructed in short sections, and the concrete is placed and rammed as the forms are built. The ends of the bottom of the forms for the girders and beams are usually supported by the column forms. To give a beveled edge to the corner of the columns, a triangular strip is fastened in the corner of the forms.

Fig. [illegible] shows the common way, or some modification of it, of constructing forms for columns. The plank may be 1 inch, 1½ inches, or 2 inches thick, and the cleats are usually 1 by 4 inches and 2 by 4 inches. The spacing of the cleats depends on the size of the columns and the thickness of the vertical plank.

Fig. 154 *B* shows column forms similar to those used in constructing the Harvard stadium. The planks forming each side of the column are fastened together by cleats, and then the four sides are fastened together by slotted cleats and steel tie-rods. These forms can be quickly and easily removed.

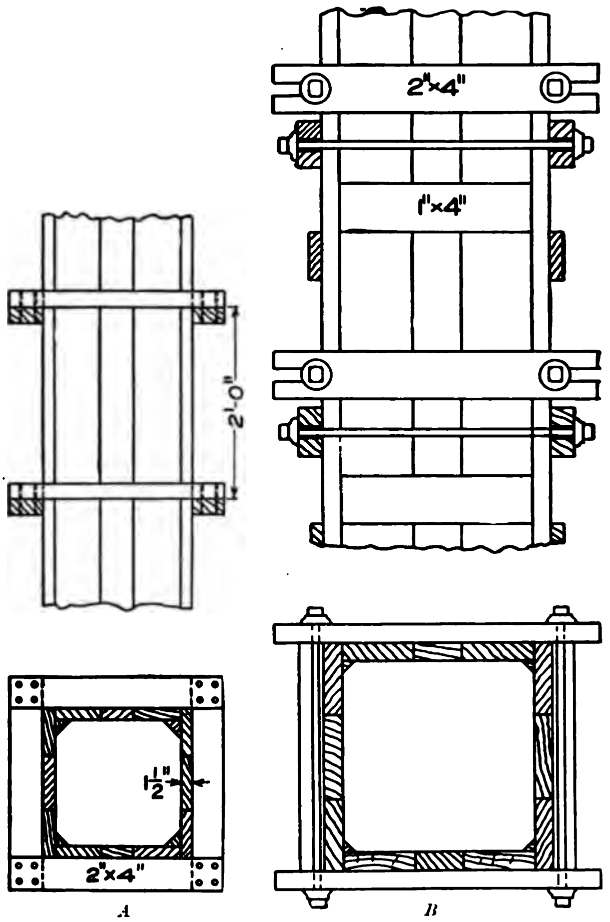


Fig. 154. Forms for Columns. *A*—Common method of construction; *B*—Method used in constructing Harvard Stadium.

Fig. 155 shows a column form in which concrete is placed and rammed as the form is constructed. Three sides are erected to the full height, and the steel is then placed. The fourth side is built up with horizontal boards as the concrete is placed and rammed.

Round columns are often desirable for the interior columns of buildings. Fig. 156 shows a form that has been used for this type of column. The columns for which these forms were used were 20 inches in diameter, and had a star-shaped core made of structural steel. The forms for each column were made in two parts and bolted together. The sides were made of 2 by 3-inch plank surfaced on all four sides, beveled on two, and held in place by the steel bands, which were ¼ by 2½ inches and spaced about 2½ feet apart. One screw in the outer plank at each band of both parts, together with a few intermediate screws, held the plank in place. The building for which these forms were made was ten stories in height. Enough forms were provided for two stories, which was sufficient, as they could be removed when the concrete had been in place one week. Later these same forms were used in constructing the interior columns of a six-story building. Some difficulty was experienced in removing these forms, owing to the concrete sticking to the plank. But had these forms been made in four sections, instead of two, and well oiled, it is thought that this trouble would have been avoided. Columns constructed with forms as shown in Fig. 156 will not have a round surface, but will consist of many flat surfaces, 2½ inches wide. If a perfectly round column is desired, it will be necessary to cut the surface of the plank next to the concrete to the desired radius. Forms for octagonal columns can be made in a somewhat similar manner to these just described.

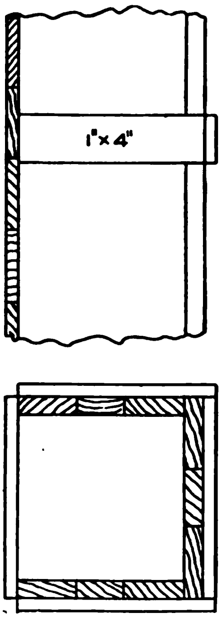


Fig. 155. Form for Column.

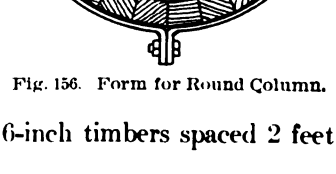


Fig. 156. Form for Round Column.

353. **Forms for Beams and Slabs.** A very common style of form for beam and slab construction is shown in Fig. 157. The size of the different members of the forms depends upon the size of the beams, the thickness of the slabs, and the relative spacing of some of the members. If the beam is 10 by 20 inches, and the slab is 4 inches thick, then 1-inch plank supported by 2 by 6-inch timbers spaced 2 feet

apart, will support the slab. The sides and bottom of the beams are enclosed by 1½-inch or 2-inch plank supported by 3 by 4-inch posts spaced 4 feet apart.

In Fig. 158 are shown the forms for a reinforced-concrete slab, with I-beam construction. These forms are constructed similarly to those just described.

A slab construction supported on I-beams, the bottom of which is not covered with concrete, may have forms constructed as shown in Fig. 159. This method of constructing forms was designed by by Mr. William F. Kearns (Taylor and Thompson, "Plain and Reinforced Concrete").

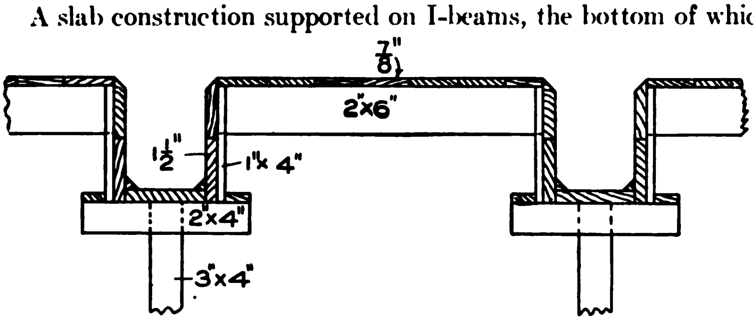


Fig. 157. Forms for Beams and Slabs.

The construction of forms for a slab that is supported on the top of I-beams is a comparatively simple process, as shown in Fig. 160. In any form of I-beam and slab construction, the forms can be constructed to carry the combined weight of the concrete and forms. When the bottom of the I-beam is to be covered with concrete, it is not so easily done as when the haunch rests on the bottom flange (Fig. 159) or is a flat plate (Fig. 160).

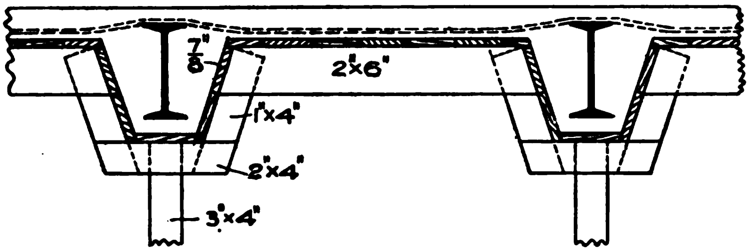


Fig. 158. Forms for Reinforced-Concrete Slab Supported by I-Beams.

354. **Forms for Locust Realty Company Building.** The forms used in constructing the building for the Locust Realty Company (the mixing plant has already been described), present one rather unusual

feature. The lumber for the slabs, beams, girders, and columns was all the same thickness, 1¼ inches. Fig. 161 shows the details of the forms for the beams and slabs. The beams are spaced about 6 feet apart, and are 8 by 16 inches; the slab is 4 inches thick. A notch is cut into the 1 by 6-inch strip on the side of the beams, to support the 2 by 4-inch strip under the plank which supports the concrete for the slab. The posts supporting the forms are 3½ by 3½-inch, and are braced by two 1 by 6-inch boards spaced about 3 feet apart, and extending in the direction of the beams.

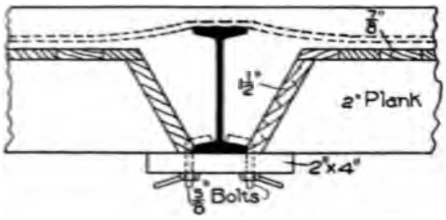


Fig. 159. Form for Reinforced-Concrete Slab between I-Beams.

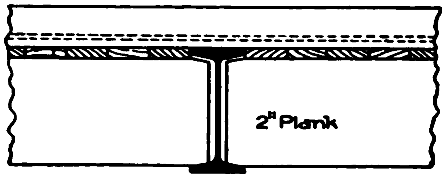


Fig. 160. Form for Floor-Slab on I-Beams.

Fig. 162 shows the forms for the columns. The planks for each side of the column are held together by the 1 by 4-inch strip, and, when erected in place, are clamped by the 2 by 4-inch strip. A large opening is left at the bottom of each column, so that all shavings and sawdust can be removed. This opening is closed just before the concrete is deposited.

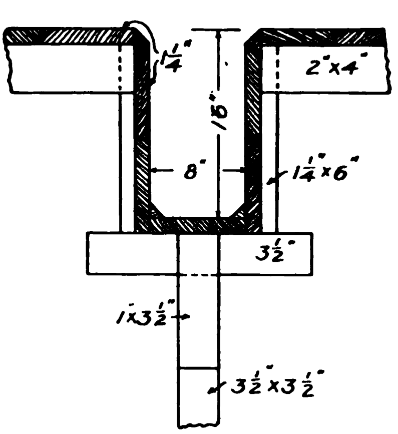


Fig. 161. Beam and Slab Forms for Locust Realty Company Building.

355. **Cost of Forms for Buildings.** An analysis of the cost of forms for an eight-story building is given by R. E. Lamb (*Concrete Engineering*, December, 1907). The basis of his estimate is made on using ⅞-inch by 6-inch tongued-and-grooved lumber for slab forms; 1¾-inch dressed plank for the sides and bottom of the beams and girders; posts 4 by 4-inch spaced 6 feet center to center; and on the fact that it cost $20.00 per thousand feet of lumber to make

and set one floor of forms; that it cost $15.00 per thousand feet to strip the forms and reset them on the next floor; and that it cost about $8.00 per thousand feet to strip the forms and lower them to the ground.

With the size of the beams and girders as shown in Fig. 163, Mr. Lamb states that it will take an average of 4 feet, board measure, to erect each square foot of floor area. The basis of his estimate is as follows: that 1.5 board feet of lumber per square foot of floor is required for the slab; that for every square foot of beam surface, including the bottom, 3.2 board feet per square foot is re-

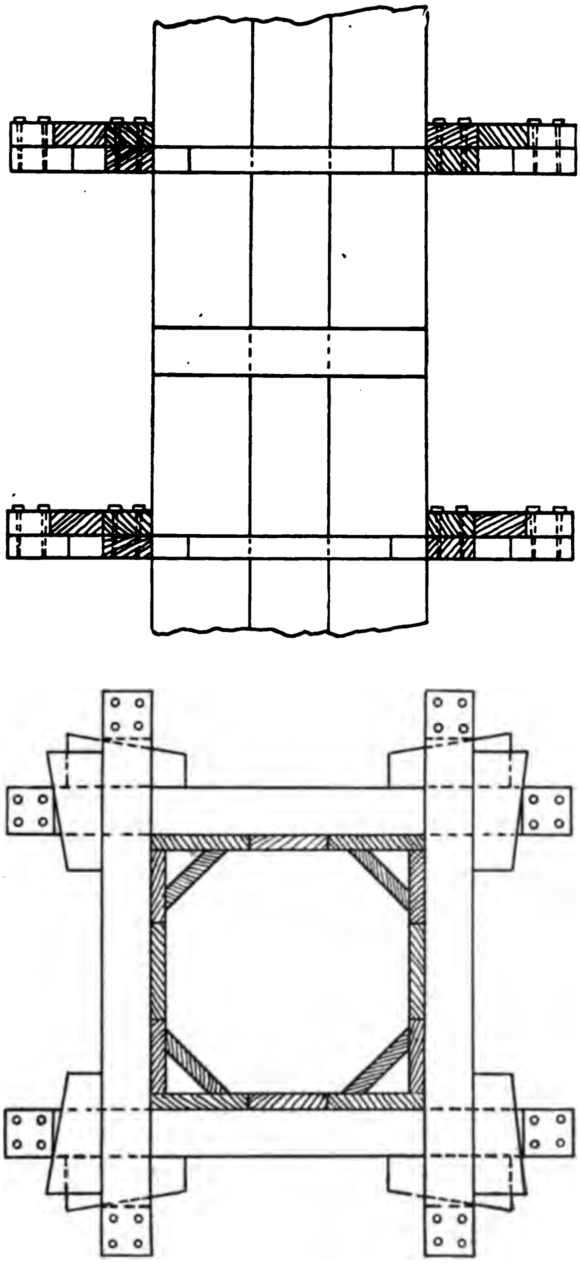


Fig. 162. Column Forms for Locust Realty Company Building.

quired; and that for each square foot of girder, including the bottom, 3.6 board feet of lumber is required. Taking these figures, for the panel shown, the slab will require 1.5 board feet per square foot; the beams, which are 8 by 18-inch, will have 3 feet 8 inches of surface per linear foot; and multiplying this by 3.2 board feet per square foot, and dividing by 7.5 feet, the distance center to center of beams, we find that 1.56 board feet per square foot of floor surface is required. Taking the girder in the same way, with 4 feet 8 inches of surface, multiplied by 3.6 board feet, and divided by 18 feet, the distance center to center of girders, we find that .94 board foot per square foot of floor is required. The total of the lumber required, then, is 1.5 board feet for the slab, 1.56 board feet for the beam, and .94 board foot for the girders—a total of 4 board feet per square foot of floor area.

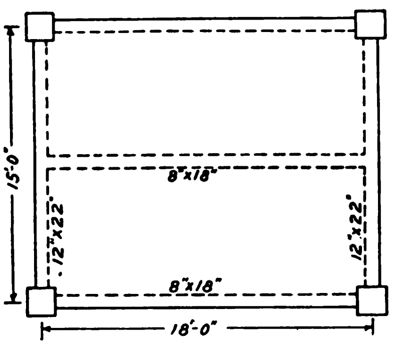


Fig. 163. Diagram of Forms.

In this estimate for an eight-story building, three sets of forms were used:

Roof: Stripping the sixth floor, resetting, altering to form valleys, and finally stripping roof and lowering forms to ground, 4 board feet at 2.6 cents		$ .104
Eighth Floor: Stripping the fifth floor, resetting, and finally stripping and lowering forms to ground, 4 board feet at 2.3 cents		.092
Seventh Floor: Stripping the fourth floor, resetting, and finally stripping and lowering forms to ground, 4 board feet at 2.3 cents		.092
Sixth Floor: Cost, same as for the fourth floor		.060
Fifth Floor: Cost, same as for the fourth floor		.060
Fourth Floor: Stripping the first floor, and resetting, 4 board feet at 1.5 cents		.060
Third Floor: Cost, same as for the first floor		.184
Second Floor: Cost, same as for the first floor		.184
First Floor: Making and setting forms, 4 board feet at 2 cents	$.080	
Material, 4 board feet at 2.6 cents	.104	.184
		9)1.020
Average cost per square foot of surface		$ .113

To this average cost of 11.3 cents, 10 per cent should be added for waste, breakage, nails, etc.; and if two sets of forms are used, the third floor would cost 6 cents per square foot, and the seventh floor 6 cents, giving an average of 9.6 cents per square foot.

In estimating the cost of the forms for the columns, it is assumed that making and placing the forms for the basement columns will cost about $26.00 per thousand; the cost of stripping and resetting, $16.00 per thousand; and 3.1 square feet of lumber is required for each square foot of column surface.

Eighth Story: Stripping sixth story, resetting and altering, finally stripping eighth story, and lowering to ground 3.1 board feet at 2.2 cents		$.068
Seventh Story: Stripping fifth story, resetting, and finally stripping and lowering to ground 3.1 board feet at 1.9 cents		.059
Sixth Story: Cost, same as second story		.050
Fifth Story: Cost, same as second story		.050
Fourth Story: Cost, same as second story		.050
Third Story: Cost, same as second story		.050
Second Story: Stripping basement columns and resetting 3.1 board feet at 1.6 cents		.050
First Story: Cost, same as for the basement columns		.162
Basement: Material, 3.1 board feet at 2.6 cents	$.081	
Making and setting 3.1 board feet at 2.6 cents	.081	
	.162	.162
		9) .701
Average cost per square foot of surface		$.077

To this average cost of 7.7 cents per square foot of column surface, should be added 10 per cent for bolts, nails, waste, etc. If three sets of forms are required, the second-story cost would be 16.2 cents, and the sixth story 5.9 cents, giving the average cost per square foot, of 9.1 cents.

The student should remember that this lumber has a value after it has been removed from the building, and that this value should be deducted from the total cost of the forms, to find the actual cost of forms.

356. **Cost of Forms for Garage.** Some interesting cost data are given by Mr. Reygondeau de Gratresse, Assoc. M. Am. Soc. C. E. (*Engineering-Contracting*, October 30, 1907), on the cost of forms used in erecting a reinforced-concrete garage in Philadelphia during the summer of 1907. The building was 53 feet wide, 200 feet long, and four stories high; also there was a mezzanine floor. Tongued-and-grooved lumber $\frac{7}{8}$ inch thick was used for the slab forms, and $1\frac{3}{4}$-inch plank for the beams and girders.

The area of the 1,740 cubic yards of concrete covered by forms was

	Sq. Ft.
Footings	4.000
Columns	20.000
Floors and girders	70.000
Total	94.000

For this work, 170,000 feet, board measure, of new lumber was bought and 50,000 feet board measure of old lumber was used, the cost being:

50,000 ft. B. M. at $13	$ 650
170,000 ft. B. M. at $26	4,420
220,000 ft. B. M. at $23	$5,070

Since 220,000 feet, board measure, were used for the 1,740 cubic yards, there were 126 feet, board measure, per cubic yard of concrete.

New forms were made for each floor, except the sides of the girders, which were used over for each floor, where the sizes would admit of this being done. The props under the girders were allowed to remain in place throughout the building until the entire job was completed. The forms for the roof were made entirely of the material used on the floors below. The area of concrete covered by the new lumber was approximately 80,000 square feet. This gives a cost for lumber of 5.5 cents per square foot.

A force of fifteen carpenters working under one foreman, framed, erected, and tore down all forms. Laborers handled all the lumber for the carpenters, except when they were at work mixing and placing concrete. The foreman was paid $35 per week, while the carpenters were paid an average of $4.40 for an 8-hour day. Laborers were paid 17 cents per hour, and worked a 10-hour day. Over the laborers was a foreman who received the same wages as the boss carpenter. The forms for a floor were erected in from 8 to 10 days. For the framing, erecting, and tearing down of the forms, the labor cost was about $3,480, which gives a cost of $2 per cubic yard. For the carrying and handling of the lumber, the cost was about $1,914, which gives a cost of $1.10 per cubic yard. This gives a total cost per cubic yard of forms as follows:

	Per Cu. Yd.
Lumber, 126 ft. B. M.	$2.90
Framing, erecting, and tearing down	2.00
Handling lumber	1.10
Total	$6.00

This cost is high, owing to the fact that so little of the lumber was used a second time, there being only from 16 to 20 per cent so used. For the 220,000 feet, board measure, of lumber used on the job, the average cost per thousand for the forms was:

	Per M.
Lumber	$23.00
Framing, erecting, and tearing down	15.67
Handling lumber	8.70
Total	$47.37

The cost per square foot of concrete for the area covered by forms was:

Lumber	$0.064
Labor	0.057
Total	$0.121

The cost per cubic yard for lumber and labor was:

Lumber	$2.90
Labcr on forms	3.10
	$6.00

It should be remembered that the lumber used in the forms had a salvage value, for which no allowance is made in the above $2.90.

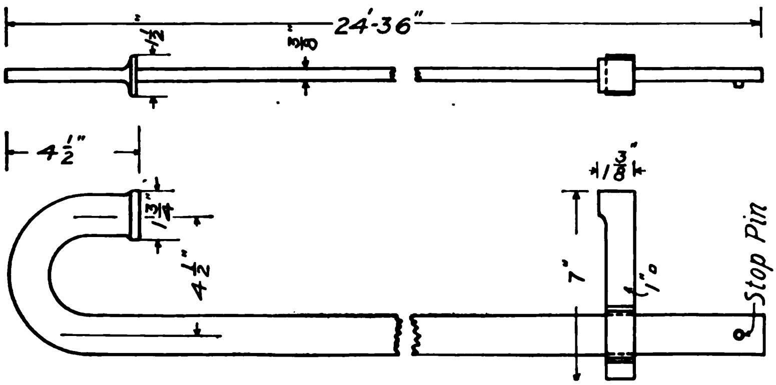


Fig. 164. Adjustable Clamp.

357. **An Adjustable Clamp.** Fig. 164 illustrates an adjustable clamp for holding forms together. It is commonly used to hold the plank forming the sides of a beam or girder in place; it is used also in clamping the opposite sides of columns. It is forged from a 1¼-inch by ⅜-inch steel bar, and is held in place by the slotted forging, 1 inch square, by driving it tight.

358. **Forms for Conduits and Sewers.** Forms for conduits and sewers must be strong enough not to give way, or to become deformed,

The Blaw collapsible steel centering is in general use for sewer and conduit construction. This centering consists of one or more steel plates about $\frac{1}{8}$ inch thick and bent to the shape required by the interior of the sewer to be constructed. The steel plates are held in shape by angle irons. When set in position, the sections are held rigid by means of turnbuckles, which also facilitate the collapsing of the sections. The adjacent sections are held together by staples and wedges, the former being riveted to the plates. The sections are usually made five feet long, and in any desired shape or size required for sewer or conduit work. When these forms are used to construct concrete sewers or conduits, the surface of the forms must be well coated with grease or soap to prevent the concrete from adhering to the steel.

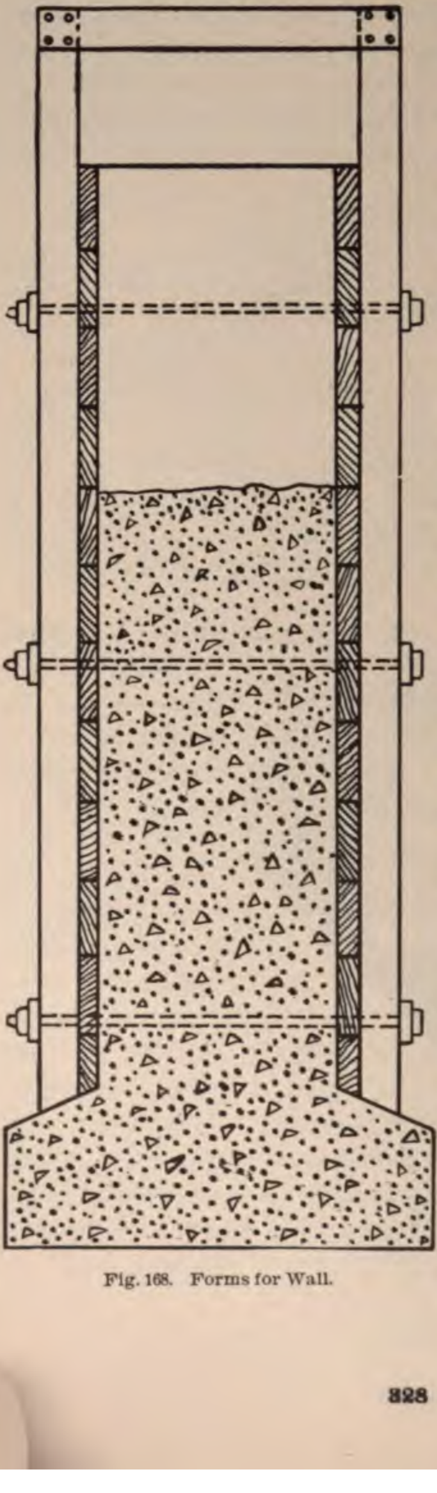


Fig. 168. Forms for Wall.

361. **Forms for Walls.** The forms for concrete walls should be built strong enough so that they will retain their correct position while the concrete is being placed and rammed. In high, thin walls, a great deal of care is required to keep the forms in place so that the wall will be true and straight.

Fig. 168 shows a very common method of constructing these forms. The plank against which the concrete is placed is seldom less than 1½ inches thick, and is usually 2 inches thick. One-inch plank is sometimes used for very thin walls; but even then, the supports must be placed close. The planks are generally surfaced on the side against which the concrete is placed. The vertical timbers that hold the plank in place will vary in size from 2 inches by 4 inches to 4 inches by 6 inches, or even larger, depending on the thickness of the wall, spacing of these vertical timbers, etc. The vertical timbers are always placed in pairs, and are held in place usually by means of bolts, except for thin walls, when heavy wire is often used. If the bolts are greased before the concrete is placed, there is usually not much trouble experienced in removing them. Some contractors place the bolts in short pieces of pipe, the diameter of the pipe being about ⅛ inch greater than that of the bolt, and the length equal to the thickness of the wall. When the bolts are removed, the holes are filled with mortar.

CENTERS FOR ARCHES

362. The centers for stone, plain concrete, and reinforced-concrete arches are constructed in a similar manner. A reinforced-concrete arch of the same span and designed for the same loading, will not be so heavy as a plain concrete or stone arch, and the centers need not be constructed so strong as for the other types of arches. One essential difference in the centering for stone arches and that for concrete or reinforced-concrete arches, is that centering for the latter types of arches serves as a mould for shaping the soffit of the arch-ring, the face of the arch-ring, and the spandrel walls.

The successful construction of arches depends nearly as much on the centers and their supports as it does on the design of the arch. The centers should be as well constructed and the supports as unyielding as it is possible to make them. When it is necessary to use piles, they should be as well driven as permanent foundation piles, and the load should not generally be heavier than that on permanent piles.

363. **Classes of Centers.** There are two general classes of centers—those which act as a truss; and those in which the support, at the intersection of braces, rests on a pile or footing. Trusses are

used when it is necessary to span a stream or roadway. Sometimes the length of the span for the centering is very short, or there are a series of short spans, or the span may be equal to that of the arch. The trusses must be carefully designed, so that the deflection and deformation due to the changes in the loading will be reduced to a minimum. By placing a temporary load on the centers at the crown, the deformation during construction may be very greatly reduced. This load is removed as the weight of the arches comes on the centers. For the design of trusses, the reader is referred to instruction papers or other treatises on Bridge Engineering and Roof Trusses.

The lagging for concrete arches usually consists of 2 by 3-inch or 2 by 4-inch plank, either set on edge or laid flat, depending on the thickness of the arch and spacing of the supports. The surface on which the concrete is laid is usually surfaced on the side on which the concrete is to be placed. The lagging is very often supported on ribs constructed of 2 by 12-inch plank, on the back of which is placed a 2-inch plank cut to a curve parallel with the intrados. These 2 by 12-inch planks are set on the timber used to cap the piles, and are usually spaced about 2 feet apart. All the supports should be well braced. The centers should be constructed to give a camber to the arch about equal to the deflection of the arch when under full load. It is therefore necessary to make an allowance for the settlement of centering, for the deflection of the arch after the removal of the centering, and for permanent camber.

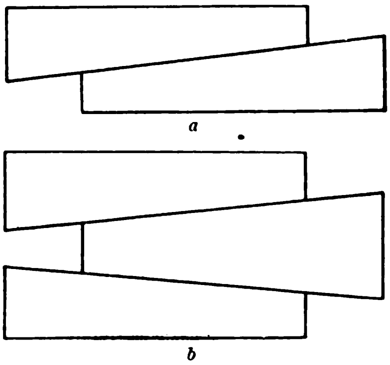


Fig. 169. Wedges Used in Placing and Removing Forms.

The centers should be constructed so that they can be easily taken down. To facilitate the striking of centers, they are usually supported on folding wedges or sand-boxes. When the latter method is used, the sand should be fine, clean, and perfectly dry, and the boxes should be sealed around the plunger with cement mortar. Striking forms by means of wedges is the commoner method. In Fig. 169, *a* shows the type of wedges generally used, although some-

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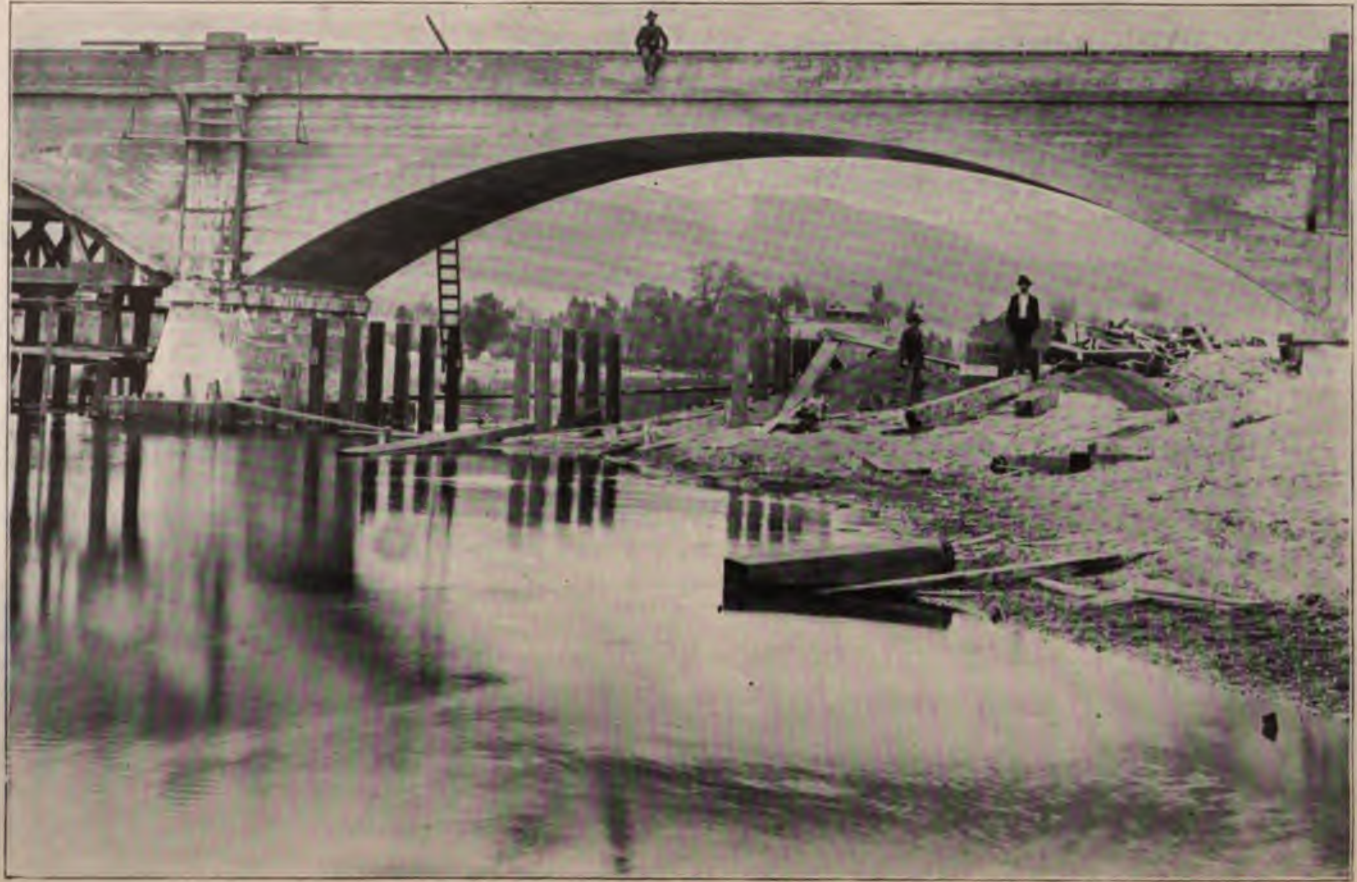

REINFORCED-CONCRETE BRIDGE OVER SAN JOAQUIN RIVER, CALIFORNIA

TABLE XXI*

Safe Load in Pounds Uniformly Distributed for Rectangular Beams, One Inch Thick, Long-Leaf Yellow Pine

Allowable fibre stress, 1,200 pounds per square inch; factor of safety, 6; modulus of rupture, 7,200 pounds per square inch.

Safe loads for other factors of safety may be obtained as follows: New safe load = Safe load from table $\times \frac{6}{\text{New factor}}$.

Span in Feet	Depth of Beam in Inches									Deflection Coefficient
	4	5	6	7	8	10	12	14	16	
4	533	833	1,200	1,633	2,133	3,333	4,800	6,533		.20
5	427	667	960	1,307	1,707	2,667	3,840	5,227		.31
6	356	556	800	1,089	1,422	2,222	3,200	4,356		.44
7	305	476	686	933	1,219	1,905	2,743	3,733		.61
8	267	417	600	817	1,067	1,667	2,400	3,267		.79
9	237	370	533	726	948	1,481	2,133	2,904	3,793	1.00
10	213	333	480	653	853	1,333	1,920	2,613	3,413	1.24
12	178	278	400	544	711	1,111	1,600	2,178	2,844	1.78
14	152	238	343	467	610	952	1,371	1,867	2,438	2.42
16	133	208	300	408	533	833	1,200	1,633	2,133	3.16
18	119	185	267	363	474	741	1,067	1,452	1,896	4.00
20	107	167	240	327	427	667	960	1,307	1,707	4.94
22	97	157	218	297	388	606	873	1,188	1,552	5.98
24	89	139	200	272	356	556	800	1,089	1,422	7.12
26		128	185	251	328	513	738	1,005	1,313	8.35
28		119	171	233	305	476	686	933	1,219	9.68
30		111	160	218	284	444	640	871	1,138	11.12

To find the safe load for beams of hemlock from the above table, the above values must be divided by 2; for beams of short-leaf yellow pine and white oak, the values must be divided by 1.2; for white pine, spruce, eastern fir, and chestnut, the values must be divided by 1.71.

times three wedges are used, as shown by *b* in the same figure. They are from one to two feet long, 6 to 8 inches wide, and have a slope of from 1 to 6 to 1 to 10. The centering is lowered by driving back the wedges; and to do this slowly, it is necessary that the wedges have a very slight taper. All wedges should be driven equally when the centering is being lowered. The wedges should be made of hardwood, and are placed on top of the vertical supports or on timbers which rest on the supports. The wedges are placed at about the same elevation as the springing line of the arch.

Tables XXI and XXII can be used to assist in the design of the different members of the centers for arches.

364. Safe Stresses in Lumber for Wooden Forms. In Table XXI are given the safe loads which may be placed on beams of long-leaf yellow pine, of various depths, on various spans.

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*From Handbook of the Cambria Steel Company.

TABLE XXII*

Strength of Solid Wooden Columns of Different Kinds of Timber

	Douglas, Oregon and Washington Yellow Fir or Pine	Southern, Long-Leaf or Georgia Yellow Pine, Canadian (Ottawa) White Pine, (Ontario) Red Pine	White Oak	Northern or Short-Leaf Yellow Pine, Red Pine, Norway Pine, Spruce, Eastern Fir, Hemlock	White Pine
F	6.000	5,000	4,500	4,000	3,500
$\frac{l}{d}$					
4	5.876	4,897	4,407	3,918	3,428
6	5.739	4,782	4,304	3,826	3,347
8	5.566	4.638	4,174	3,710	3,247
10	5.368	4,474	4,026	3,579	3,132
12	5,156	4,297	3,867	3,438	3,008
14	4.937	4,114	3,703	3,291	2,880
16	4.716	3,930	3,537	3,144	2,751
18	4.498	3,748	3,373	2,998	2,624
20	4.286	3,571	3,214	2,857	2,500
22	4.082	3,402	3,061	2,721	2,381
26	3,703	3,086	2,777	2,469	2,160
30	3.366	2,805	2,524	2,244	1,963
36	2.934	2,445	2,200	1,956	1,711
40	2,690	2,241	2,017	1,793	1,569
50	2,203	1,835	1,652	1,468	1,285

To find the load that a wooden column will support per square inch of sectional area, from the above table, the length of the column in inches is divided by the least diameter of the column, and the result is the ratio of length to diameter of the column. From this ratio is found the ultimate strength per square inch of section of a column of any kind of wood given in the table. A factor of safety of 5 should be used in finding the size of column required; that is, the working load should not be greater than one-fifth of the values given in the table.

The values given in Table XXI are the safe loads in pounds uniformly distributed, exclusive of the weight of the beam itself, for rectangular beams one inch thick. The safe load for a beam of any thickness may be found by multiplying the values given in the tables by the thickness of the beam in inches. From the last column, the deflection may be obtained, corresponding to the given span and safe load, by dividing the coefficient by the depth of the beam in inches, which will give approximately the deflection in inches.

365. *Example.* If a beam is required to support a uniformly distributed load of 4,000 pounds on a span of 10 feet, what would be the dimensions of the beam of long-leaf yellow pine, and what would be the deflection?

Solution. Following the line for beams of 10-foot span, it is found that a beam 8 inches deep and 5 inches wide (853 × 5 = 4,265) would support the load of 4,000 pounds, and the deflection would be 1.24 ÷ 8 = .16 inch. A second solution would be to use a beam 12 inches deep and 2 inches wide (1,920 × 2 = 3,840); but according to the table, this beam would not be quite strong enough, as it would only support a load of 3,840 pounds.

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*From Handbook of the Cambria Steel Company.

366. Safe Loads on Wooden Columns. The values given in Table XXII are based on the formula:

$$P = F \times \frac{700 + 15c}{700 + 15c + c^2},$$

in which,

P = Ultimate strength of timber in pounds per square inch;
F = Ultimate crushing strength of timber;
l = Length of column, in inches;
d = Least diameter, in inches;
$c = \frac{l}{d}$.

Example. If a column 10 feet long is required to support a load of 20,000 pounds, what would be the size of the column required if short-leaf yellow pine was used?

Solution. Dividing the length of the beam in inches by the assumed least diameter, 6 inches, we have 120 ÷ 6 = 20, which gives the ratio of the length to the diameter. By the table it is shown that 2,857 pounds is the ultimate strength for a column of short-leaf pine, when $l \div d = 20$. Assuming a factor of safety of 5, and dividing 2,857 by 5, the working load is found to be 571 per square inch. Dividing 20,000 by 571, it is found that a column whose area is 35 square inches is required to support the load. The square root of 35 is 5.9. Therefore a column of short-leaf yellow pine 6 inches square will support the load.

367. Form for Arch at 175th Street, New York. In constructing the 175th Street Arch in New York City, the forms were built so that they could be easily moved. The arch is elliptical and is built of hard-burned brick and faced with granite. The span of the arch is 66 feet; the rise is 20 feet; the thickness of the arch-ring is 40 inches and 48 inches at the crown and springing line, respectively; and the arch is built on a 9-degree skew. The total length of this arch is 800 feet.

This arch is constructed in sections, the centering being supported on 11 trusses placed perpendicular to the axis of the arch and having the form and dimensions shown in Fig. 170. The trusses are placed 5 feet on centers, and are supported at the ends and middle by three lines of 12 by 12-inch yellow pine caps. The caps are supported by 12 by 12-inch posts spaced five feet center to center, and rest on timber sills on concrete foundations. The upper and lower chord members of the trusses are of long-leaf yellow pine, but the diagonals and verticals are of short-leaf yellow pine. The lagging is 2¾ by 6-inch long-leaf yellow pine plank. The connections of the timbers are made by means of ⅜-inch steel plates and ⅞-inch bolts arranged as shown in the illustration. As it was absolutely necessary to have the forms alike, so that they could be moved along the arch and at all times fit the brickwork, they were built on the ground from the same pattern, and hoisted to their place by two guyed derricks with 70-foot booms.

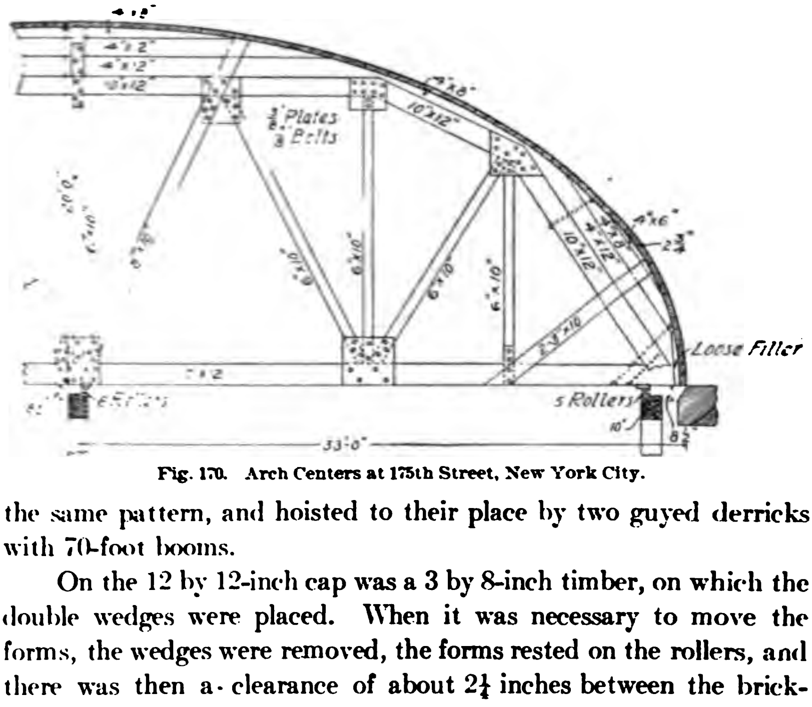

Fig. 170. Arch Centers at 175th Street, New York City.

On the 12 by 12-inch cap was a 3 by 8-inch timber, on which the double wedges were placed. When it was necessary to move the forms, the wedges were removed, the forms rested on the rollers, and there was then a clearance of about 2¼ inches between the brickwork and the lagging. The timber on which the rollers ran was faced with a steel plate ¼ inch by 4 inches. The forms were moved forward by means of the derricks. The settlement of the forms under the first section constructed was ¼ inch; and the settlement of the arch-ring of that section after the removal of forms, was ¼ inch.*

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*Engineering Record, October 5, 1907.

368. Forms for Bridge at Canal Dover, Ohio.* The details of the centering used in erecting one of the 106-foot 8-inch spans of a reinforced-concrete bridge over the Tuscarawas River at Canal Dover, Ohio, are shown in Figs. 171*a* and 171*b*. Besides this span, the bridge consisted of two other spans of 106 feet 8 inches each, and a canal span of 70 feet. The centering for the canal span was built in six bents, each bent having seven piles. A clear waterway of 18 feet was required in the canal span by the State Canal Commissioner, and this passage was arranged under the center of the arch. The piles were driven by means of a scow. The cap for the piles was a 3 by 12-inch timber. Plank 2 inches thick were sawed to the correct curvature, and nailed to the 2 by 12-inch joists, which were spaced about 12 inches apart. The lagging was one inch thick, and was nailed to the curved plank. The wedges were made and used as shown. The centering was constantly checked; this was found important after a strong wind. The centering for the other two of the main arches was constructed similarly to that of the arch shown.

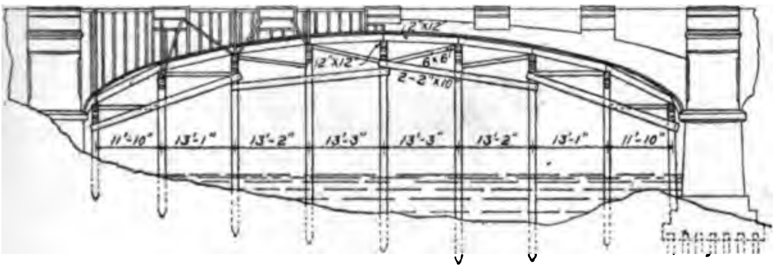

Fig. 171 *a*. Centers for Bridge at Canal Dover, Ohio.

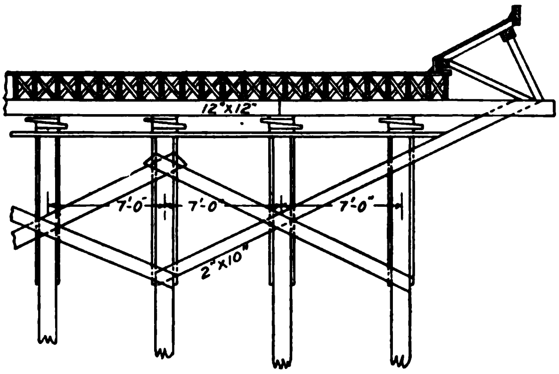

Fig. 171 *b*. Centers for Bridge at Canal Dover, Ohio.

After some difficulty had been experienced in keeping the forms in place during the concreting of the first arch, the concrete for the other arches was placed as shown in Fig. 172, and no difficulty was encountered. Sections 1 and 1 were first placed, then 2 and 2, finishing with section 6.

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*Engineering Record, February 9, 1907.

The concreting on the canal span was begun November 1, and finished November 12; and the forms were lowered by means of the wedges five weeks later. The deflection at the crown was 0.5 inch, and after the spandrel walls were built and the fill made, there was an additional deflection of 0.4 inch. In building the forms, an allowance of $\frac{1}{800}$ part of the span was made, to allow for this deflection. The deflections at the crown of the other three arches were 0.6 inch, 1.45 inches, and 1.34 inches.

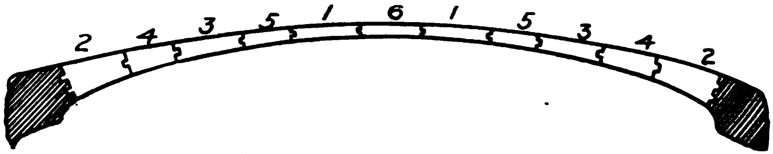


Fig. 172. Diagram of Order of Placing Concrete in Bridge at Canal Dover, Ohio.

BENDING OR TRUSSING BARS

369. **Bending Details.** The full bending details of the bars should be made before the reinforcing steel is ordered for any reinforced-concrete work that is to be constructed. It has been the common practice for contractors to make these details, if they are made; and they may or may not submit them to the designing architects or engineers for their approval. Very often the plans or specifications do not state how long the bars are to be, or even state what lap of the bars is required; or they may not be very definite in the number of bars to be turned up in the beams and girders. If architects and engineers would make these details and submit them with their general drawings, the contractors could then make a very definite estimate on the amount of steel required for the work, and these details should also assist the contractor in estimating the cost of the bending of the bars. With the assistance of these details being made very definite, it should not only assist the contractor in making his bid on the work, but would often result in better work being done.

The angle at which the diagonal bars are turned up, varies from about 10 degrees to 45 degrees, and sometimes to a greater angle than

45 degrees. A great deal depends upon the length and depth of the beam or girder. If the beam is very short and deep, the bars are usually turned up at an angle of about 45 degrees, or perhaps a little greater; but if the beam is long and shallow, the angle at which these bars are turned is very small. This angle, in the average practice, is about 30 degrees.

The bending of the bars is usually a simple matter, and generally can be easily and quickly done. If bends of 30 degrees or more, with short radii, are required of large bars—1 inch to 1¼ inches square—it is usually necessary to heat the bars. This makes the bending more expensive, as it requires the use of forges and blacksmiths to do the work.

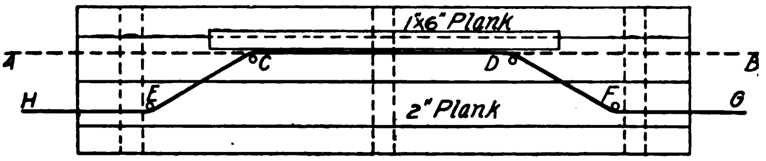


Fig. 173. Plan of Bending Table.

370. Tables for Bending Bars. The usual outfit for bending the bars cold consists of a strong table, a vise, and a lever with two short prongs. The outline to which the bar is to be bent is laid out on the table, and holes are bored at the point where the bends are to be made. Steel plugs 5 inches to 6 inches long are then placed in these holes. Short pieces of boards are nailed to the table where necessary, to hold the bar in place while being bent. The bar is then placed in the position *A-B*, Fig. 173, and bent around the plugs *C* and *D*, and then around the plugs *E* and *F*, until the ends *EH* and *FG* are parallel to *AB*. When bends with a short radius are required, the bars are placed in the vise, near the point where the bend is wanted, and the end of the bar is pulled around until the required angle is secured. The vise is usually fastened to the table. The lever shown in Fig. 174 is also used in making bends of short radii. This is

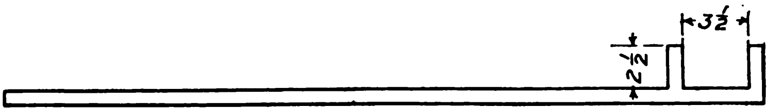


Fig. 174. Lever Bender.

done by placing the bar between the prongs and pulling the end of the lever around until the required shape is secured.

371. **The Hunt Bender.** The bar-bending device shown in Fig. 175 was devised by Mr. R. S. Hunt, C. E., and has been used by him to bend 1¼-inch bars. In bending bars of this size, it is not necessary to heat them; and the size of bars that can be shaped by this bender depends largely on the proportions of the materials of which the bender is constructed.

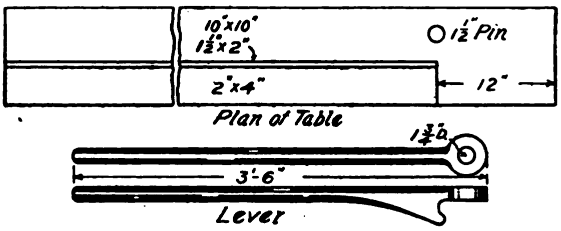


Fig. 175. The Hunt Bender.

In constructing this device, a timber 10 inches by 10 inches and about 10 feet long is supported on posts and well braced, the top of the timber being about 3 feet high. A 2 by 4-inch plank is spiked on one edge of the 10 by 10-inch timber, the smaller timber extending to within 12 inches of the end of the larger, as shown in the figure. On the edge of the 2 by 4-inch timber, is fastened a ¼-inch by 2-inch steel strap, which is the same length as the timber to which it is fastened. Opposite the end of the timber, and 3 inches from the timber, is a steel pin 1½ inches in diameter. The lever is usually about 3½ feet long, and made as shown in the figure.

To bend a bar with this device, the bar is placed against the steel strap with the point of the bar at which the bend is to be made opposite the steel pin. The lever is hooked on the pin, while being held at right angles to the bar to be bent. The lug on the lever rests against the bar; and by moving the lever towards the end of the

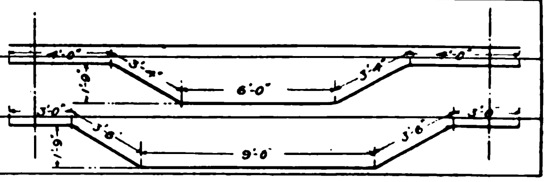
Mk	Nº of Beams	Nº of Bars in each Beam	Shape
		3-¾" 20'-0"	
B2	60	1-¾" 20'-8"	
		2-¾"	

Fig. 176. Bending Detail of Bars for a Beam or Girder.

timber, the required bend is given to the bar. For smaller sizes of bars, a washer should be placed over the pin so as to reduce the space between the pin and the bar to be bent.

372. **Beams and Girders.** Fig. 176 shows the bending details of the bars for a beam or girder in which six bars are required for the reinforcement three of which are turned up, one at a distance of 3 feet, and two at a distance of 4½ feet from the center of the span. The light lines indicate the depth of the beam, including the thickness of the slab; the vertical dash-and-dot lines, the center of the supports of the beam; and the heavy full lines, the bars.

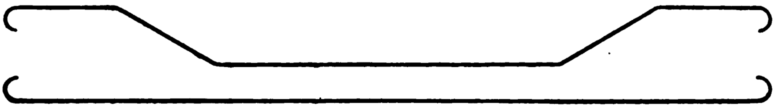


Fig. 177. Bars with Hooked Ends.

When plain bars are used for reinforced concrete, architects and engineers very often require that the ends of all the bars in the beams and girders shall be hooked as shown in Fig. 177. This is done to prevent the bars from slipping before their tensile strength is fully developed.

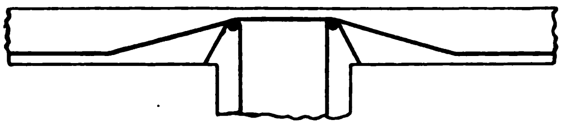


Fig. 178. Slab Bars.

373. **Slab Bars.** To secure the advantage of a continuous slab, it is very often required that a percentage of the slab bars, usually one-half, shall be turned up over each beam. Construction companies have different methods of bending and holding these bars in place; but the method shown in Fig. 178 will insure good results, as the slab bars are well supported by the two longitudinal bars which are wired to the tops of the stirrups. Fig. 179 shows the bending details of slab bars, the beams being spaced six feet center to center.

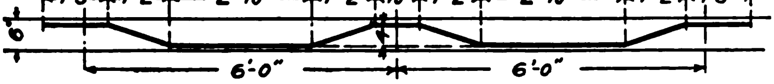


Fig. 179. Bent Bars for Slabs.

374. Stirrups. Fig. 180 shows the bending of the bars f stirrups. The ends of the stirrups rest on the forms and support tl beam bars, which assist in keeping these bars in place. The ends the stirrups seldom show on the bottom of the slab of the finish

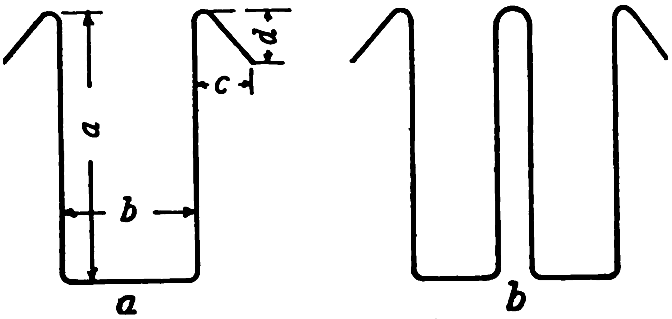


Fig. 180. Bending of Bars for Stirrups.

floor. Sufficient mortar seems to get under the ends of the stirru to cover them. Type *a* is much more extensively used than type The latter type is generally used when a large amount of steel required for st rups, or if the s rups are made very small bars.

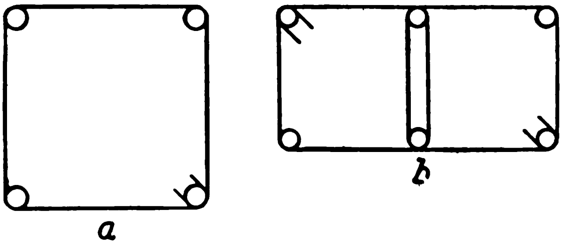


Fig. 181. Column Bands.

375. Colu Bands. In Fig. two types of colu bands are sho Type *a* shows ba for a square or a round column; and type *b*, bands for a rectang column. The bar which forms the band is bent close around e vertical bar in the columns, and therefore assists in holding th bars in place. The bands for the rectangular column *b* are made up of two separate bands.

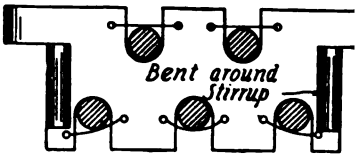


Fig. 182. Spacer.

376. Spacers. Spacers, Fig. 182, for holding the bars in place in beams and girders, have been successfully used. These spacers are made of heavy sheet iron. They are fastened to the stirrups means of the loops in the spacers. The ends of the spacers which project out to the forms of the sides of the beams, should be made blunt or rounded. This will prevent the ends of the spacers being driven into the forms when the concrete is being tamped. The number of these spacers required will depend on the lengths of the beams; usually 2 to 4 spacers are used in each beam.

377. **Unit-Frames.** Among the patented methods of fastening the bars together for beams and girders, is the *Unit Girder Frame System*. The loose bars are bent and made into a frame as shown in Fig. 183. All this work is done in a shop; and the frames are sent to where the building is being constructed, ready to be placed. The stirrups are made of round or flat bars, and are hot-shrunk on the longitudinal rods. The girder, beam, or column unit is shipped to the site of the building being constructed, bearing a tag numbered to correspond with a number on the plan showing the proper position of the reinforcement.

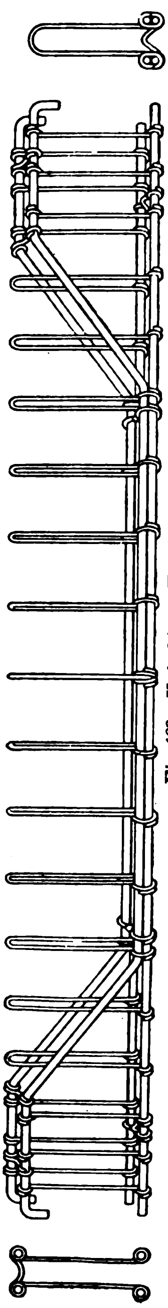


Fig. 183. Unit Girder Frame.

BONDING OLD AND NEW CONCRETE

The place and manner of making breaks or joints in floor construction at the end of a day's work, is a subject that has been much discussed by engineers and construction companies. But there has not been any general agreement yet as to the best method and place of constructing these joints. Wherever joints are made, great care should be exercised to secure a bond between the new and the old concrete.

378. **Methods of Making Bonds.** *First Method.* Fig. 184 shows a sectional view of one method of making a break at the end of the day's work, which has been used very extensively and successfully. The stirrups and slab bars form the main bond between the old and the new work, if the break is left more than a few hours. Short bars in the top of

the slab will also assist in making a good bond; also, an additional number of stirrups should be used in the beam where the break is to be made. Before the new concrete is placed, the old concrete should be well scraped, thoroughly soaked with clean water, and given a thin coat of neat cement grout. An objection to this method of forming a joint is that the shrinkage in the concrete may cause a separation of the concrete placed at the two different times, so that water will find a passage. The top coat that is generally placed later will greatly assist in overcoming this objection.

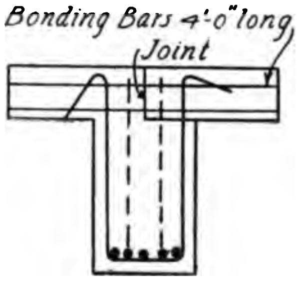


Fig. 184. Break in Slab.

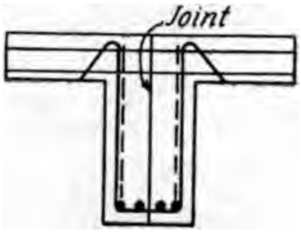


Fig. 185. Break in Beam.

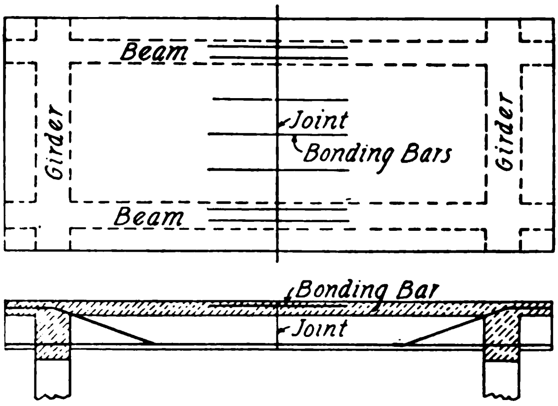


Fig. 186. Break in Center of Span.

379. *Second Method.* Another method of forming stopping-places is by dividing the beam vertically—that is, making two L-beams instead of one T-beam, Fig. 185. Theoretically this is a very good method, but practically it is found difficult to construct the forms dividing the beam, as the steel is greatly in the way.

380. *Third Method.* The method of stopping the work at the center of the span of the beams and parallel to the girders, has been used to some extent. Fig. 186 illustrates this method. Theoretically the slab is not weakened; and as the maximum bending moment occurs at this point, the shear is zero, and therefore the

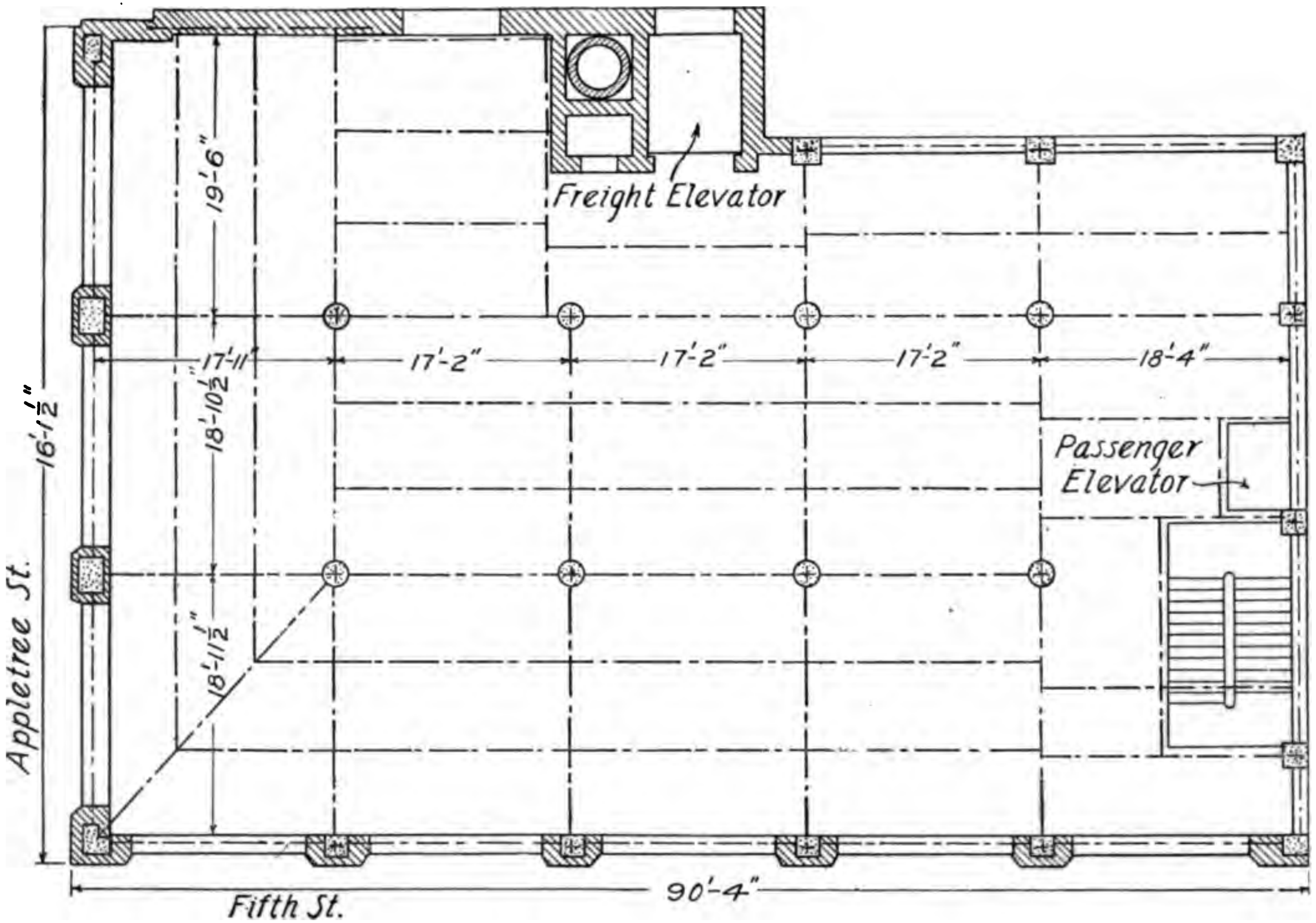


Fig. 187. Typical Structural Floor-Plan of Buck Building, Philadelphia, Pa.

beams are not supposed to be weakened, except for the loss of concrete in tension, and this is not usually considered in the calculation. The bottoms of the beams are tied together by the steel that is placed in the beams to take the tensile stresses; and there should be some short bars placed in the top of these beams, as well as in the top of the slab, to tie them together. The objection made in the description of the first method—in that any shrinkage in the concrete at the joint will permit water to pass through—is greater in the second and third methods than in the first.

REPRESENTATIVE EXAMPLES OF REINFORCED-CONCRETE WORK

381. **Buck Building.** Fig. 187 shows the typical structural floor-plan, above the first floor, of a building constructed for J. C. Buck at Fifth and Appletree Streets, Philadelphia. The architects were Ballinger & Perrot, and the building was constructed by Cramp & Company, Philadelphia. The building has a frontage of 90 feet on Fifth Street, and a depth of 61 feet on Appletree Street, and is seven stories high. The building was constructed structurally of reinforced concrete, except the first floor and the columns in the lower floors. The floors were all designed to carry 200 pounds per square foot. The side walls were constructed of light-colored brick, and trimmed with terra-cotta. The first floor was constructed especially to suit the requirements of the chemical company that is to occupy the building for several years. If this company should leave the building when their present lease expires, it will very probably be necessary to reconstruct the first floor; and therefore it was constructed of structural steel, as it will be much easier to remove a

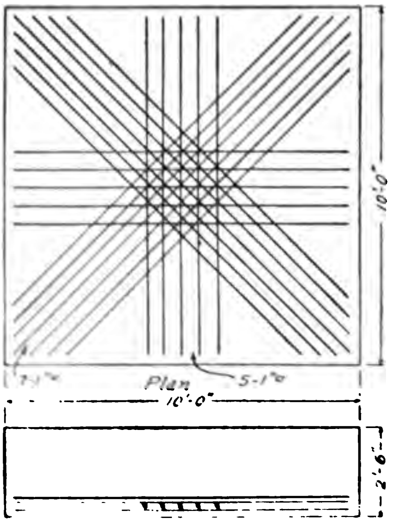


Fig. 188. Interior Column Footing for Buck Building, Philadelphia, Pa.

floor constructed of structural steel than one constructed of reinforced concrete.

The footings for each of the interior columns were designed as single footings. They were 10 feet square, 30 inches thick, and were reinforced as shown in Fig. 188.

The columns in the basement, first, and second floors, were of structural steel, and fireproofed with concrete. The wall columns were either square or rectangular in shape; and the interior columns were round, being twenty inches in diameter. The stress allowed in the structural steel of these columns was 16,000 pounds per square inch of the steel section; but no allowance was made for the four small bars placed in the column. These steel cores were provided with angle brackets to support the beams, and with spread bases to transmit the stress in the steel to the foundation. The cores are composed of angles and plates, and are riveted together in the usual manner. The columns were built in sections of a length equal to the height of two stories. The extra metal required in this practice was very small; and the expense of half the joints, if a change of section had been made at each floor, was saved.

The general outline and details of these steel cores are illustrated in Fig. 189. In the exterior columns, the steel cores were used in the basement and the first, second, and third floors, where necessary; in the interior columns, they were used also in the fourth story, and in two columns the structural steel extended to the sixth floor line. The exterior columns

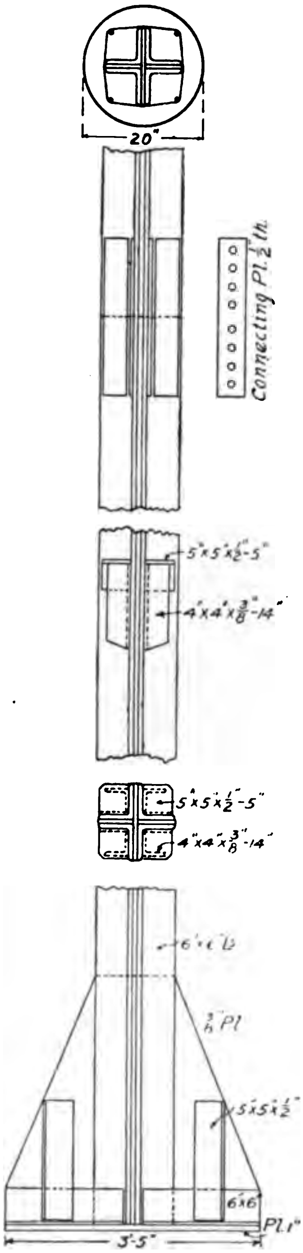


Fig. 189. Steel Column Core for Buck Building, Philadelphia, Pa.

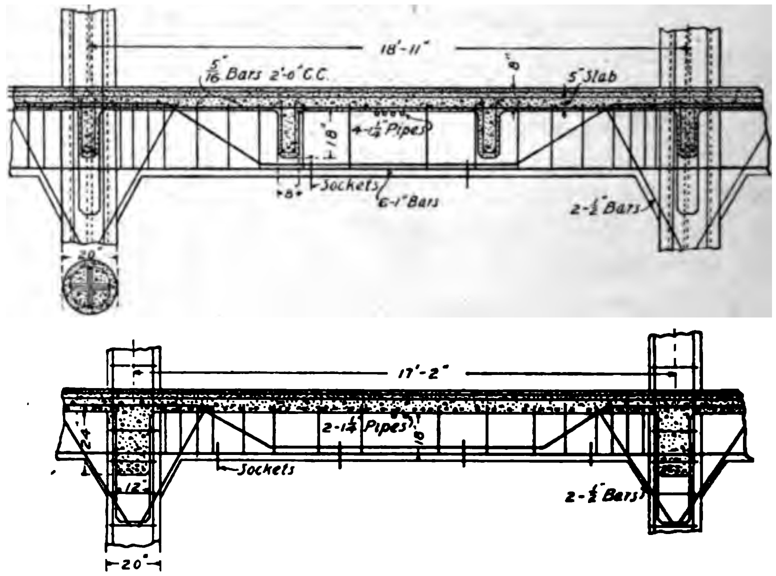


Fig. 190. Detail of Beams and Girders for Buck Building, Philadelphia, Pa.

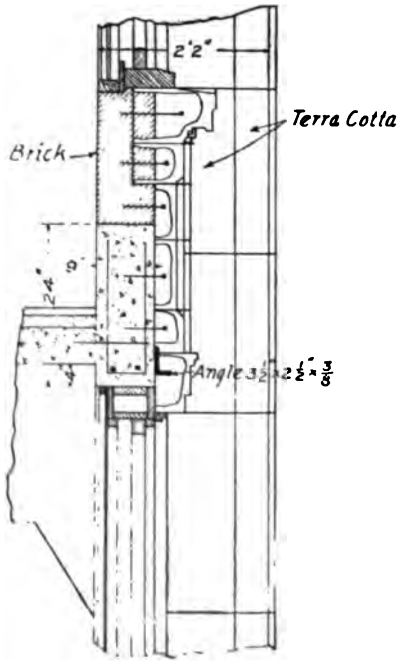
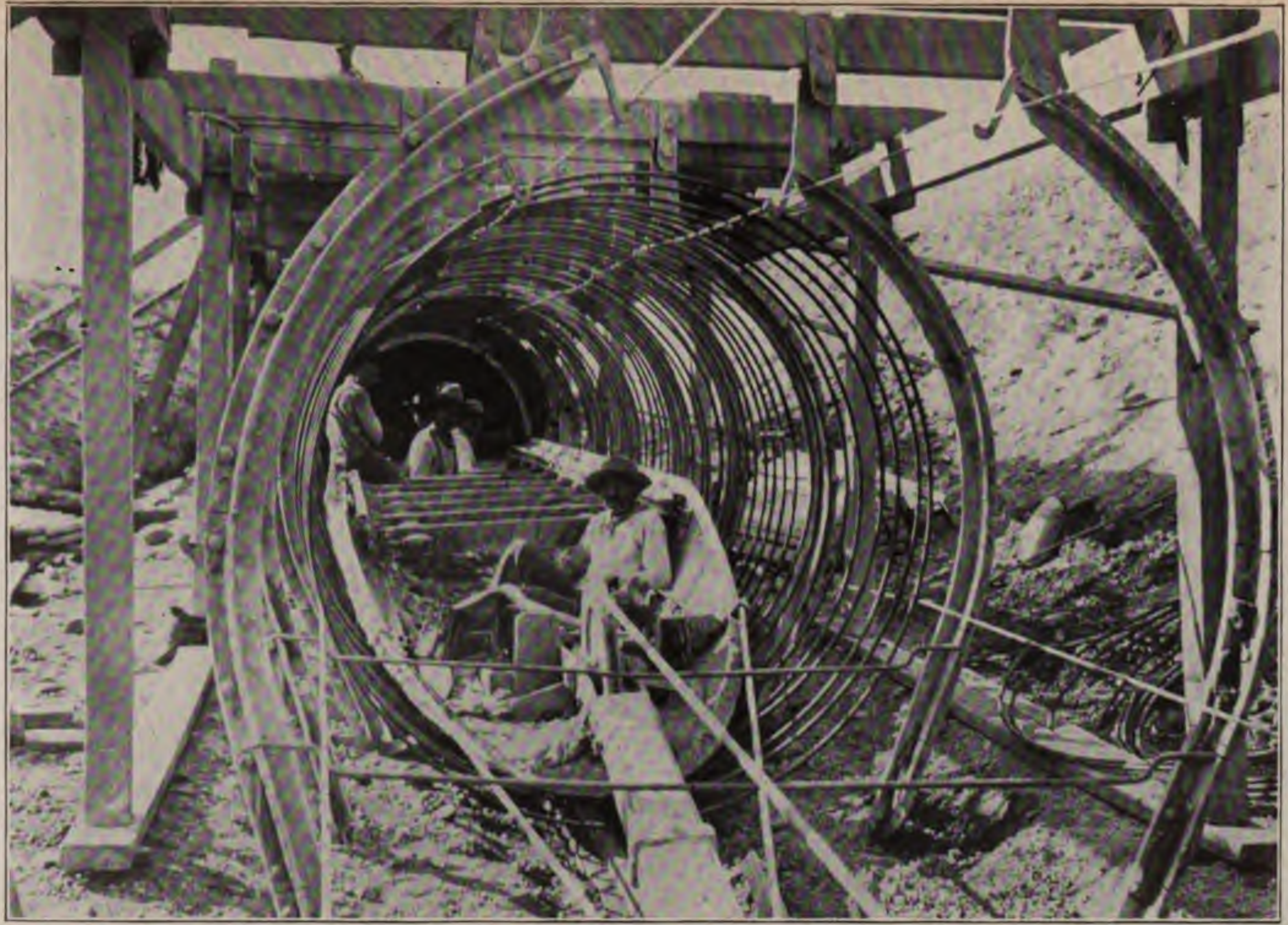


Fig. 191. Wall Beams, Buck Building, Philadelphia, Pa.

above the structural steel, and also the columns in which structural steel was not required, were in general reinforced with 8 bars 1 inch square, in the lower floors; and this amount of steel was gradually reduced to 4 bars 1 inch square, in the seventh story. In the interior columns, the reinforcement above the steel cores consisted of 8 bars ⅞ inch square, in the floor just above the structural steel; and the number of these bars was gradually reduced to 4 in the seventh floor.

The floor-slab was 5 inches thick, and was reinforced with ⅜-inch square bars spaced 6 inches on centers, and $\frac{5}{16}$-inch bars



BUILDING THE REINFORCING SKELETON OF A CONCRETE FLUME

spaced 24 inches on centers, the latter being placed at right angles to the former. The roof slab was designed to carry a live load of 40 pounds per square foot, and was $3\frac{1}{2}$ inches thick. The reinforcement consisted of $\frac{5}{16}$-inch bars spaced 6 inches, and the same sized bars spaced 24 inches at right angles.

The floor-beams were in general 8 inches wide, and the depth below the slab was 18 inches. The amount of reinforcement in the beams varied, depending on the length of the beams. Most of the beams were reinforced with 2 bars 1 inch square, and 1 bar $1\frac{1}{8}$ inches square. The $1\frac{1}{8}$-inch bar was turned up or trussed at the ends, and the 1-inch bars were straight. The roof beams were 6 by 12-inch below the slab, and were reinforced with 2 bars $\frac{7}{8}$ inch square, except in the longest beams, in which 2 bars 1 inch square were required. A $\frac{3}{4}$-inch bar, 5 feet long, was placed in the top of all floors and roof beams, where they were framed into a girder. The ends of these bars were turned down. The stirrups were made of $\frac{3}{8}$-inch round bars, and were spaced as shown in the detail of the beam. See Fig. 190.

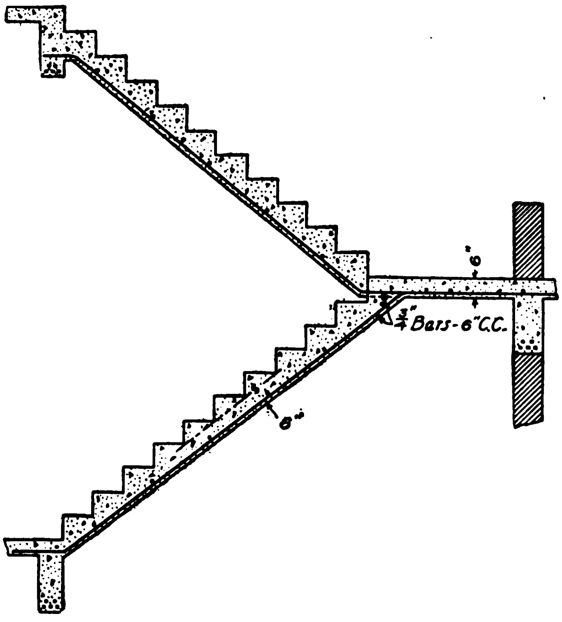


Fig. 192. Stairs for Buck Building, Philadelphia, Pa.

The floor girders were 12 by 24-inch below the slab. The span of the girders varied from about 18 feet to about 20 feet; and they were all reinforced with 6 bars 1 inch square, three of the bars being turned up at the ends. Two $\frac{3}{4}$-inch square bars were placed in the top of the girders over the supports. These bars were 5 feet long, and they were *hooked* at the ends. Bars $\frac{3}{8}$ inch square, 5 feet long, were placed in the slab near the top, at right angles to the girders. The bars were 12 inches center to center, and were placed over the center of the girders.

The wall beams or lintels on the Fifth Street and Appletree Street sides of the building, are shown in section in Fig. 191. They are 9 inches by 24 inches, and are reinforced with 2 bars 1 inch square. The wall girders in the side of the building opposite Appletree Street are 14 inches by 24 inches, and are reinforced with 6 bars 1 inch square.

The stairs were constructed as shown in Fig. 192. The structural concrete slab was 6 inches thick, and was reinforced with ¾-inch bars. Safety-treads 5½ inches in width, and 12 inches shorter than the width of the stairs, were set in each step.

The concrete for the beams, girders, slabs, and footings was a 1 : 2½ : 5 mixture; and for the columns, a 1 : 2 : 4 mixture was required. The stone used in this concrete was trap rock. The concrete was mixed in a batch mixer, and the consistency of the mixture was what is commonly known as a *wet mixture*. Square twisted bars were used as the reinforcing steel.

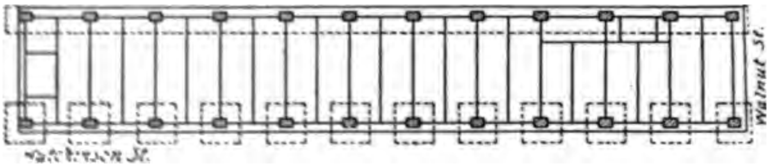


Fig. 193. Structural Floor-Plan of Mershon Building, Philadelphia, Pa.

The first, second, and third floors were finished with 1¼-inch maple flooring. The stringers, 2 inches by 3 inches, were spaced 16 inches apart, and the space between the stringers was filled with cinder concrete. The other floors were finished with a one-inch coat of *cement finish*. A cinder fill 2 inches thick was laid on the concrete floor-slab, on which was laid the cement finish. The cinder concrete consisted of 1 part Portland cement, 3 parts sand, and 7 parts cinders. The cement finish was composed of 1 part Portland cement, 1 part sand, and 1 part ¼-inch crushed granite.

382. **Mershon Building.** Fig. 193 shows the plan of the foundations and the typical layout of the structural members for each floor of a building constructed by Cramp & Company on the

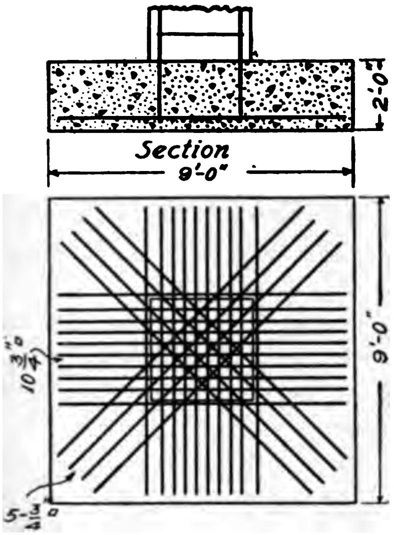


Fig. 194. Detail of Column Footing, Mershon Building, Philadelphia, Pa.

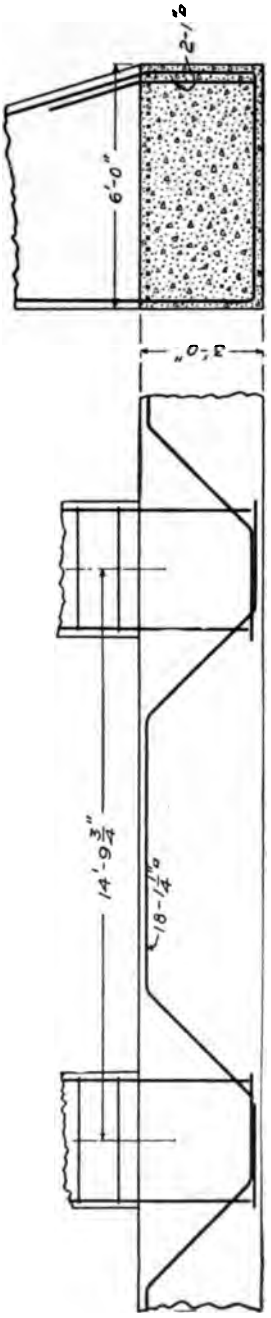


Fig. 195. Details of Continuous Footing, Mershon Building, Philadelphia, Pa.

south side of Walnut Street, between Ninth and Tenth Streets, Philadelphia. This building was erected during the summer of 1907. It has a frontage of 27 feet on Walnut Street, and a depth of 165 feet on Hutchinson Street, and is eight stories high. It was constructed for manufacturing and storage purposes, and the floors were designed to carry a uniformly distributed live load of 200 pounds per square foot.

At the time that this building was constructed, the Building Code of Philadelphia permitted a working stress of 500 pounds per square inch in compression in concrete, and a tensile strength of 16,000 pounds per square inch in the reinforcing steel. The concrete could be made of any desired proportions that would insure an ultimate strength of 2,000 pounds per square inch. A thick-

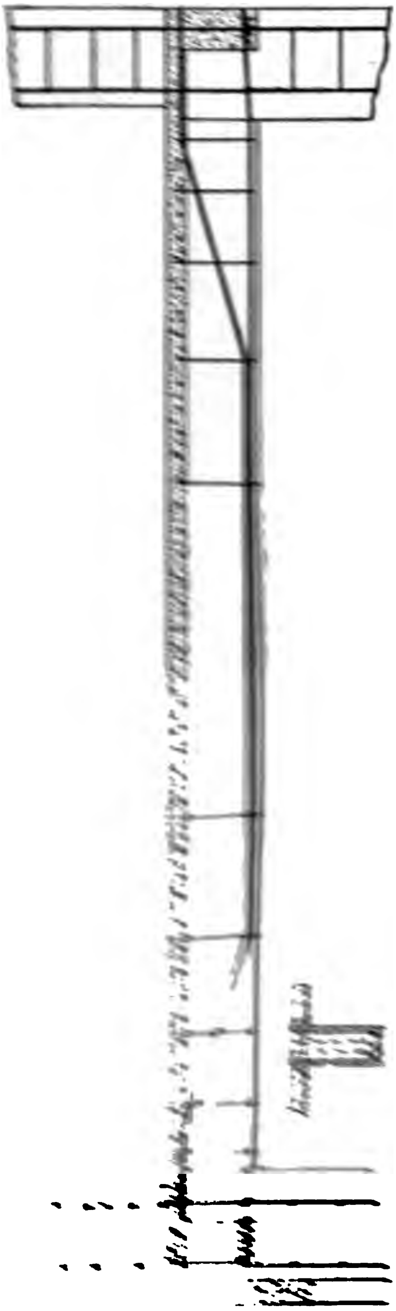


Fig. 193. Detail of Beam. [illegible] Building, Philadelphia, Pa.

ness of 2 inches of concrete was required on the outside of the reinforcing steel in columns, girders, and beams, and 1 inch on the bottom of floor-slabs. The Building Code required that all girders, beams, and slabs should be considered as simple beams supported at the ends, no allowance being made for continuous construction over supports. Owing to the building being only 27 feet wide, interior columns were not required, and therefore footings were needed only along the two sides of the building. The footings along the Hutchinson Street side of the building were designed as isolated footings, as shown in the general plans, and detailed in Fig. 194. But this type of construction could not be used to support the columns of the opposite side of the building, owing to the adjacent property, and therefore a continuous footing was used. This footing, which is 3 feet deep, 4 feet wide, and reinforced with 18 twisted bars 1¼ inches square, is really an inverted beam with a span of 14 feet 6 inches. In designing this inverted beam, the load was considered the same as the load permitted on the soil, which was 3½ tons per square foot. See Fig. 195.

In designing the columns, a working stress of 500 pounds per square inch was allowed for the whole section of the column. The steel reinforcement consists of round bars, banded every 12 inches with a ⅜-inch bar. The area of the longitudinal bars was less than one per cent of the area of the section of the column. The columns decreased in size from 32 by 36 inches in the basement to 12 by 28 inches at the eighth floor.

All the floor-beams were the same size, being 10 inches wide and 18 inches in depth below the slab; but the amount of reinforcement was varied. In the cross-beams between the columns, the reinforcement consisted of 5 twisted bars 1 inch square; but 6 bars 1 inch square were required for the cross-beams between the longitudinal beams, as the span was 4 to 5 feet longer for most of the floors. The detail of the beams between the columns is shown in Fig. 196*a*. The longitudinal beams between the columns were reinforced with 4 twisted bars 1 inch square, the details of which are given in Fig. 196*b*. The stirrups for all the beams were made of ⅜-inch round steel bars. The beams were connected by a 5-inch slab reinforced with ⅜-inch square bars spaced 5 inches.

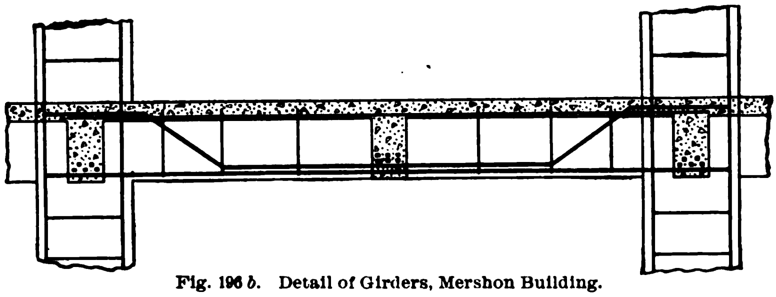
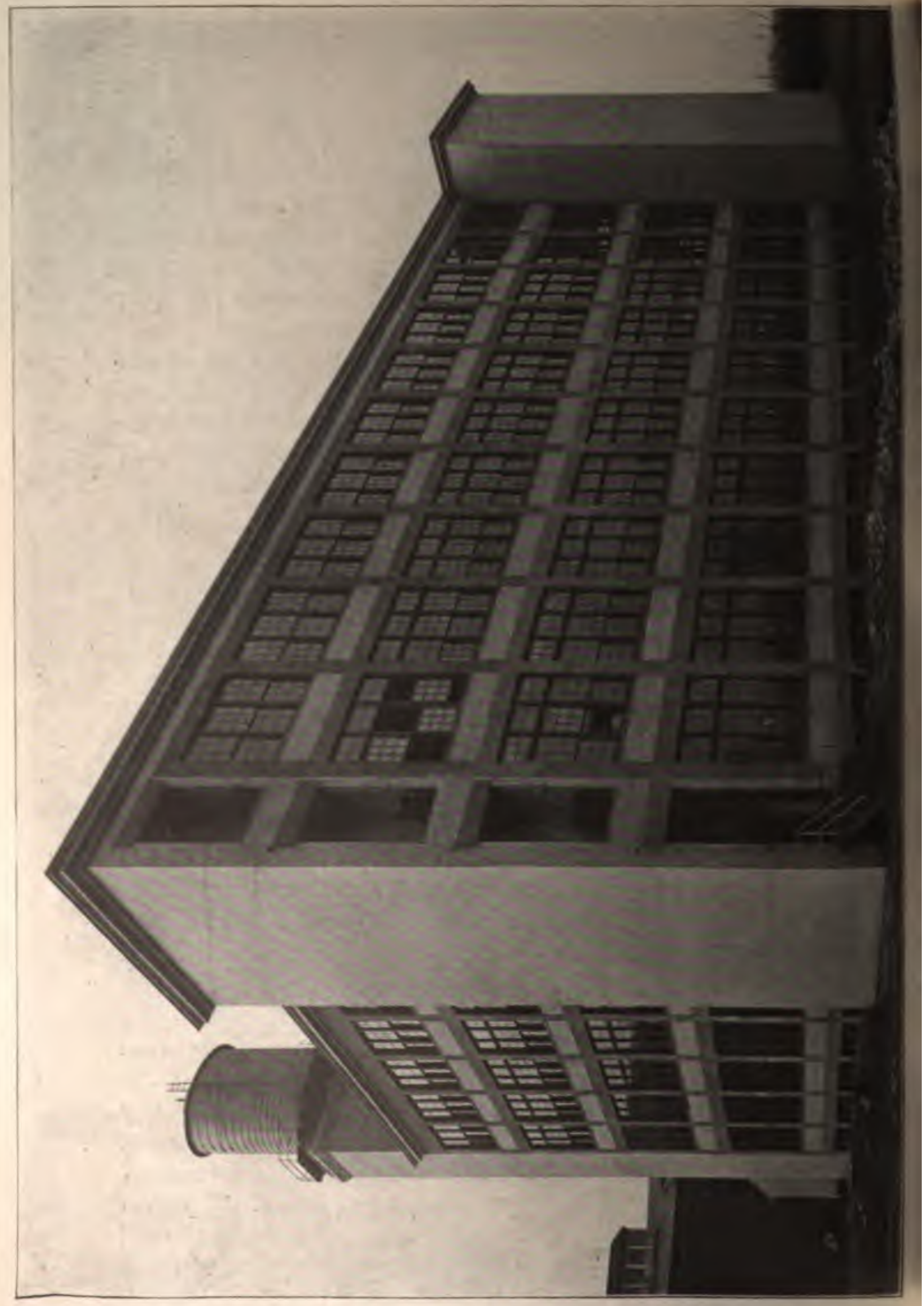


Fig. 196 *b*. Detail of Girders, Mershon Building.

383. **Erben-Harding Company Building.** The exterior and interior of a factory building, designed and constructed by Wm. Steele & Sons Company for the Erben-Harding Company, Philadelphia, are shown in Figs. 197 and 198. This building is 100 feet by 153 feet, and was constructed structurally of reinforced concrete, except that structural steel was used in the columns. The floors and columns were designed to support safely a live load of 120 pounds per square foot.



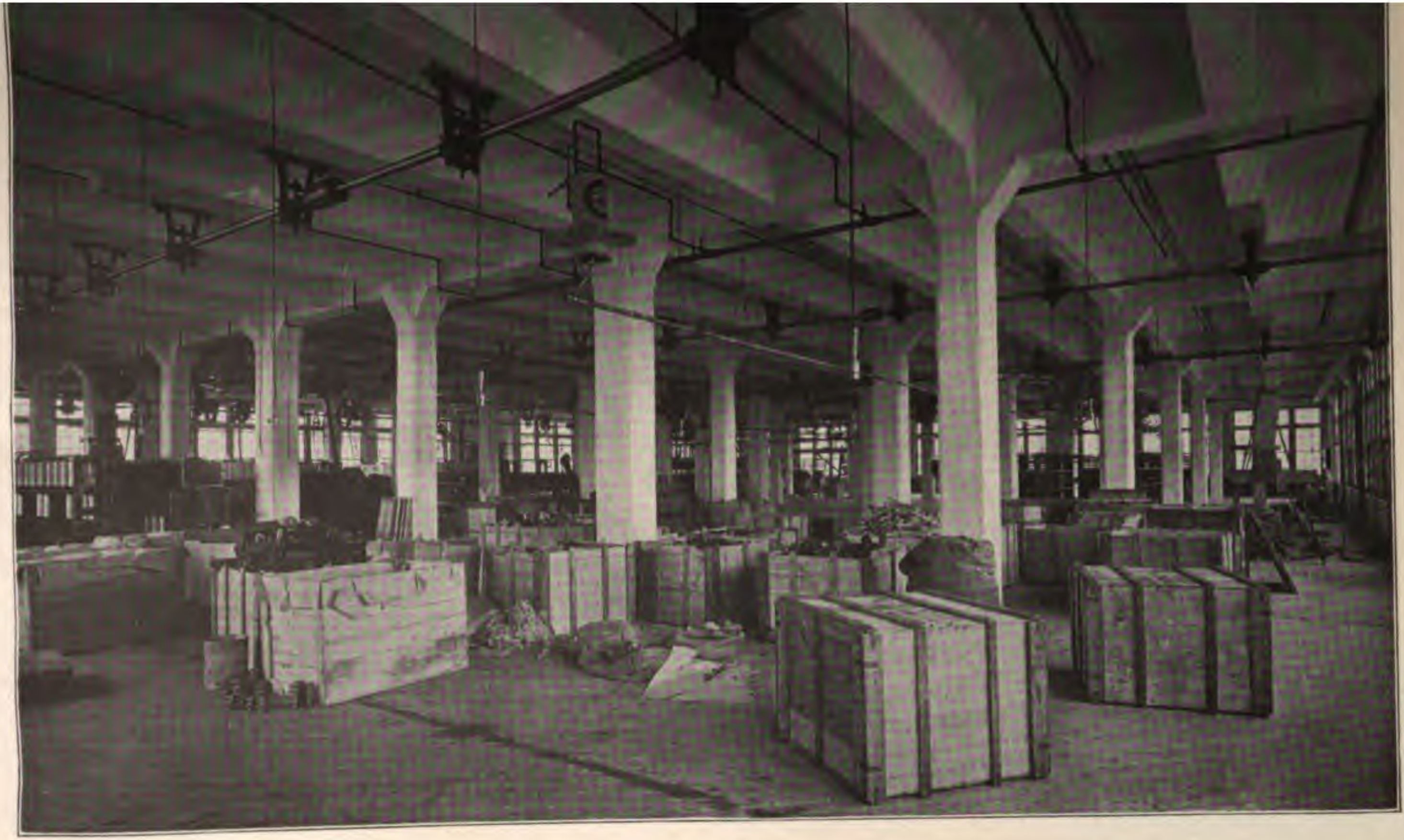


Fig. 198. Interior of Factory Building for the Erben-Harding Company, Philadelphia, Pa.

The floor-panels were about 12 feet by 25 feet, the girders having a span of about 12 feet, and the beams a span of 25 feet. One intermediate beam was placed in each panel, as shown in the interior view. The girders were 12 inches wide and 20 inches deep below the slab, and were reinforced with 4 bars $1\frac{1}{16}$ inches in diameter. The beams were 12 by 18-inch, and were reinforced with 4 bars $1\frac{1}{4}$ inches in diameter. The floor-slab was 4 inches thick, and was reinforced with 3-inch mesh, No. 10 gauge, expanded metal.

The columns were all 18 by 18-inch; but the structural steel (4 angles arranged as shown in Fig. 199) in the columns was designed to support the entire load on the columns. Four $\frac{3}{4}$-inch bars were placed in the columns and wrapped with expanded metal. The exterior columns were exposed to view on both the exterior and the interior of the building. The entire width between the wall columns was filled by triple windows. The wall beams were constructed flush with the exterior surface of the wall columns, as shown in Fig. 197. The space between the bottom of the windows and the wall beams was filled with white brick. The two fire towers located at the corners of the building were also constructed of white brick.

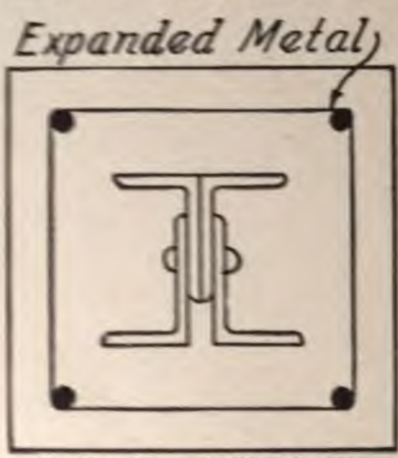


Fig. 199. Column for Erben-Harding Building, Philadelphia, Pa.

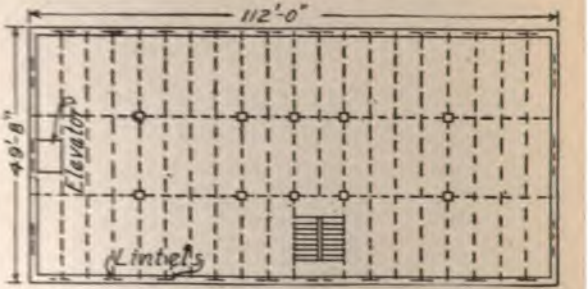


Fig. 200. Plan of Shop Building, Swarthmore College, Swarthmore, Pa.

The floor finish of this building is somewhat unusual. Sills 2 by 4 inches were laid on the structural floor-slab of concrete, and the space between these sills was filled with cinder concrete. On these sills was laid a covering of 2-inch tongued-and-grooved plank; and on these planks was laid a floor of $\frac{7}{8}$-inch maple, the latter being laid perpendicular to the 2-inch plank.

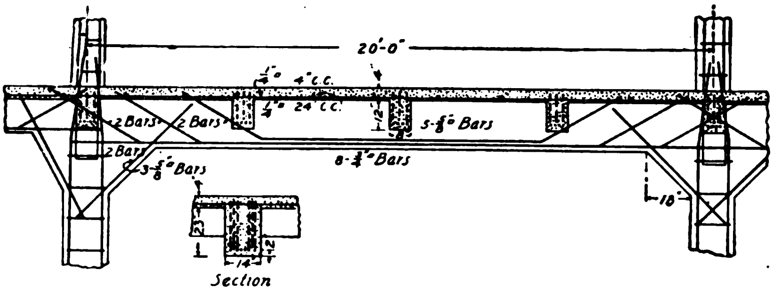


Fig. 201. Detail of Girder for Shop Building of Swarthmore (Pa.) College.

384. **Swarthmore Shop Building.** In constructing the new shop building at Swarthmore College, Swarthmore, Pennsylvania, in 1906, concrete blocks were used for the side walls, and the floors were constructed of reinforced concrete. This building is 49 feet 8 inches by 112 feet, and is 3 stories high. The floors were designed to carry a live load of 150 pounds per square foot. A factor of safety of 4 was used in all the reinforced-concrete construction.

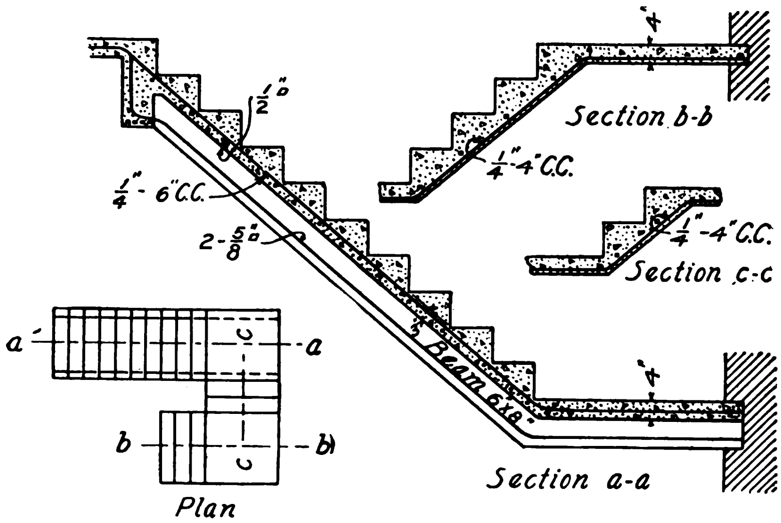


Fig. 202. Stairway in Shop Building, Swarthmore College, Swarthmore, Pa.

The columns are located as shown in Fig. 200. The span of the girders is 20 feet, except for the three middle bays, in which the span is only 10 feet. The 20-foot girders are 14 inches wide, and the depth below the slab is 23 inches. The reinforcement consists of 8 bars $\frac{3}{4}$ inch square. The details of these girders are given in Fig. 201. The beams are spaced 5 feet center to center. The span of these beams is about 16 feet, the width 8 inches, and the depth 12 inches below the slab; and the reinforcement consists of 5 bars $\frac{5}{8}$ inch square. The slab is 4 inches thick, including the top coat of 1 inch, which was composed of 1 part Portland cement and 1 part sand. This finishing coat was put on before the other concrete had set, and was figured as part of the structural slab. The slab reinforcement consisted of $\frac{1}{4}$-inch bars spaced 4 inches on centers, and $\frac{1}{4}$-inch bars spaced 24 inches at right angles to the bars spaced 4 inches. The columns ranged in size from 10 by 10-inch to 18 by 18-inch, and were reinforced by placing a bar in each corner of the column, which bars are tied together by $\frac{1}{4}$-inch bars spaced 12 inches. The amount of this steel was about one per cent of the total area of the column.

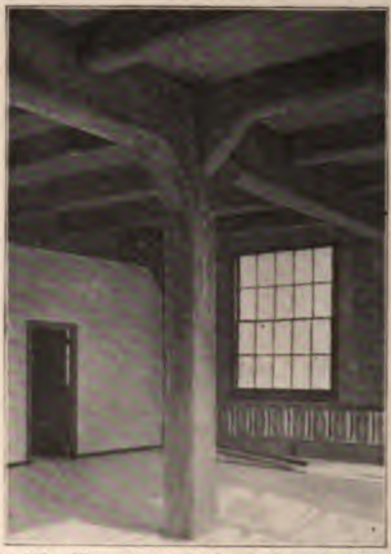


Fig. 203. Floor Construction in Shop Building of Swarthmore (Pa.) College, Showing Connection of Girder Beams with Column.

Fig. 202 shows the plans of the stairway. The lintels were moulded on the ground, and placed when the side walls had been built to the proper height for the lintels to be placed. The size of the lintels was varied on the different floors to conform with the architectural features of the building. The width of the lintels was made the same as the thickness of the walls, and therefore both sides of the lintels were exposed to view. The lintels were reinforced with 3 bars $\frac{1}{2}$ inch square.

The concrete was composed of 1 part Portland cement, 3 parts sand, and 5 parts stone. The stone was graded in size from $\frac{1}{4}$ inch o 1 inch. "Johnson" corrugated bars were used as the reinforcing steel. A panel, 16 by 20 feet, of one of the floors, was tested by placing a load of 300 pounds per square foot over this area. The deflection was so slight that it could not be conveniently measured. In Fig. 203 is given a view of the under side of a floor, showing the connection of the girder and beams with the column.

There is a criticism that may be made in the details of the girder shown in Fig. 201. The bars, which are turned up at the end, should have been long enough so that the bars could be again bent parallel to the floor line and be extended through the column. This would have tied the girders together in a more secure manner; and these bars, being near the top of the slab, would have resisted any negative bending moment.

385. **Apartment House.** In designing a reinforced-concrete apartment house which was constructed at Juniper and Spruce Streets, Philadelphia, it was desirable to have a floor system that was flat on the under side, except for the beams connecting the columns, so as to avoid the expense of a suspended ceiling. The greatest span of the flat construction necessary to avoid having beams in the ceiling of the rooms, was about 18 feet. It was at first intended to use a slab of reinforced concrete to connect the beams; but, as the Philadelphia Building Code requires that the depth of reinforced concrete must be at least three-fifths of an inch per foot of span, to fulfil this condition a slab much thicker than necessary for structural purposes was required. The Building Code requires that the floors of apartment houses shall be designed to carry safely 70 pounds per square foot.

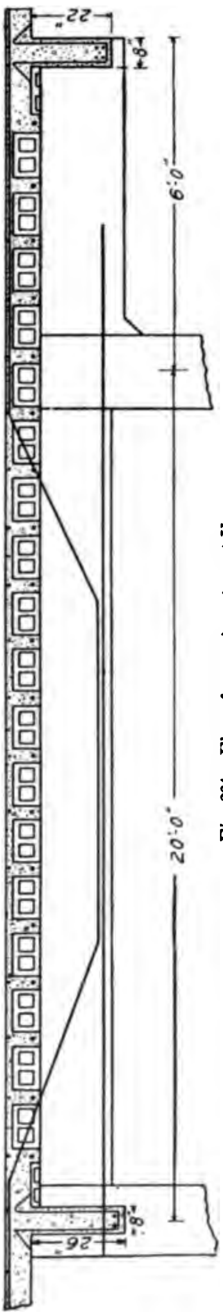


Fig. 204. Floor for an Apartment House.

This apartment house is 40 feet by 127 feet, and eight stories in height. There is also a basement under the entire building. In taking bids on this building, it was found that a steel frame, not including the fireproofing, cost more than a reinforced-concrete structure. It was therefore decided to construct the building of reinforced concrete. The walls were of brick, except the eighth story, which was concrete. The concrete wall is hollow, having a total thickness of 16 inches; and it is composed of two slabs, each six inches in thickness, with an air space of four inches between the slabs. These slabs are reinforced with steel bars placed longitudinally and vertically.

The type of floor construction used is shown in Fig. 204. Reinforced-concrete girders were constructed, connecting the columns; and the space between them was filled with small reinforced-concrete beams and plaster blocks. The girders were designed, when possible, as T-beams; and as a certain amount of concrete was required in the slab to take the compression, the hollow block construction was omitted for a sufficient width on each side of the girder to allow for this compression. This feature is shown in Fig. 204. The beams were 4 inches wide, 6 inches to 8 inches deep, depending on the span, and were connected with a 2-inch slab of concrete. The beams were spaced 16 inches center to center, and each beam was reinforced with a 1-inch round bar. The two-inch slab was reinforced with ¼-inch bars spaced 24 inches; and over the girders and at right angles to the girders, ⅜-inch bars 6 feet long were spaced 16 inches; that is, one of these bars was placed in the top of each of the beams. The span of these beams varied from 12 feet to 18 feet.

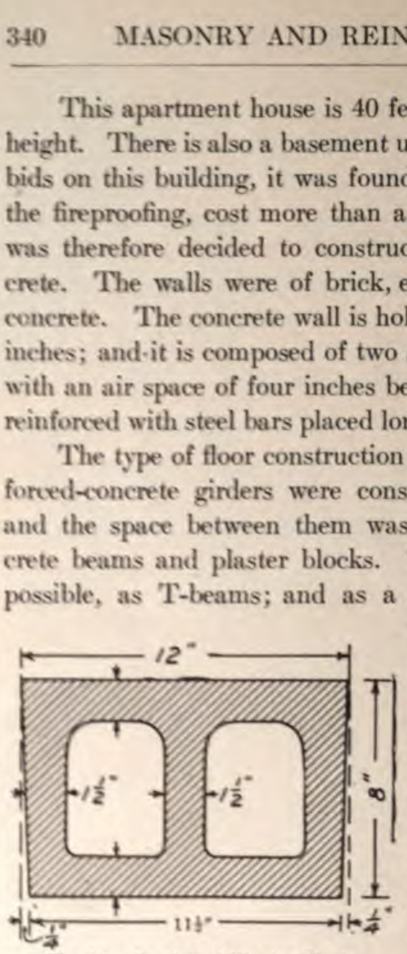


Fig. 205. Section of Plaster Block.

A hollow plaster block, 12 inches wide, was used as a filler between the concrete beams. These blocks were made of the required depth, 6 and 8 inches, and were 12 inches wide at the top and 11½ inches wide at the bottom. The object in sloping the sides of the blocks was to key the blocks between the beams. The block, in section, is shown in Fig. 205, and is known as the *Keystone Fireproof*

Block. The coefficient of expansion of plaster blocks is very small compared with that of the terra-cotta block; and also the plaster block is more efficient as a non-conductor of heat. The blocks were spaced 4 inches apart, and therefore served as the forms for the sides of the beams. The planks on which the blocks were placed were spaced 8 inches apart, which made a saving in the amount of lumber required for forms. It was found necessary to wet the blocks thoroughly by means of a hose, before the concrete was placed, as the dry blocks quickly absorbed the water from the concrete. About one per cent of the blocks were broken in handling them. The partitions in the building were made with the blocks. When the floor forms were removed, the ceilings and walls were plastered.

On the Juniper Street side, balconies were constructed nearly the full length of the house. They are 4 feet wide. Structurally they were constructed as cantilever beams, and consist of slabs of concrete 6 inches thick and reinforced with $\frac{1}{2}$-inch round bars spaced 6 inches center to center. These balconies are constructed at each floor level. In Fig. 204 is shown a cantilever beam with a span of 6 feet. It is 12 inches wide and 26 inches in depth, and is reinforced with 4 bars $\frac{7}{8}$ inch in diameter. This cantilever supports the exterior wall and one end of a simple beam of a span of about 16 feet.

The exterior and interior columns were constructed of concrete, and reinforced with plain round bars. The roof construction was similar to the floor construction.

The concrete consisted of a mixture of 1 part Portland cement, 3 parts sand, and 5 parts stone. The stone was trap rock, broken to pass through a $\frac{3}{4}$-inch ring, dust screened out; and the sand is known as *Jersey gravel,* which is a bank sand. The reinforcing bars were plain round bars of medium steel.

386. **The McNulty Building.*** The columns used in the construction of the McNulty Building, New York City, are a very interesting feature in this building. The building is 50 feet by 96 feet, and is 10 stories high, and was one of the first *small-column* reinforced-concrete buildings erected in New York. The plan of all the floors is the same. A single row of interior columns is placed in the center of the building, about 22 feet center to center.

The columns are of the hooped type, and were designed from the

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**Engineering Record,* August 15, 1907.

formula approved by the building laws of New York City. The formula used was $P = 1{,}600\, r^2 + (160{,}000\, Ah \div P) \times r + 6{,}000\, As$, in which P = the total working load, r = radius of the helix, As = the total area of the vertical steel, Ah = sectional area of the hooping wire, P = the pitch of the helix.

The interior columns are cylindrical in form, except those supporting the roof, which are 12 by 12-inch and are reinforced with 4 bars ¾ inch in diameter. In all the other stories except the ninth, they are 27 inches in diameter. Below the fifth floor the reinforcement in each of these columns consists of 2-inch round vertical bars, ranging in number from seven in the fifth floor to thirty in the basement, and banded by a 24-inch helix of ½-inch wire with a pitch of 1½ inches. The vertical bars were omitted between the sixth and tenth floors; and the diameter of the helix was gradually decreased, while the pitch was increased. In the ninth floor the diameter was reduced to 21 inches.

The wall columns are in general 30 by 26 inches, and support loads from 48,000 pounds in the tenth floor to 719,750 pounds in the basement. In the sixth story, the reinforcement in these columns consists of 3 round vertical bars 2 inches in diameter; and in each of the floors below, the number of bars was increased in these columns there being 24 in the basement columns. These are spirally wound with $\frac{5}{16}$-inch steel wire forming a helix 23 inches in diameter, with a pitch of 2½ inches. Above the seventh floor, the columns are reinforced with 4 bars ¾ inch in dameter, and tied together by $\frac{5}{16}$-inch wire spaced 18 inches apart. The columns rest on cast-iron shoes, which are bedded on solid rock about 2½ feet below the basement floor.

The main-floor girders extend transversely across the building, and have a clear span of 21 feet. The floor-beams are spaced about 6 feet apart, and have a span of about 20 feet 6 inches. The sides of the beams slope, the width at the bottom being two inches less than the width at the under surface of the slab. The reinforcement consists of plain round bars. The bars for the girders and beams were bent and made into a truss (the *Unit System*) at the shops of the contractor, and were shipped to the work ready to be put in place. The stirrups were hot-shrunk on the longitudinal bars. The helices for the columns were wound and attached to some of the vertical rods at the shop, to preserve the pitch. The vertical rods in each column project 6 inches above the floor line, and are connected to the bar placed on it, by a piece of pipe 12 inches long.

The concrete was a 1:2:4 mixture. Giant Portland cement was used, and ¾-inch trap rock. The placing of concrete was begun about the middle of August, 1906, and the building was completed December 20.

387. **The McGraw Building.** The McGraw Building, New York City, completed in 1907, is a good example of a reinforced-concrete building. The building has a frontage of 126 feet and a depth of 90 feet, and is 11 stories in height. The height of the roof is about 150 feet above the street level. The building was designed to resist the vibration of heavy printing machinery. The first and second floors were designed for a live load of 250 pounds per square foot; for the third floor, 150 pounds per square foot; for the fourth floor and all floors above the fourth floor, 125 pounds per square foot.

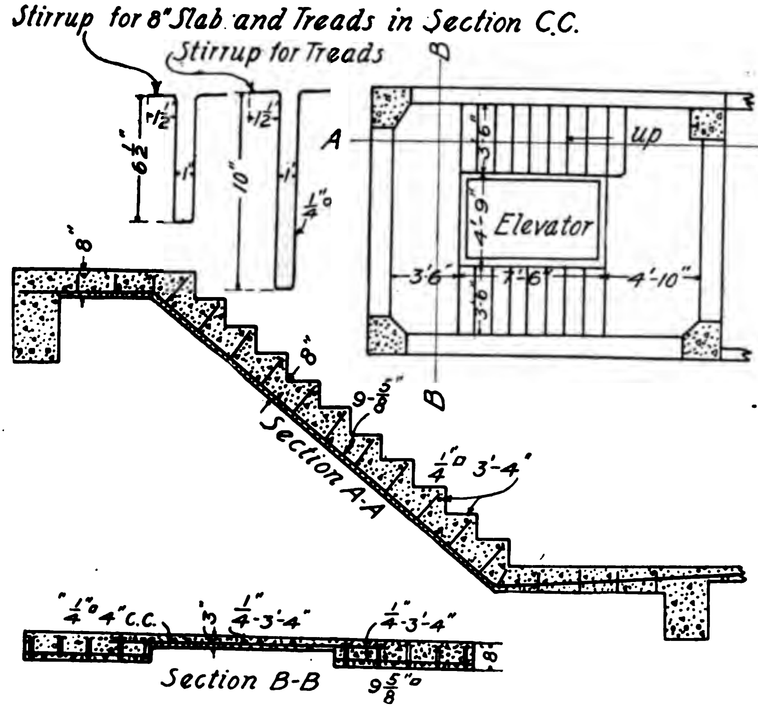


Fig. 206. Stairs for Fridenberg Building.

All beams and girders were designed as continuous beams, even where supported on the outside beams. There was twice as much steel over the supports as in the center of the spans. The Building Code of the City of New York requires that the moment for continuous beams be taken as $\frac{Wl}{10}$ at the center of the span, and as $\frac{Wl}{5}$ over the support. These values are more than twice the theoretical value as computed for continuous beams.

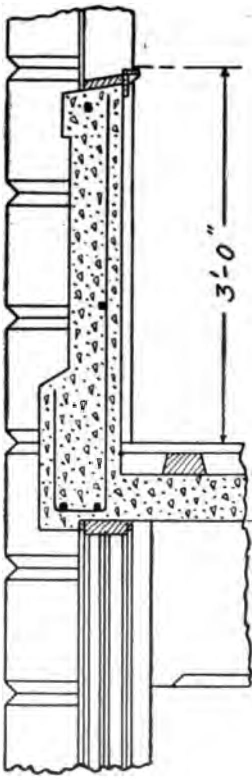


Fig. 207. Detail of Lintel.

One very interesting feature of this building is that it was constructed during the winter. The first concrete was laid during September, and the concrete work was completed in April. During freezing weather, the windows of the floors below the floor that was being constructed were closed with canvas; and salamanders (open stoves) were distributed over the completed floor, and kept in constant operation. Coke was used as the fuel for the salamanders. The concrete was mixed with hot water, and the sand and the stone were also heated. After two or three stories had been erected, and the construction force was fully organized, a floor was completed in about 12 days. Three complete sets of forms were provided and used. They were usually left in place nearly three weeks.

388. **Fridenberg Building.** In Fig. 206 are shown the plans of stairs constructed in the Fridenberg building at 908 Chestnut Street, Philadelphia. This building is 24 feet by 60 feet, and is seven stories high. Structurally the building was constructed of reinforced concrete. The stair and elevator tower is located in the rear of the main building.

The plans of the stairs are interesting on account of the long-span (about 16 feet) slab construction. The stairs were designed to carry

Large Concrete Sewer in [illegible] Canal.

Wakeling Street Concrete Sewer, 16 ft. by 10 ft. 6 in.

TWO VIEWS OF SEWERS IN THE CITY OF PHILADELPHIA, PA.

Courtesy of Geo. S. Webster, Chief Engineer, Bureau of Surveys, Dept. of Public Works.

safely a live load of 100 pounds per square foot; and in the theoretical calculations the slab was treated as a flat slab with a clear span of 16 feet. The shear bars were made and spaced as shown in the details. The calculations showed a low shearing value in the concrete, but stirrups were used to secure a good bond between the steel and concrete.

The concrete was a 1:2:4 mixture, and was mixed wet. The reinforcing steel consisted of square deformed bars, except the stirrups, which were made of ¼-inch plain round steel.

389. **General Electric Company Building.** An interesting feature of a large reinforced-concrete building constructed for the General Electric Company at Fort Wayne, Ind., is the design of the lintels. As shown in Fig. 207, the bottom of the lintel is at the same elevation as the bottom of the slab. The total space between the columns is filled with double windows; and the space between the bottom of the windows and the floor is filled with lintels and a thin wall of reinforced concrete, as shown in the figure.

390. **Water-Basin and Circular Tanks.** Figs. 208 and 209 illustrate sections of the walls of the pure water basin and the 50-foot circular tanks which have been partly described in Part I under the heading of *Waterproofing.*

The pure water basin was 100 feet by 200 feet, and 13 feet deep, giving a capacity of 1,500,000 gallons. The counterforts are spaced 12 feet 6 inches center to center, and are 12 inches thick, except every fourth one, which was made 18 inches thick. The 18-inch counterforts were constructed as two counterforts 9 inches thick, as the vertical joints in the walls were made at this point; that is, the concrete between the centers of two of the 18-inch counterforts was placed in one day. On the two ends and one side of the basin, the counterforts were constructed on the exterior of the basin to support about 10 feet of earth. But on one side it would have been necessary to remove rock 6 to 8 feet in thickness to make room for the counterforts, had they been constructed on the exterior of the basin. Therefore they were constructed inside of the basin. If both faces of the vertical wall had been reinforced, the same as the one shown, then the wall would have been able to resist an outward or inward pressure, and the "piers" would act as counterforts or buttresses, depending on whether they were in tension or in compression.

The concrete used consisted of 1 part Portland cement, 3 parts sand, and 5 parts crushed stone. The stone was graded in size from ¼-inch as the minimum to ¾-inch as the maximum size. Square-sectioned deformed bars were used as the steel reinforcement. The forms were constructed in units so that they could be put up and taken down quickly.

The size and spacing of the bars in the walls of the circular tanks, are shown in Fig. 209. The framework of the forms to which the lagging was fastened was cut to the desired curve at a planing mill.

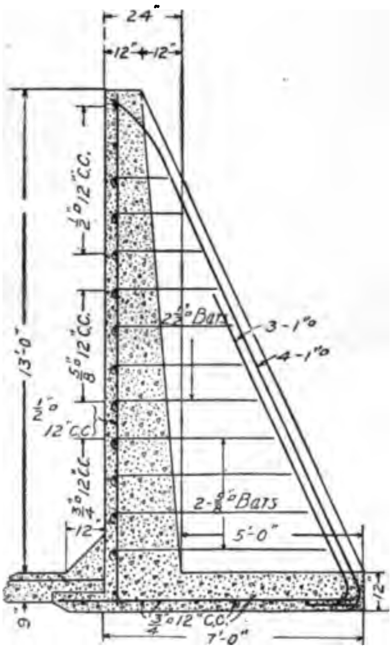


Fig. 208. Section of Water Basin Wall.

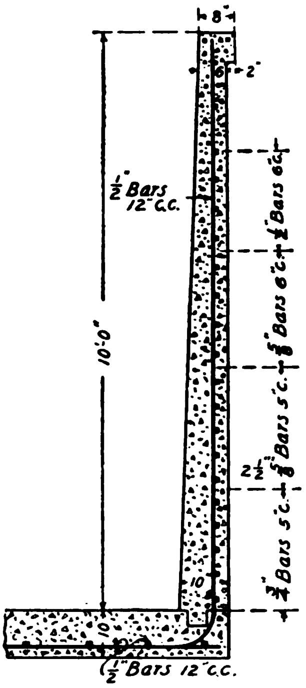


Fig. 209. Section of Tank.

This framing was cut from 2 by 12-inch lumber. The lagging was ⅞ inch thick, and surfaced on one side.

391. **Main Intercepting Sewer.** In the development of sewage purification work at Waterbury, Conn., the construction of a main intercepting sewer was a necessity. This sewer is three miles long. It is of horseshoe shape, 4 feet 6 inches by 4 feet 5 inches, and is constructed of reinforced concrete. The details are illustrated in Fig. 210.

The trench excavations were principally through water-bearing gravel, the gravel ranging from coarse to fine. Some rock was encountered in the trench excavations. It was a granite-gneiss of irregular fracture, and cost, with labor at 17½ cents per hour, about $2.00 per cubic yard to remove it. Much trench work has varied in depth from 20 to 26 feet. Owing to the varying conditions, it was necessary to vary the sewer section somewhat. Frequently the footing course was extended. However, the section shown in the figure is the normal section.

The concrete was mixed very wet, and poured into practically water-tight forms. The proportions used were 1 part Atlas Portland cement to 7½ parts of aggregate, graded to secure a dense concrete. Care was used in placing the concrete, and very smooth surfaces were secured. Plastering of the surfaces was avoided. Any voids were grouted or pointed and smoothed with a wooden float. Expanded metal and square twisted bars were used in different parts of the work. In Fig. 210, the size and spacing of the bars are shown. The bars were bent to their required shape before they were lowered into the excavation.

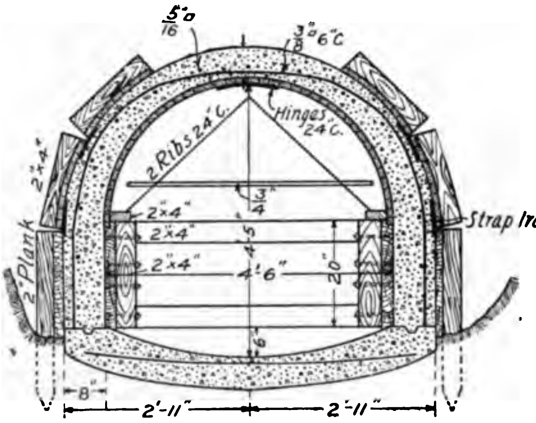


Fig. 210. Intercepting Sewer at Waterbury, Conn.

The forms in general were constructed as shown in the figure. The inverted section was built as the first operation; and after the surface was thoroughly troweled, the section was allowed to set 36 to 48 hours before the concreting of the arch section was begun. The lagging was ⅞ inch thick, with tongued-and-grooved radial joints, and toe-nailed to the 2-inch plank ribs. The exterior curve was planed and scraped to a true surface. The vertical sides of the inner forms are readily removable, and the semicircular arch above is hinged at the soffit and is collapsible. The first cost of these forms has averaged

$18.00 for 10 feet of length; and the cost of the forms per foot of sewer built, including first cost and maintenance, averaged 10 cents. Petrolene, a crude petroleum, was found very effective in preventing the concrete from adhering to the forms.

A mile and a-half of the sewer has been completed (May, 1908), and is in use, all of which has been constructed in water-bearing soil; and the greater part of it has been 4 to 12 feet below the ground-water level. The interior surface in this length subjected to percolation is 118,000 square feet. The total seepage from this area has been less than 0.03 cubic foot per second.

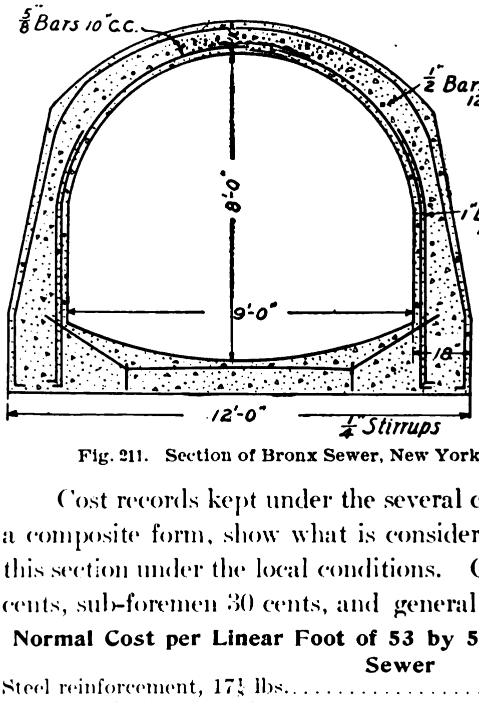


Fig. 211. Section of Bronx Sewer, New York.

Cost records kept under the several contracts and assembled into a composite form, show what is considered to be the normal cost of this section under the local conditions. Common labor averaged 17½ cents, sub-foremen 30 cents, and general foremen 50 cents per hour.

Normal Cost per Linear Foot of 53 by 54-Inch Reinforced-Concrete Sewer

Steel reinforcement, 17½ lbs.	$ .43
Making and placing reinforcement cages	.14
Wooden interior forms, cost, maintenance, and depreciation	.12
Wooden exterior forms, cost, maintenance, and depreciation	.05
Operation of forms	.16
Coating oil	.01
Mixing concrete	.30
Placing concrete	.27
Screeding and finishing invert	.08
Storage, handling, and cartage of cement	.08
0.482 bbl. cement at $1.53	.74
0.17 cu. yd. sand at $0.50	.09
0.435 cu. yd. broken stone at $1.10	.47
Finishing interior surface	.01
Sprinkling and wetting completed work	.02
Total cost per linear foot	$2.97

This is equivalent to a cost of $9.02 per cubic yard.

392. **Bronx Sewer, New York.** In Fig. 211 is shown a section of one of the branch sewers that is being constructed in the Borough of the Bronx, New York City. A large part of this sewer is located in a salt marsh where water and unstable soil make construction work very difficult. The general elevation of the marsh is 1.5 feet above mean high water. In constructing this sewer in the marsh, it is necessary to construct a pile foundation to support the sewer The foundation is capped with reinforced concrete; and then the sewer, as shown in the section, is constructed on the pile foundation. The concrete for this work is composed of 1 part Portland cement, 2½ parts sand, and 5 parts trap rock. The rock was crushed to pass a ¾-inch screen. "Ransome" twisted bars were used for the reinforcement in this work.

393. **Girder Bridge.** The reinforced-concrete bridge shown in Fig. 212 was constructed near Allentown, Pennsylvania, in 1907.

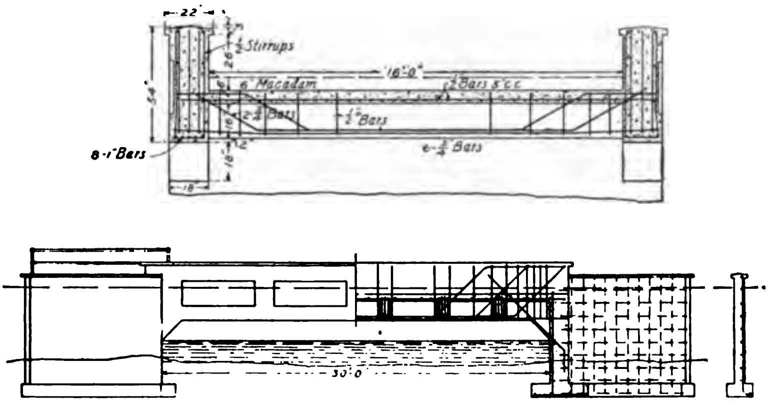


Fig. 212. Girder Bridge near Allentown, Pa.

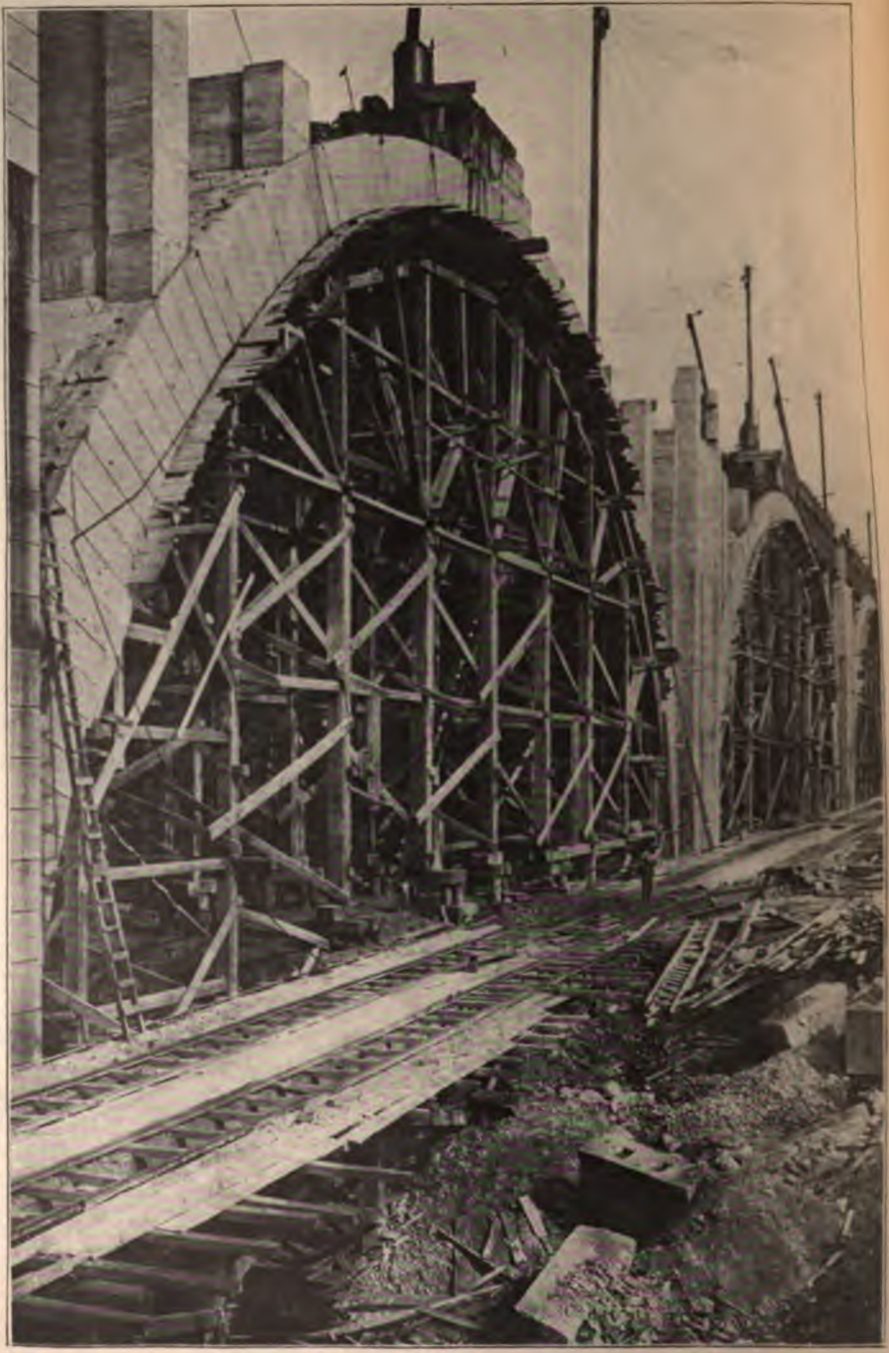
This type of bridge has been found to be economical for short spans. Worn-out wooden and steel highway bridges are in general being replaced with reinforced-concrete bridges, and usually at a cost less than that of a steel bridge of the same strength. Steel bridges should be painted every year; and plank floors, as commonly used in highway bridges, require almost constant attention, and must be entirely renewed several times during the life of a bridge. A reinforced-concrete bridge, however, is entirely free of these expenses, and its life should at least be equal to that of a stone arch. From an architectural standpoint, a well-finished concrete bridge compares very favorably with a cut-stone arch.

The bridge shown in Fig. 212 is 16 feet wide, and has a clear span of 30 feet. It was designed to carry a uniformly distributed load of 150 pounds per square foot, or a steel road-roller weighing 15 tons, the road-roller having the following dimensions: The width of the front roller is 4 feet; and of each rear roller, 20 inches; the distance apart of the two rear rollers is 5 feet, center to center; and the distance between front and rear rollers is 11 feet, center to center; the weight on the front roller is 6 tons, and 4.5 tons on each of the rear rollers.

In designing this bridge, the slab was designed to carry a live load of 4.5 tons on a width of 20 inches, when placed at the middle of the span, together with the dead load consisting of the weight of the macadam and the slab. The load considered in designing the cross-beams, consisted of the dead load—weight of the macadam, slab, and beam—and a live load of 6 tons placed at the center of the span of the beam, which was designed as a T-beam. In designing each of the longitudinal girders, the live load was taken as a uniformly distributed load of 150 pounds per square foot over one-half of the floor area of the bridge. The live load was increased 20 per cent over the live load given above, to allow for impact.

In a bridge of this type, longitudinal girders act as a parapet, as well as the main members of the bridge. The concrete for this work was composed of 1 part Portland cement, 2 parts sand, and 4 parts 1-inch stone. Corrugated bars were used as the reinforcing steel.

When there is sufficient headroom, all the beams can be constructed in the longitudinal direction of the bridge, and are under the slab. The parapet may be constructed of concrete; or a cheaper method is to construct a handrailing with $1\frac{1}{2}$-inch or 2-inch pipe.



CONNECTICUT AVENUE BRIDGE OVER ROCK CREEK, WASHINGTON, D. C.

Largest concrete bridge in world without steel reinforcement. The five principal arches have spans of 150 feet; highest point of bridge above gorge, 150 feet; each abutment pier comprises two smaller arches. Total length between abutments, 1,341 feet.

MASONRY AND REINFORCED CONCRETE

PART V

THEORY OF ARCHES

394. The mechanics of the arch are almost invariably solved by a graphical method, or by a combination of the graphical method with numerical calculations. This is done, not only because it simplifies the work, but also because, although the accuracy of the graphical method is somewhat limited, yet, with careful work, it may easily be made even more accurate than is necessary, considering the uncertainty as to the true ultimate strength of the masonry used. The development of this graphical method must necessarily follow the same lines as in Statics. It is here assumed that the student has a knowledge of Statics, and that he already understands the graphical method of representing the magnitude, direction, and line of application of a force. Several of the theorems or general laws regarding the composition and resolution of forces will be briefly reviewed as a preliminary to the proof of those laws of graphical statics which are especially applied in computing the stresses in an arch.

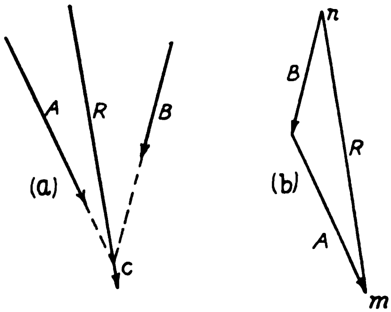


Fig. 213. Resultant of Two Forces.

395. **Resultant of Two Forces.** The resultant of two forces, *A* and *B*, which are not parallel, whose lines of action are as shown in Fig. 213*a*, and which are measured by the *lengths* of the lines *A* and *B* in diagram *b*, is readily found by producing the lines of action to their intersection at *c*. The two known forces are drawn in

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diagram b so that their direction is parallel to the known directions of the forces, and so that the point of one force is at the butt end of the other. Then the line R joining the points m and n in diagram b gives the direction of the resultant; and a line through c parallel to that direction, gives the actual line of that resultant. The line mn also measures the amount of the resultant. Note that diagram b is a *closed figure.* If an arrow is marked on R so that it points *upward,* the arrows on the forces would *run continuously* around the figure. If R were acting upward, it would represent the force which would just hold A and B in equilibrium; pointing downward, it is the resultant or combined effect of the two forces. We may thus define the *resultant* of two (or more) forces as the force which is the *equal* and *opposite* of that force which will just hold that combination of forces in equilibrium.

396. **Resultant of Three or More Forces.** This may be solved by an extension of the method previously given as shown in Fig. 213. The resultant of B and C (see Fig. 214) is R'; and this is readily combined with A, giving R'' as the resultant of all three forces. The same principle may be extended to any number of non-parallel forces acting in a plane. The resultant of four non-parallel forces is best determined by finding, first, the resultant of each pair of the forces taken two and two. Then the resultant of the two resultants is found, just as if each resultant were a single force.

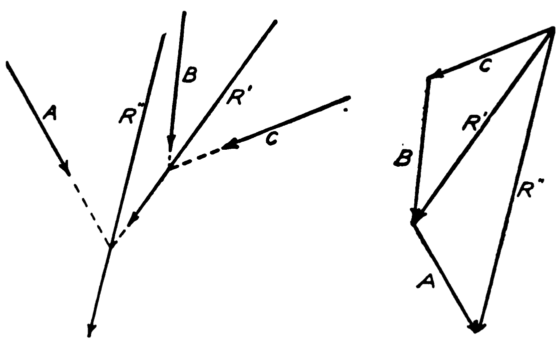


Fig. 214. Resultant of Three Forces.

397. **Resultant of Two or More Parallel Forces.** When the forces are all parallel, the *direction* of the resultant is parallel to the component forces; the *amount* is equal to the *sum* of the component forces; but the *line of action* of the resultant is not determinable as in the above cases, since the forces do not intersect. It is a principle of Statics which is easily appreciated, that it does not alter the statics of any combination of forces to assume that two *equal* and *opposite*

forces are applied along any line of action. From Fig. 215 *b*, we see that the forces *F* and *G* will hold *A* in equilibrium; that *G* and *H* will hold *B* in equilibrium; and that *H* and *K* will hold *C* in equilibrium. But the force *G* required to hold *A* in equilibrium is the *equal* and *opposite* of the force *G* required to hold *B* in equilibrium; and similarly the force *H* for *B* is the equal and opposite of the *H* for *C*. We thus find that the forces *A*, *B*, and *C* can be held in equilibrium by an unbalanced force *F*, two equal and opposite forces *G*, two equal and opposite forces *H*, and the unbalanced force *K*. The net result, therefore, is that *A*, *B*, and *C* are held in equilibrium by the two forces *F* and *K*. The resultant *R* is the *sum* of *A*, *B*, and *C*; and therefore the combined-load line represents the resultant *R*. The external lines of diagram *b* show that *F*, *K*, and *R* form a closed

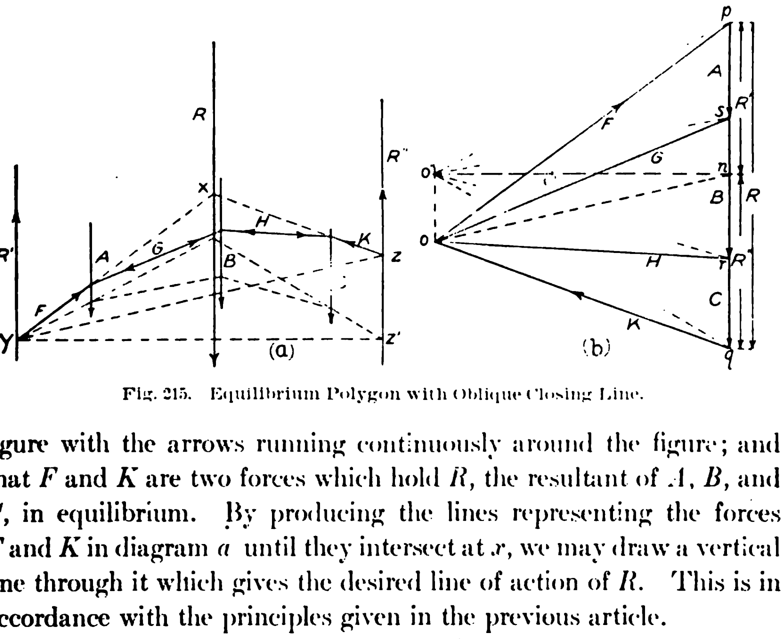


Fig. 215. Equilibrium Polygon with Oblique Closing Line.

figure with the arrows running continuously around the figure; and that *F* and *K* are two forces which hold *R*, the resultant of *A*, *B*, and *C*, in equilibrium. By producing the lines representing the forces *F* and *K* in diagram *a* until they intersect at *x*, we may draw a vertical line through it which gives the desired line of action of *R*. This is in accordance with the principles given in the previous article.

Nothing was said as to how *F*, *G*, *H*, and *K* were drawn in *a* and *b*. These forces simply represent one of an infinite number of combinations of forces which would produce the same result. The point *o* is chosen at random, and lines (called *rays*) are drawn to the extremities of all the forces. The lines of force (*A*, *B*, and *C*) in diagram *b* (which is called the *force diagram*), are together called the *load line*. The line of forces (F, G, H, and K) in diagram a, together with the *closing line yz*, is called an *equilibrium polygon*.

398. **Statics of a Linear Arch.** We shall assume that the lines in Fig. 215 by which we have represented forces F, G, H, and K represent struts which are hinged at their intersections with the forces A, B, and C, which represent loads; and that the two end struts F and K are hinged at two abutments located at y and z. Then all of the struts will be in compression, and the rays of the force diagram will represent, at the same scale as that employed to represent forces or loads A, B, and C, the compression in each of the struts. In the force diagram, draw a line from o, parallel with the line yz. It intersects the load line in the point n. Considering the triangle opn as a force diagram, op represents the force F, while pn and on may represent the direction and amount of two forces which will hold F in equilibrium. Therefore pn would represent the amount and direction of the vertical component of the abutment reaction at y, and on would represent the component in the direction of yz. Similarly we may consider the triangle onq as a force diagram; that nq represents the vertical component R'', and that on represents the component in the direction zy. Since on is common to both of these force triangles, they neutralize each other, and the net resultant of the two forces F and K is the two vertical forces R and R''; but since the resultant R is the resultant of F and K, we may say that R' and R'' are two vertical forces whose combined effect is the equal and opposite of the force R. Although an indefinite number of combinations of forces could begin and end at the points y and z, and could produce equilibrium with the forces A, B, and C, the forces R' and R'' are independent of that particular combination of struts, F, G, H, and K.

399. **Graphical Demonstration of Laws of Statics by Student.** The student should test all this work in Statics by drawing figures very carefully and on a large scale, in accordance with the general instructions as described in the sections, and should purposely make some variation in the relative positions and amounts of the forces from those indicated by the figures. By this means the student will be able to obtain a virtual demonstration of the accuracy of the laws of Statics as formulated. The student should also remember that the laws are theoretically perfect; and when it is stated, for example,

that certain lines should be parallel, or that a certain line drawn in a certain way should intersect some certain point, the mathematical laws involved are perfect; and if the drawing does not result in the expected way, it either proves that a blunder has been made, or it may mean that the general method is correct, but that the drawing is more or less inaccurate.

400. **Equilibrium Polygon with Horizontal Closing Lines.** In Fig. 216, have been drawn the same forces *A*, *B*, and *C*, having the same relative positions as in Fig. 215. The lines of action of the two vertical forces R′ and R″ have also been drawn in the same relative position as in Fig. 215. The point *n* has also been located on the load line in the same position as in Fig. 215. Thus far the lines are a repetition of those already drawn in Fig. 215, the remainder of the figure being omitted for simplicity. Since the point *n* in Fig. 215 is the end of the line from the trial pole *o*, which is parallel to the closing line *yz*, and since the point *n* is a definitely fixed point and determines the abutment reactions regardless of the position of the trial pole *o*, we may draw from *n* an indefinite horizontal line, such as *no′*, and we know that the pole of any force diagram must be on this line if the closing line of the corresponding equilibrium polygon is to be a horizontal line. For example, we shall select a point o′ on this line at random. From *o′* we shall draw rays to the points *p*, *s*, *r*, and *q*. From the point *y*, we shall draw a line parallel to *o′p*. Where this line intersects the force *A*, draw a line parallel to the ray *o′s*. Where this intersects the force *B*, draw a line parallel to the ray *o′r*. Where this intersects the force *C*, draw a line parallel to the ray *o′q*. This

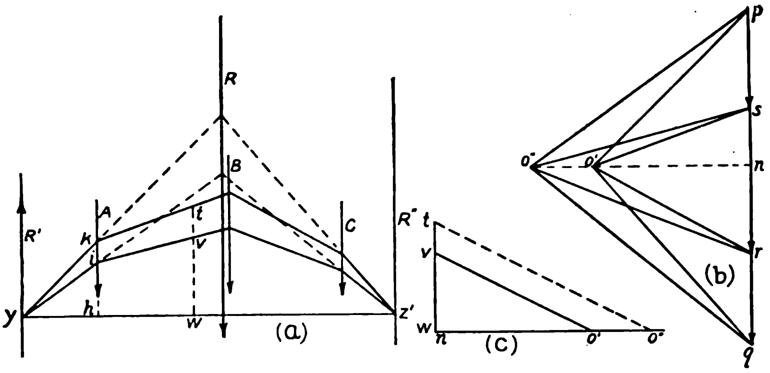


Fig. 216. Equilibrium Polygon with Horizontal Closing Line.

line must intersect the point z', which is on a horizontal line from y. The student should make some such drawing as here described, and should demonstrate for himself the accuracy of this law. This equilibrium polygon is merely one of an infinite number which, if acting as struts, would hold these forces in equilibrium, but it combines the special condition that it shall pass through the points y and z'. There are also an infinite number of equilibrium polygons which will hold these forces in equilibrium and which will pass through the points y and z'.

We may also impose another condition, which is that the first line of the equilibrium polygon shall have some definite direction, such as yl. In this case the ray from the point p of the force diagram must be parallel to yl; and where this line intersects the horizontal line no' (produced in this case), is the required position for the pole o''. Draw rays from o'' to s, r, and q, continuing the equilibrium polygon by lines which are respectively parallel to these rays. As a check on the work, the last line of the equilibrium polygon which is parallel to $o''q$ should intersect the point z'. The triangles ykh and $o'pn$ have their sides respectively parallel to each other, and the triangles are therefore similar, and their corresponding sides are proportional, and we may therefore write the equation:

$$o'n : yh :: pn : kh.$$

Also, from the triangles ylh and $o''pn$, we may write the proportion:

$$o''n : yh :: pn : lh.$$

From these two proportions we may derive the proportion:

$$o'n : o''n :: lh : kh;$$

but $o'n$ and $o''n$ are the pole distances of their respective force diagrams, while kh and lh are intercepts by a vertical line through the corresponding equilibrium polygons. The proportion is therefore a proof, in at least a special case, of the general law that the perpendicular distances from the poles to the load lines of any two force diagrams are inversely proportional to any two intercepts in the corresponding equilibrium polygons. The above proportions prove the theorem for the intercepts hk and hl. A similar combination of proportions would prove it for any vertical intercept between y and h. The proof of this general theorem for intercepts which pass through other lines of the equilibrium polygon, is more complicated and tedious, but is equally conclusive. Therefore, if we draw any vertical intercept, such as tvw, we may write out the general proportion:

$$o''n : o'n :: tw : vw \dots\dots\dots\dots (46)$$

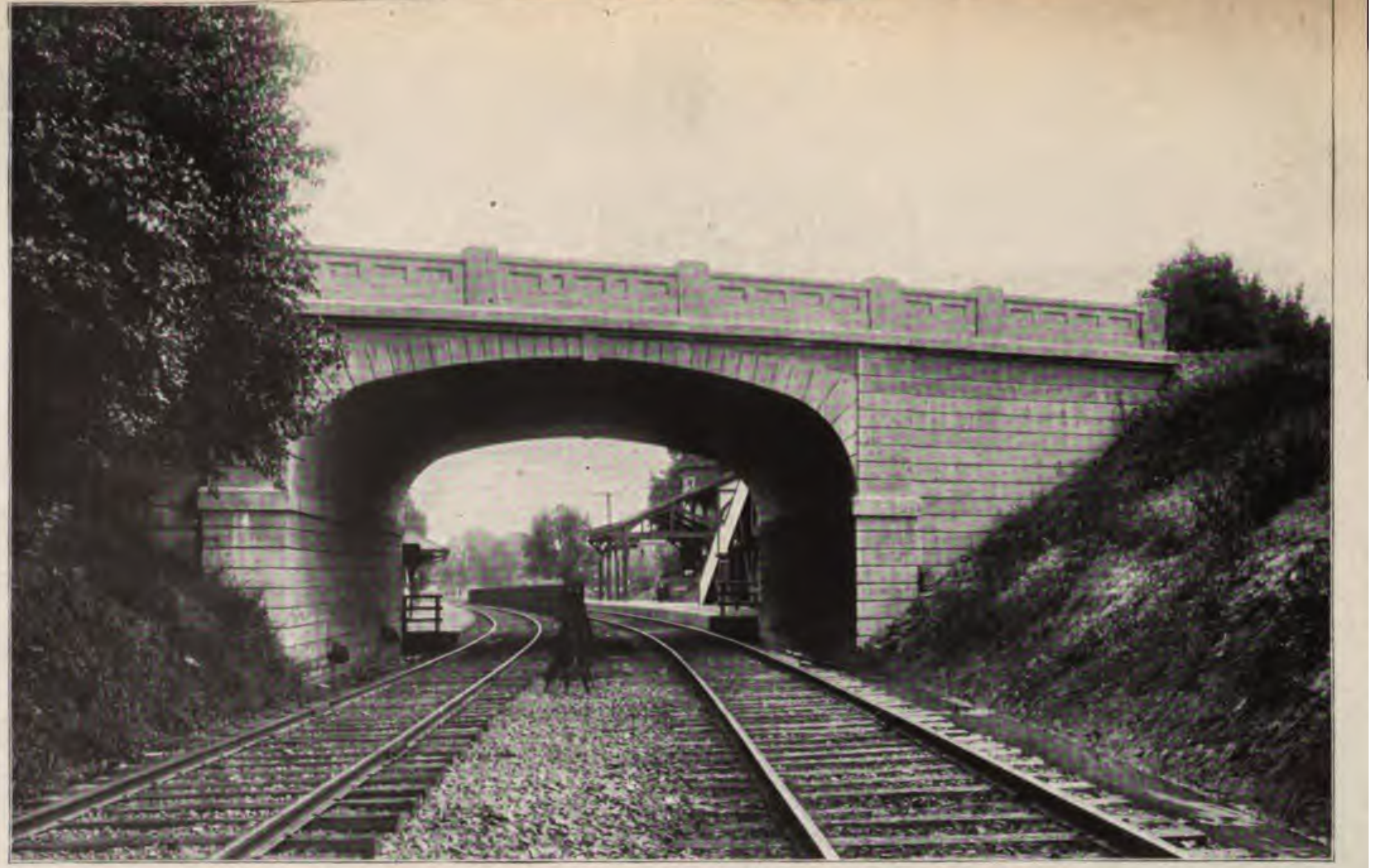
In this proportion, if $o''n$ were an unknown quantity, or the position of o'' were unknown, it could be readily obtained by drawing two random lines as shown in diagram *c*, and laying off on one of them the distance no', and on the other line the distances vw and tw. By joining v and o' in diagram *c*, and drawing a line from t parallel to vo', it will intersect the line no' produced, in the point o''. As a check, this distance to o'' should equal the distance no'' in diagram *b*. A practical application of this case, and one that is extensively employed in arch work, is the requirement that the equilibrium polygon shall be drawn so that it shall pass through three points, of which the abutments are two, and some other point (such as v) is the third. After obtaining a trial equilibrium polygon whose closing line passes through the points y and z', the proper position for the pole o'' which shall give the equilibrium polygon that will pass through the point v, may be easily determined by the method described above.

The process of obtaining an equilibrium polygon for parallel forces which shall pass through two given abutment points and a third intermediate point, may be still further simplified by the application of another property, and without drawing two trial equilibrium polygons before we can draw the required equilibrium polygon. It may be demonstrated that if the pole distance from the pole to the load line is unchanged, all the vertical intercepts of any two equilibrium polygons drawn with these same pole distances are equal. For example, in Fig. 215, a line is drawn from o, vertically upward until it intersects the horizontal line drawn through n in the point o''. This point is the pole of another equilibrium polygon whose closing line will be horizontal, because the pole lies on a horizontal line from the previously determined point n in the load line. Any vertical intercept of this equilibrium polygon will be equal to the corresponding intercept on the first trial equilibrium polygon; therefore, in order to draw a special equilibrium polygon for a given set of vertical loads, the polygon to pass through two horizontal abutment points and a definite third point between them, we need only draw first a trial equi-

librium polygon, the rays in the force diagram being drawn through *any* point chosen as a pole. Then, if we draw a line from the trial pole which shall be parallel with the closing line of this trial equilibrium polygon, the line will intersect the load line in the point n. Drawing a horizontal line from the point n in the load line, we have the *locus* of the pole of the desired special equilibrium polygon. Then draw a vertical through the point through which the special equilibrium polygon is to pass. The vertical distance of this point above the line joining the abutments, is the required intercept of the true equilibrium polygon. The intersection of that vertical with the upper line and the closing line of the trial equilibrium polygon, is the intercept of the trial polygon. The pole distance of the true equilibrium polygon is then obtained by the application of Equation **46**, by which the pole distances are declared inversely proportional to any two corresponding intercepts of the equilibrium polygons.

Another useful property, which will be utilized later, and which may be readily verified from Figs. 215 and 216, is that, no matter what equilibrium polygon may be drawn, the two extreme lines of the equilibrium polygon, if produced, intersect in the resultant R; therefore, when it is desired to draw an equilibrium polygon which shall pass through any two abutment points, such as yz or yz', we may draw from these two abutment points, two lines which shall intersect at any point on the resultant R. We may then draw two lines which will be respectively parallel to these lines from the extremities p and q of the load lines, their intersection giving the pole of the corresponding force diagram.

401. **Equilibrium Polygon for Non-Vertical Forces.** The above method is rendered especially simple, owing to the fact that the forces are all vertical. When the forces are not vertical, the method becomes more complicated. The principle will first be illustrated by the problem of drawing an equilibrium polygon which shall pass through the points y, z, and v in Fig. 217. We shall first draw the two non-vertical forces in the force diagram. The resultant R of the forces A and B is obtained as shown in Fig. 213. Utilizing the property referred to in the previous article, we may at once draw two lines through y and z which intersect at some assumed point e on the resultant R. Drawing lines from p and q parallel respectively to ez and ey, we determine the point o' as the trial pole for our force



CONCRETE ARCH AT GRAVER'S LANE, PHILADELPHIA, PENNSYLVANIA

Courtesy of Geo. S. Webster, Chief Engineer, Bureau of Surveys, Dept. of Public Works.

diagram. As a check on the drawing, the line joining the intersections b and c should be parallel to the ray $o's$, thus again verifying one of the laws of Statics. If the line bc is produced until it intersects the line yz produced, and a line is drawn from the intersection x through the required point v, it will intersect the forces A and B in the points d and g. Then dg will be one of the lines of the required equilibrium polygon. By drawing lines from q and p parallel to yd and zg, we find their intersection o'', which is the pole of the required force diagram. There are two checks on this result: (1) the line so'' is parallel to dg; and (2) the line $o'o''$ is horizontal.

If the line bc is horizontal or nearly so, the intersection (x) of bc and yz produced is at an infinite distance away, or is at least off the

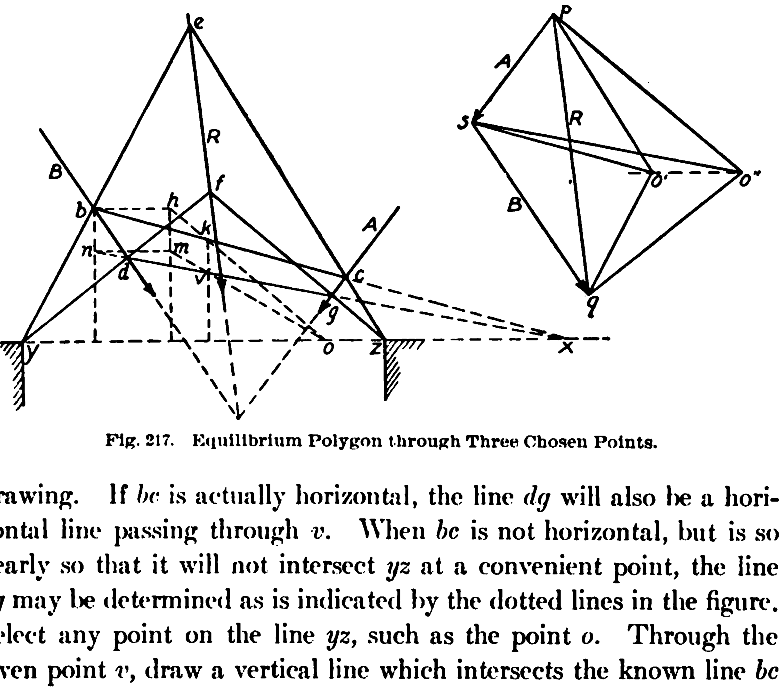


Fig. 217. Equilibrium Polygon through Three Chosen Points.

drawing. If bc is actually horizontal, the line dg will also be a horizontal line passing through v. When bc is not horizontal, but is so nearly so that it will not intersect yz at a convenient point, the line dg may be determined as is indicated by the dotted lines in the figure. Select any point on the line yz, such as the point o. Through the given point v, draw a vertical line which intersects the known line bc in the point k. From some point in the line bc (such as the point b), draw the horizontal line bh and the vertical line bn. The line from o through k intersects the horizontal line from b in the point h. From the point h, drop a vertical; this intersects the line ov produced, in the point m. From m, draw a horizontal line which intersects the vertical line from b. This intersection is at the point n. The line vn forms part of the required line *dg*. As a check on the work, the lines *zg* and *yd* should intersect at some point *f* on the force *R*. Another check on the work, which the student should make, both as a demonstration of the law and as a proof of the accuracy of his work, is to select some other point on the line *yz* than the point *o*, and likewise some other point on the line *bc* than the point *b*, and make another independent solution of the problem. It will be found that when the drawing is accurate, the new position for the point *n* will also be on the line *dg*.

In applying the above principle to the mechanics of an arch, the force *A* represents the resultant of all the forces acting on the arch on one side of the point *v* through which the desired equilibrium polygon is required to pass; and the force *B* is the resultant of all the forces on the other side of that point. A practical illustration of this method will be given later.

CONSTRUCTIVE FEATURES OF MASONRY ARCHES

402. Definitions of Terms Pertaining to Arch Masonry. The following are definitions of technical terms frequently used in connection with the subject of arch masonry (see Fig. 218):

Abutment—The masonry which supports an arch at either end, and which is so designed that it can resist the lateral thrust of an arch.

Arch Sheeting—That portion of an arch which lies between the ring stones.

Backing—Masonry which is placed outside of or above the extrados, with the sole purpose of furnishing additional weight on that portion of the arch; it is always made of an inferior quality of masonry and with the joints approximately horizontal.

Coursing Joint—A joint which runs continuously from one face of the arch to the other.

Crown—The highest part of an arch ring.

Extrados—The upper or outer surfaces of the voussoirs which compose the arch ring.

Haunch—That portion of an arch which is between the crown and the skewback; although there is no definite limitation, the term applies generally to that portion of the arch ring which is approximately half-way between the crown and the skewback.

Heading Joint—A joint between two consecutive stones in any string course. In order that the arch shall be properly bonded together, such joints are purposely made *not* continuous.

Intrados—The inner or lower surface of an arch. The term is frequently restricted to the line which is the intersection of the inner surface by a plane which is perpendicular to the axis of the arch.

Keystone—The voussoir which is placed at the crown of an arch.

Parapet—The wall which is usually built above the spandrel walls and above the level of the roadway.

Rise—The vertical height of the bottom of the keystone above the plane of the skewbacks.

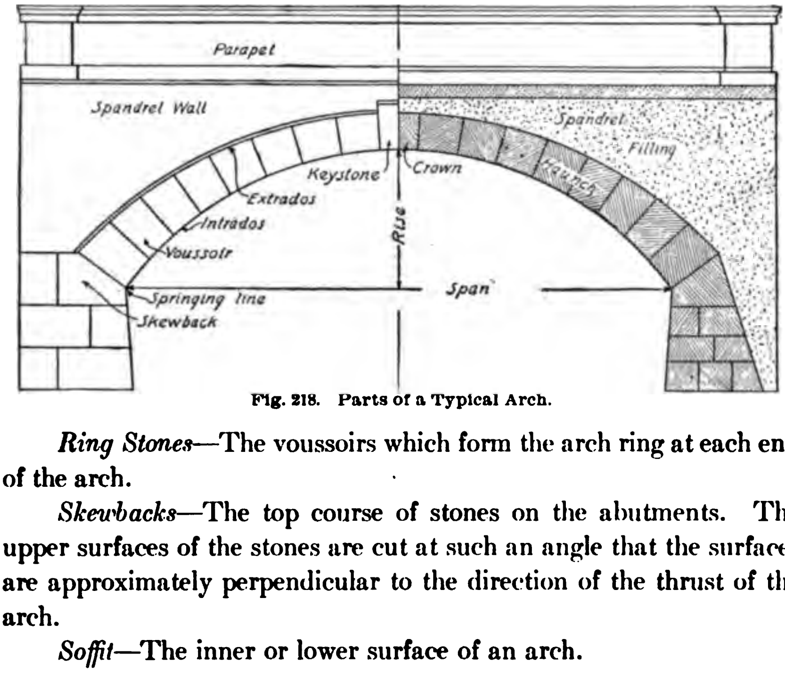


Fig. 218. Parts of a Typical Arch.

Ring Stones—The voussoirs which form the arch ring at each end of the arch.

Skewbacks—The top course of stones on the abutments. The upper surfaces of the stones are cut at such an angle that the surfaces are approximately perpendicular to the direction of the thrust of the arch.

Soffit—The inner or lower surface of an arch.

Span—The perpendicular distance between the two springing lines of an arch.

Spandrel—The space between the extrados of an arch and the roadway. The walls above the ring stones at the ends of the arch, are called *spandrel walls*. The material deposited between the spandrel walls and in this spandrel space, is called the *spandrel filling*.

Springer—The first arch stone above a skewback.

Springing Line—The upper (and inner) edge of the line of skewbacks on an abutment.

String Course—A course of voussoirs of the same width (perpendicular to the axis of the arch), which extends from one arch face to the other.

Voussoirs—The separate stones forming an arch ring.

KINDS OF ARCHES

403. Arches are variously described according to the shape of the intrados, and also according to the form of the soffit:

Basket-Handled Arch—One whose intrados consists of a series of circular arcs tangent to each other. They are usually *three-centered* or *five-centered*, as described below.

Catenarian Arch—One whose intrados is the mathematical curve known as a *catenary*.

Circular Arch—One whose intrados is the arc of a circle.

Elliptical Arch—One whose intrados is a portion of an ellipse.

Hydrostatic Arch—One whose intrados is of such a form that the equilibrium of the arch is dependent upon such a loading as would be made by water.

Pointed Arch—One whose intrados consists of two similar curves which meet at a point at the top of the arch.

Relieving Arch—An arch which is built above a lintel, which relieves the lintel of the greater portion of its load.

Right Arch—An arch whose soffit is a cylinder, and whose ends are perpendicular to the axis of the arch.

Segmental Arch—One whose intrados is a circular arc which is less than a semicircle.

Semicircular Arch—One whose intrados is a full semicircle. Such an arch is also called a *full-centered arch*.

Skew Arch—An arch whose soffit may or may not be cylindrical, but whose ends are not perpendicular to the axis of the arch. They are also called *oblique arches*.

VOUSSOIR ARCHES

404. **Definition.** A voussoir arch is an arch composed of separate stones, called *voussoirs,* which are so shaped and designed that the line of pressures between the stones is approximately perpendicular to the joints between the stones. So far as it affects the mechanics of the problem, it is assumed that the mortar in the joints between the voussoirs acts merely as a cushion, and that the mortar has no tensile strength whatever, even if the pressure at any joint should be such as to develop tensile action. It is this feature which constitutes the distinction between a voussoir arch and an elastic arch, which is assumed to be an arch of such material that tensile or transverse stresses may be developed.

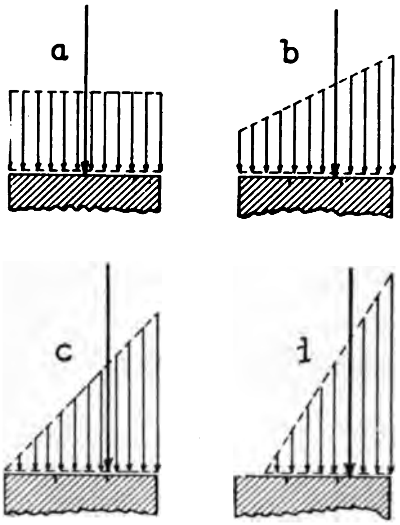


Fig. 219. Distribution of Pressure

405. **Distribution of the Pressure between Two Voussoirs.** The unit-pressure on any joint is assumed to vary in accordance with the location of the center of pressure, as is illustrated in Fig. 219. In the first case, where the center of pressure is over the center of the face of the joint and is perpendicular to it, the pressure will be uniformly distributed, and may be represented, as in Fig. 219*a*, by a series of arrows which are all made equal, thus representing equal unit-pressures. As the center of pressure varies from the center of the joint, the unit-pressure on one side increases and the unit-pressure on the other side decreases, as shown in Fig. 219 *b*. The trapezoid in this diagram has the same area as the rectangle of the first diagram (*a*), and the center of pressure passes through the center of gravity of the trapezoid. As the center of pressure continues to move away from the center of the joint, the unit-pressure on one side becomes greater, and on the other side less, until the center of pressure is at a point $\frac{1}{6}$ of the width of the joint away from the center. In this case (*c*), the center of pressure is at the extreme edge of the *middle third* of

the joint. The group of pressures illustrated in diagram *c* becomes a triangle, which means that the pressure at one side of the joint has become just equal to zero, and that the maximum pressure at the other side of the joint is twice the average pressure. If the line of pressure varies still further from the center of the joint, the diagram of pressures will always be a triangle whose base is always three times the distance of the center of pressure from the nearest edge of the joint. If the *total* pressure on that joint remains constant, then the intensity of pressure on one side of the joint becomes extreme, and may be sufficient to crush the stone. Also, since the elasticity of the stone (or of the mortar between the stones) will cause the stone (or mortar) to yield, the yielding being proportional to the pressure, the joint will *open* at the other side, where there is no pressure. In accordance with this principle of the distribution of pressure, it is always specified that a design for an arch cannot be considered safe unless it is possible to draw a line of pressure (an equilibrium polygon) which shall at every joint pass through the *middle third* of that joint. If the line of pressure at any joint does not pass through the middle third, it means that such a joint will inevitably open, and make a bad appearance, even though the unit-pressure on the other end of that joint is not so great that the masonry is actually crushed.

Since the actual crushing strength of stone is a rather uncertain and variable quantity, a larger factor of safety is usually employed with stone than with other materials of construction. This factor is usually made *ten;* and therefore, whenever the line of pressures passes through the edge of the middle third, the *average* unit-pressure on the joint should not be greater than $\frac{1}{20}$ of the crushing strength of the stone.

A table of these ultimate values has been given in Table I, Part I (page 10). They vary from about 3,000 pounds per square inch, for a sandstone found in Colorado, up to 28,000 pounds per square inch for a granite found in Minnesota. The weaker stone would hardly be selected for any important work. Usually a stone whose ultimate strength is 10,000 pounds per square inch or more, would be selected for a stone arch. Such a stone could be used with a working pressure of 500 pounds per square inch at any joint, assuming that the line of pressure does not pass outside of the middle third at any joint.

406. External Forces Acting on an Arch. There is always some uncertainty regarding the actual external forces acting on ordinary arches. The ordinary stone arch consists of a series of voussoirs, which are overlaid usually with a mass of earth or cinders having a depth of perhaps several feet, on top of which may be the pavement of a roadway. The spandrel walls over the ends of the arch, especially when made of squared stone masonry, also develop an arch action of their own which materially modifies the loading on the arch rings. As this, however, invariably assists the arch, rather than weakens it, no modification of plan is essential on this account. The actual pressure of the earth filling, together with that caused by the live load passing over the arch, on any one stone, is uncertain in very much the same way as the pressure on a retaining wall is uncertain, as previously explained.

The simplest plan is to consider that each voussoir is carrying a load of earth equal to that indicated by lines from the joints in the voussoir vertically upward to the surface. The development of the graphical method makes it more convenient to draw what is called a *reduced load line* on top of the arch, in which the depth of earth above the arch is reduced in the ratio of the relative weights per cubic foot of the earth filling and of the stone of which the arch is made (see Fig. 220). Even the live load on the arch is represented in the same manner, by an additional area on top of the reduced line for the earth pressure, the depth of that area being made in proportion to the intensity of the live load compared with the unit-weight of stone. For example, if the earth filling weighs 100 pounds per cubic foot, and the stone of the arch weighs 160 pounds per cubic foot, then each ordinate for the earth load would be $\frac{100}{160}$ of the actual depth of the earth. Likewise, if the live load per square foot on the arch equals 120 pounds, then the area representing the live load would be $\frac{120}{160}$ of a foot, according to the scale adopted for the arch. The weight of the paving, if there is any, should be similarly allowed for. If we draw from the upper end of each joint a vertical line extending to the top of the reduced load line, then the area between

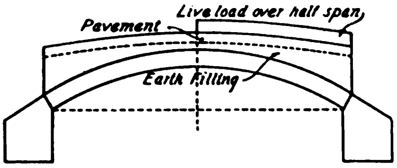


Fig. 220. Determination of Reduced Load Line.

line must intersect the point z', which is on a horizontal line from y. The student should make some such drawing as here described, and should demonstrate for himself the accuracy of this law. This equilibrium polygon is merely one of an infinite number which, if acting as struts, would hold these forces in equilibrium, but it combines the special condition that it shall pass through the points y and z'. There are also an infinite number of equilibrium polygons which will hold these forces in equilibrium and which will pass through the points y and z'.

We may also impose another condition, which is that the first line of the equilibrium polygon shall have some definite direction, such as yl. In this case the ray from the point p of the force diagram must be parallel to yl; and where this line intersects the horizontal line no' (produced in this case), is the required position for the pole o''. Draw rays from o'' to s, r, and q, continuing the equilibrium polygon by lines which are respectively parallel to these rays. As a check on the work, the last line of the equilibrium polygon which is parallel to $o''q$ should intersect the point z'. The triangles ykh and $o'pn$ have their sides respectively parallel to each other, and the triangles are therefore similar, and their corresponding sides are proportional, and we may therefore write the equation:

$$o'n : yh :: pn : kh.$$

Also, from the triangles ylh and $o''pn$, we may write the proportion:

$$o''n : yh :: pn : lh.$$

From these two proportions we may derive the proportion:

$$o'n : o''n :: lh : kh;$$

but $o'n$ and $o''n$ are the pole distances of their respective force diagrams, while kh and lh are intercepts by a vertical line through the corresponding equilibrium polygons. The proportion is therefore a proof, in at least a special case, of the general law that the perpendicular distances from the poles to the load lines of any two force diagrams are inversely proportional to any two intercepts in the corresponding equilibrium polygons. The above proportions prove the theorem for the intercepts hk and hl. A similar combination of proportions would prove it for any vertical intercept between y and h. The proof of this general theorem for intercepts which pass through other lines of the sions of the voussoirs to the span of the arch, the errors involved by the approximations are harmless, while the additional labor necessary for a more accurate solution would not be justified by the inappreciable difference in the final results.

407. **Depth of Keystone.** The proper depth of keystone for an arch should theoretically depend on the total pressure on the keystone of the arch as developed from the force diagram; and the depth should be such that the unit-pressure shall not be greater than a safe working load on that stone. But since we cannot compute the stresses in the arch, until we know, at least approximately, the dimensions of the arch and its thickness, from which we may compute the dead weight of the arch, it is necessary to make at least a trial determination of the thickness. The mechanics of such an arch may then be computed, and a correction may subsequently be made, if necessary. Usually the only correction which would be made would be to increase the thickness of the arch, in case it was found that the unit-pressure on any voussoir would become dangerously high. Trautwine's Handbook quotes a rule which he declares to be based on a very large number of cases that were actually worked out by himself, the cases including a very large range of spans and of ratios of span to rise. The rule is easily applied, and is sufficiently accurate to obtain a trial depth of the keystone. It will probably be seldom, if ever, that the depth of the keystone, as determined by this rule, would need to be altered. The rule is as follows:

$$\text{Depth of Keystone, in feet} = \frac{\sqrt{\text{Rad.} + \text{half-span}}}{4} + 0.2 \text{ foot} \quad (47)$$

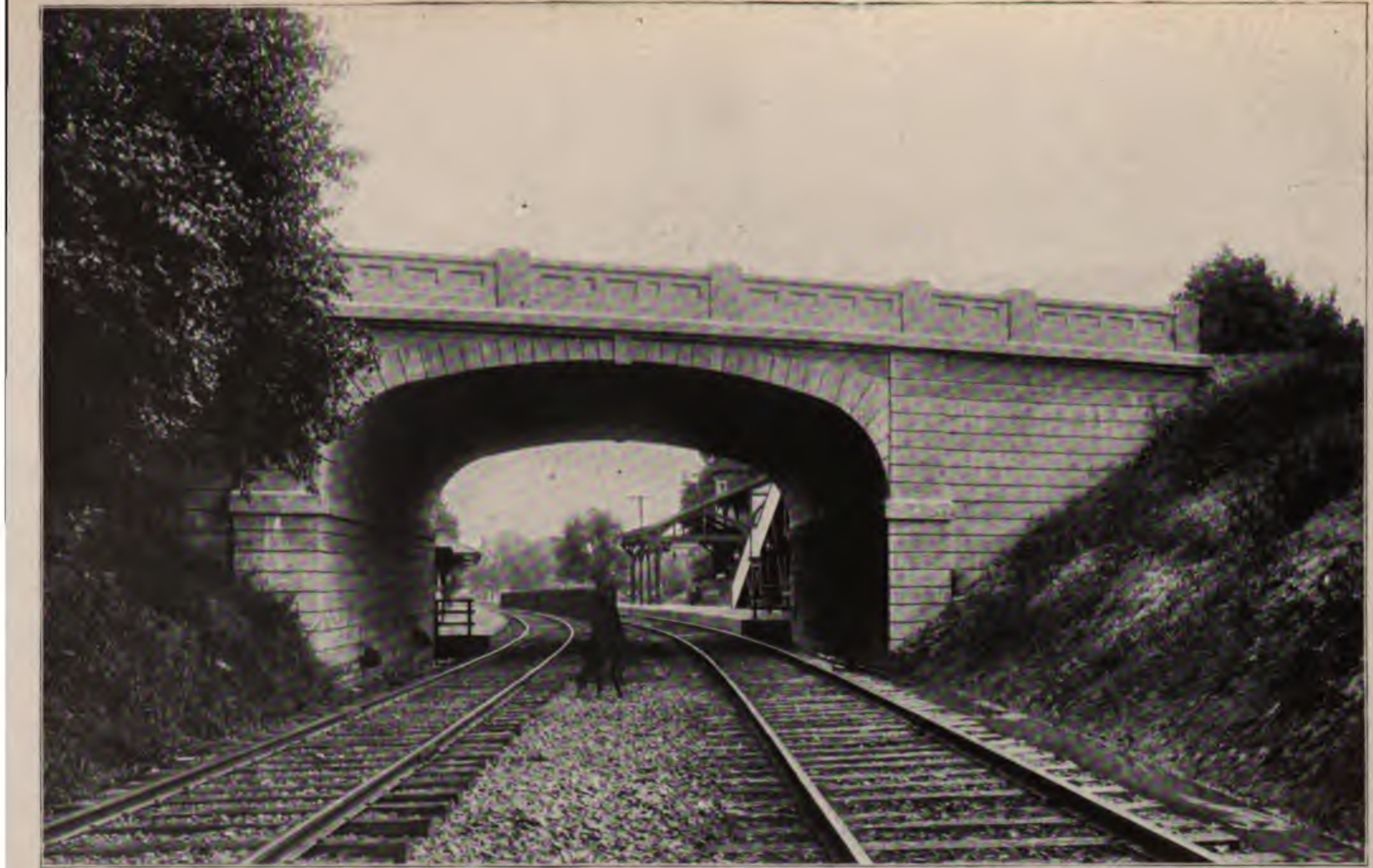
For architectural reasons, the actual keystone of an arch is usually made considerably deeper than the voussoirs on each side of it, as illustrated in Fig. 218. When computing the maximum permissible pressure at the crown, the actual depth of the voussoirs on each side of the keystone is used as the depth of the keystone; or perhaps it would be more accurate to say that the extrados is drawn as a regular curve over the keystone (as illustrated in Fig. 223), and then any extra depth which may subsequently be given to the keystone should be considered as mere ornamentation and as not affecting the mechanics of the problem.

408. **Numerical Illustration.** The above principles will be applied to the case of an arch having a span of 20 feet and a rise of

librium polygon, the rays in the force diagram being drawn through *any* point chosen as a pole. Then, if we draw a line from the trial pole which shall be parallel with the closing line of this trial equilibrium polygon, the line will intersect the load line in the point n. Drawing a horizontal line from the point n in the load line, we have the *locus* of the pole of the desired special equilibrium polygon. Then draw a vertical through the point through which the special equilibrium polygon is to pass. The vertical distance of this point above the line joining the abutments, is the required intercept of the true equilibrium polygon. The intersection of that vertical with the upper line and the closing line of the trial equilibrium polygon, is the intercept of the trial polygon. The pole distance of the true equilibrium polygon is then obtained by the application of Equation **46**, by which the pole distances are declared inversely proportional to any two corresponding intercepts of the equilibrium polygons.

Another useful property, which will be utilized later, and which may be readily verified from Figs. 215 and 216, is that, no matter what equilibrium polygon may be drawn, the two extreme lines of the equilibrium polygon, if produced, intersect in the resultant R; therefore, when it is desired to draw an equilibrium polygon which shall pass through any two abutment points, such as yz or yz', we may draw from these two abutment points, two lines which shall intersect at any point on the resultant R. We may then draw two lines which will be respectively parallel to these lines from the extremities p and q of the load lines, their intersection giving the pole of the corresponding force diagram.

401. **Equilibrium Polygon for Non-Vertical Forces.** The above method is rendered especially simple, owing to the fact that the forces are all vertical. When the forces are not vertical, the method becomes more complicated. The principle will first be illustrated by the problem of drawing an equilibrium polygon which shall pass through the points y, z, and v in Fig. 217. We shall first draw the two non-vertical forces in the force diagram. The resultant R of the forces A and B is obtained as shown in Fig. 213. Utilizing the property referred to in the previous article, we may at once draw two lines through y and z which intersect at some assumed point e on the resultant R. Drawing lines from p and q parallel respectively to ez and ey, we determine the point o' as the trial pole for our force



CONCRETE ARCH AT GRAVER'S LANE, PHILADELPHIA, PENNSYLVANIA

Courtesy of Geo. S. Webster, Chief Engineer, Bureau of Surveys, Dept. of Public Works.

librium polygon, the rays in the force diagram being drawn through *any* point chosen as a pole. Then, if we draw a line from the trial pole which shall be parallel with the closing line of this trial equilibrium polygon, the line will intersect the load line in the point n. Drawing a horizontal line from the point n in the load line, we have the *locus* of the pole of the desired special equilibrium polygon. Then draw a vertical through the point through which the special equilibrium polygon is to pass. The vertical distance of this point above the line joining the abutments, is the required intercept of the true equilibrium polygon. The intersection of that vertical with the upper line and the closing line of the trial equilibrium polygon, is the intercept of the trial polygon. The pole distance of the true equilibrium polygon is then obtained by the application of Equation **46**, by which the pole distances are declared inversely proportional to any two corresponding intercepts of the equilibrium polygons.

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the scaled length of the perpendicular which is midway between the verticals. The weight of the voussoir itself may be computed as accurately as necessary, by multiplying the radial thickness by the length between the joints as measured on the curve lying half-way between the intrados and the extrados.

For example, the load for full loading of the arch which is over voussoir No. 1, is measured as follows: The width between the perpendiculars is 2.0 feet; the height measured on the middle vertical is 4.05 feet; the area is therefore 8.10 feet, which, multiplied by 160, equals 1,296 pounds, which is the load on this voussoir for every foot of width of the arch parallel with the axis. The radial thickness of voussoir No. 1 is 1.90 feet, and the length is 2.15 feet; this gives an area of 4.085 feet, which, multiplied by 160, equals 653.6 pounds. The weight of the voussoir is therefore almost exactly one-half that of the live and dead loads above it; therefore the resultant of these two weights will be almost precisely one-third of the distance between the center of this stone and the vertical through the center of the loading. By drawing this line, we have the line of action of the resultant of these two forces, and this value is the sum of 1,296 and 654, or 1,950 pounds.

In order to simplify the figure, the arrows representing the lines of force of the loading on the voussoir and the weight of the voussoir have been omitted from the figure, and only their resultant is drawn in. It was of course necessary to draw in these forces in pencil and obtain the position of the resultant, as explained in Fig. 221; and then, for simplicity, only the resultant was inked in.

The loads on the other voussoirs are computed similarly. The numerical values for the loads on the various voussoirs (including the weights of the voussoirs), are tabulated as follows:

FIRST CONDITION OF LOADING

Voussoir No.	Load	Weight of Voussoir	Total
1 and 11	1,296	654	1,950
2 " 10	1,135	592	1,727
3 " 9	1,010	528	1,538
4 " 8	927	483	1,410
5 " 7	880	456	1,336
6	867	455	1,322

For this first condition of loading, the total loads for voussoirs Nos. 7, 8, 9, 10, and 11 will be the same as those for voussoirs 5, 4, 3, 2, and 1 respectively.

The loads for the second condition of loading are found by using the same load on the first five voussoirs, but with only half of the live load on voussoir No. 6, which means that the load for the first condition of loading (1,322 pounds) is reduced by 200 pounds, making it 1,122 pounds. Voussoirs Nos. 7 to 11 are each reduced by 400 pounds. The total load for each voussoir is as tabulated below.

The loads for the third condition of loading are found by using the same loads as were employed for the second condition, except that for voussoirs Nos. 3 and 4, 1,600 pounds should be added to each load. These loads are also tabulated below:

SECOND CONDITION OF LOADING		THIRD CONDITION OF LOADING	
VOUSSOIR NO.	TOTAL LOAD	VOUSSOIR NO.	TOTAL LOAD
1	1,950	1	1,950
2	1,727	2	1,727
3	1,538	3	3,138
4	1,410	4	3,010
5	1,336	5	1,336
6	1,122	6	1,122
7	936	7	936
8	1,010	8	1,010
9	1,138	9	1,138
10	1,327	10	1,327
11	1,550	11	1,550

Fig. [illegible] was originally drawn at the scale of ½ inch = 1 foot, and with the force diagram at the scale of [illegible] pounds per inch. The photographic reproduction has of course changed these scales somewhat. The student should redraw the figure at these scales, and should obtain substantially the same final results.

[illegible] **Drawing the Load Line for the First Condition of Loading.** When the live load is uniformly distributed over the entire arch, the load is symmetrical and we need to consider only one-half of the arch. The [illegible] of the load line for the force diagram corresponding to this condition of loading [illegible] in Article 397. [illegible] to be entirely [illegible] are at the same elevation, we need only draw a horizontal line from the lower end of the *half*-load line, and select on it a trial position (o_1) for the pole, drawing the rays as previously explained; the trial equilibrium polygon passes through the center vertical at the point a'. Drawing a horizontal line from a' until it intersects the first line (produced) of the trial equilibrium polygon, and drawing through it a vertical line, we have the line of action of the resultant (R_1) of all the forces on that half of the arch. If we draw through a, the center of the keystone, a horizontal line, its intersection with R_1 gives a point in the first line (produced) of the true equilibrium polygon. A line from the upper end of the load line parallel to this first section of the true equilibrium polygon, intersects the horizontal line through the middle of the load line at o_1', which is the position of the true pole. Drawing the rays from the true pole to the load line, and drawing the segments of the true equilibrium polygon parallel to these rays, we may at once test whether the true equilibrium polygon always passes through the middle third of each joint. As is almost invariably the case, it is found that for full loading, the true equilibrium polygon passes within the middle third at every joint.

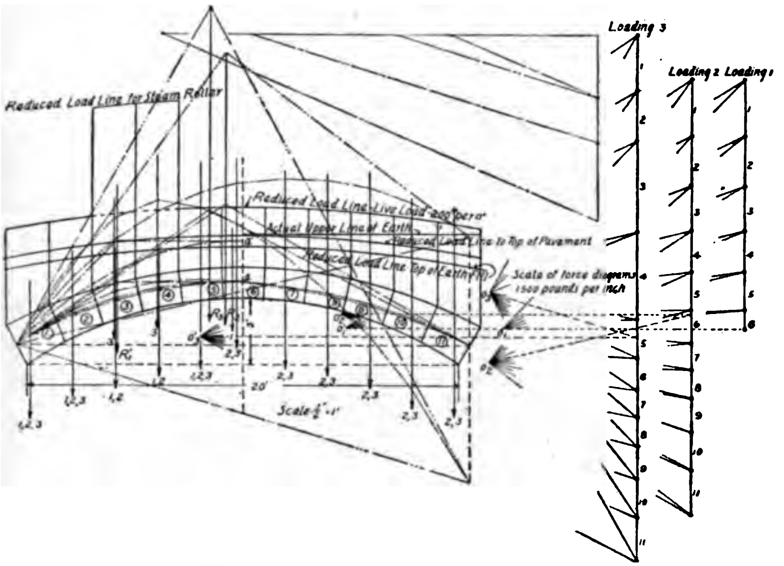
Loading 3
Loading 2
Loading 1
Reduced Load Line for Steam Roller
Reduced Load Line–Live Load 200# per □'
Actual Upper Line of Earth
Reduced Load Line to Top of Pavement
Reduced Load Line Top of Earth Fill
Scale of force diagrams 1500 pounds per inch
20'
Scale ½" = 1'
R₁
1,2,3
1,2
2,3

Fig. 223. Stresses in a 20-Foot Arch.
Reproduced from an original drawn at scale of ½ inch = 1 foot.

The student should carefully check over all these calculations, drawing the arch at the scale of one-half inch to the foot, and the load line of the force diagram at the scale of 1,500 pounds per inch; then the rays of the true equilibrium polygon will represent at that scale the pressure at the joints. Dividing the total depth of any joint by the pressure found at that joint, gives the *average* pressure. In the case of the joint at the crown, the total pressure at the joint is 13,900 pounds. The depth of the joint is 1.5 feet, and the area of the joint is 216 square inches; therefore the average unit-pressure is 64 pounds per square inch; if it is assumed that the line of pressure passes through either edge of the middle third, then the pressure at the edge of the joint is twice the average, or is 128 pounds per square inch. This is a very low pressure for any good quality of building stone.

Similarly, the maximum pressure at the skewback is scaled from the force diagram as 16,350 pounds; but since the arch is here two feet thick, and the area is 288 square inches, it gives an average pressure of 57 pounds per square inch. Since this equilibrium polygon is supposed to start from the center of this joint, this represents the actual pressure.

Usually it is only a matter of form to make the test for uniform full loading. Eccentric loading nearly always tests an arch more severely than uniform loading. The ability to carry a full uniform load is no indication of ability to carry a partial eccentric loading, except that if the arch appeared to be only just able to carry the uniform load, it might be predicted that it would probably fail under the eccentric load. On the other hand, if an arch will safely carry a heavy eccentric load, it will certainly carry a load of the same intensity uniformly distributed over it.

411. **Test for the Second Condition, or Loading of Maximum Load over One-Half of the Arch.** Since the arch has a dead load over the entire arch, and a live load over only one-half of the arch, the load line for the entire arch must be drawn. The load line for the loaded half of the arch will be identical with that already drawn for the previous case. The load line for the remainder of the arch may be similarly drawn. This case is worked out by precisely the same general method as that already employed in the similar case given in detail in Article 410. As in that article, we select a trial pole

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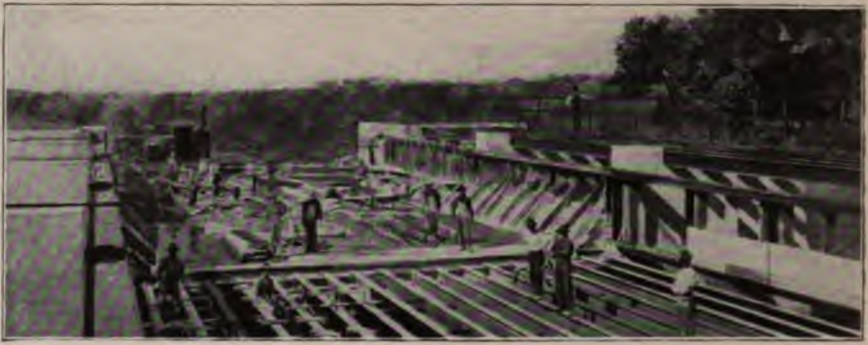
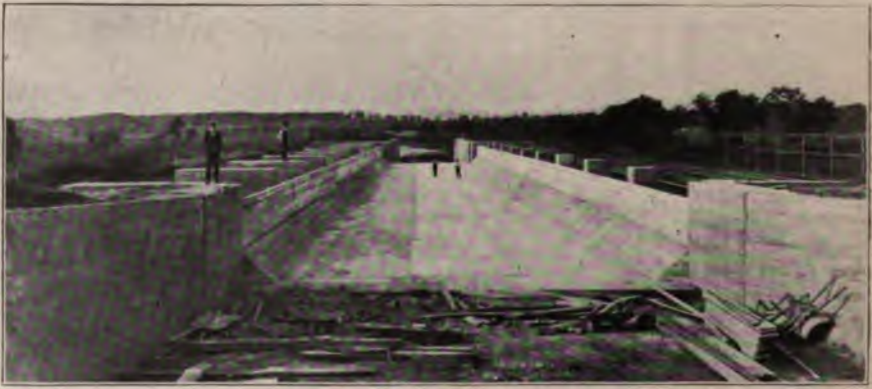
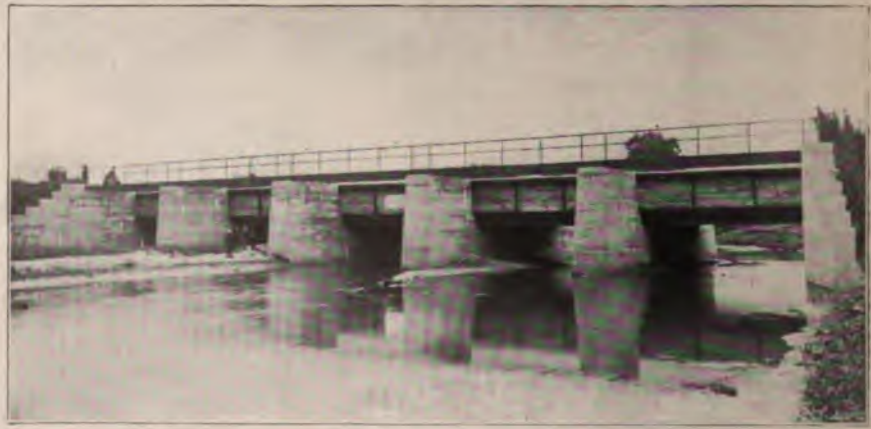


Depositing Concrete for Lining of Aqueduct No. 7.



Lining of Aqueduct, Completed.



View of Completed Aqueduct.

CONSTRUCTION OF AN AQUEDUCT ON LINE OF ILLINOIS AND MISSISSIPPI CANAL which in general will give an oblique closing line for the equilibrium polygon. This closing line must be brought down to the horizontal by the method already explained in Article 400; then a second trial must be made in order to shift the polygon so that it shall pass through the middle third at the crown joint. This line should pass through the middle of the crown joint; then the real test is to determine how it passes through the haunches of the arch. As in the previous case, the total pressure at any joint will be determined by the corresponding lines in the force diagram, and the unit-pressure at the joint may be determined from the area of the joint and the position of the line of force with respect to the center of the joint. Even though a line of force passed slightly outside of the middle third, it would not necessarily mean that the arch will fail, provided that the maximum intensity of pressure, determined according to the principles enunciated in Article 405, does not exceed the safe unit-pressure for the kind of stone used.

An inspection of the force diagram with the pole at o_2', shows that the rays are all shorter than those of the force diagram for the first condition of loading—with pole at o_1'. This means that the actual pressure at any joint is less than for the first case; but since the true equilibrium polygon for this case does not pass so near the center of the joints as it does for the first condition of loading, the intensity of pressure at the edges of the joints may be higher than in the first case. However, since the equilibrium polygon for this second case is always well within the middle third at every joint, and since even twice the average joint pressure for the first case is well within the safe allowable pressure on any good building stone, we may know that the second condition of loading will be safe, even without exactly measuring and computing the maximum intensity of pressure produced by this loading.

412. Test for the Third Condition, Involving Concentrated Load. The method of making this test is exactly similar to that previously given; but on account of a load eccentrically placed, the force diagram will be more distorted than in either of the cases previously given, and there is greater danger that the arch will prove to be unstable on such a test. An inspection of the equilibrium polygon for this case shows that the critical point is the joint between voussoirs Nos. 3 and 4. This is what might be expected, since it is the joint under the heavy concentrated load. The ray in the force diagram which is parallel to the section of the equilibrium polygon passing through this joint, is the ray which reaches the load line between loads 3 and 4. This ray, measured at the scale of 1,500 pounds per square inch, indicates a pressure of 15,625 pounds on the joint. The line of pressure is 4¾ inches from the upper edge of the joint; it is outside of the middle third; and therefore the joint will probably open somewhere under this loading. According to the theory of the distribution of pressure over a stone joint, the pressure will be maximum on the upper edge of this joint, and will be zero at three times 4¾ inches, or 14.25 inches, from the upper edge. The area of pressure for a joint 12 inches wide will be 14.25 × 12 = 171 square inches. Dividing 171 into 15,625, we have an *average* pressure of 91 pounds, or a maximum pressure of twice this, or 182 pounds, per square inch at the edge of the joint. But this is such a safe working pressure for such a class of masonry as cut-stone voussoirs, that the arch certainly would not fail, even though the elasticity of the stone caused the joint to open slightly at the intrados during the passage of the steam roller.

413. Correcting a Design. The above general method of testing an arch consists of first designing the arch, and then testing it to see whether it will satisfy all the required conditions. In case some condition of loading is found which will cause the line of pressure to pass outside of the middle third or to introduce an excessive unit-pressure in the stones, it is theoretically necessary to begin anew with another design, and to make all the tests again on the basis of a new design; but it is usually possible to determine with sufficient closeness just what alterations should be made in the design so that the modified design will certainly satisfy the required conditions. For example, if the line of pressure passes on the upper side of the middle third at the haunches of the arch, a thickening of the arch at that point until the line of pressure is within the middle third of the revised thickness, will usually solve the difficulty. The effect of the added weight on the haunch of the arch will be to make the line of pressure move upward slightly; but the added thickness can allow for this. As another illustration, the unit-pressure, as determined for the crown of the arch, might be considerably in excess of a safe pressure for the arch, and it might indicate a necessity to thicken the arch, not only at the center, but also throughout the length of the arch.

For example, in the above numerical case, although it is probably not really necessary to alter the design, the arch might be thickened on the haunches, say 3 inches. This would add to the weight on the haunches one-fourth of the *difference* of the weights per cubic foot of stone and earth, or $\frac{1}{4}(160 - 100) = 15$ pounds per square foot. This is so utterly insignificant compared with the actual total load of about 750 pounds per square foot, that its effect on the line of pressure is practically inappreciable, although it should be remembered that the effect, slight as it is, will be to raise the line of pressure. A thickening of 3 inches will leave the line of pressure nearly $7\frac{3}{4}$ inches (or say $7\frac{1}{2}$ inches, to allow generously for the slight raising of the line of pressure) from the extrados, while the thickness of the arch is increased from 19 inches to 22 inches. But the line of pressure would now be within the middle third.

414. **Location of True Equilibrium Polygon.** In the above demonstration, it is assumed that the true equilibrium polygon will pass through the center of each abutment, and also through the center of the keystone; and the test then consists in determining whether the equilibrium polygon which is drawn through these three points will pass within the middle third at every joint, or at least whether it will pass through the joints in such a way that the maximum intensity of pressure at either edge of the joint shall not be greater than a safe working pressure. With any system of forces acting on an arch, it is possible to draw an infinite number of equilibrium polygons; and then the question arises, which polygon, among the infinite number that can be drawn, represents the true equilibrium polygon and will represent the actual line of pressure passing through the joints. On the general principle that forces always act along the line of least resistance, the pressure acting through any voussoir would tend to pass as nearly as possible through the center of the voussoir; but since the forces of an equilibrium polygon, which represent a *combination* of lines of pressure, must all act *simultaneously*, it is evident that the line of pressure will pass through the voussoirs by a course which will make the summation of the intensity of pressures at the various joints a minimum. It is not only possible but probable that the true equilibrium polygon does *not* pass through the center of the keystone, but at some point a little above or below,

through which a polygon may be drawn which will give a less summation of pressures than those for a polygon which does pass through the point *a*. The value and safety of the method given above, lie in the fact that the true equilibrium polygon always passes through the voussoirs in such a way that the summation of the intensities of the pressures is the least possible combination of pressures; and therefore any polygon which can be drawn through the voussoirs in such a way that the pressures at all the joints are safe, merely indicates that the arch will be safe, since the true combination of pressures is something less than that determined. In other words, the true system of pressures is never greater, and is probably less, than the system as determined by the equilibrium polygon which is assumed to be the true polygon.

When an equilibrium polygon for eccentric loading passes through the arch at some distance from the center of the joint at one part of the arch, and very near the center of the joint in all other sections, it can be safely counted on, that the true polygon passes a little nearer the center at the most unfavorable portion, and a little further away from the center at some other joints where there is a larger margin of safety. For example, the true equilibrium polygon for the third condition of loading (see Fig. 223) probably passes a little nearer the center on the left-hand haunch, and a little farther away from the center on the right-hand haunch, where there is a larger margin; in other words, the whole equilibrium polygon is slightly lowered throughout the arch. No definite reliance should be placed on this allowance of safety; but it is advantageous to know that the margin exists, even though the margin is very small. The margin, of course, would reduce to zero in case the equilibrium polygon chosen actually represented the true equilibrium polygon. While it would be convenient and very satisfactory to be able to obtain always the true equilibrium polygon, it is sufficient for the purpose to obtain a polygon which indicates a safe condition when we know that the true polygon is still safer.

415. **Design of Abutments.** The force diagram of Fig. 223, which shows the pressures between the voussoirs of the arch, also gives, for any condition of loading, the pressure of the last voussoir against the abutment. A glance at the diagram shows that the maximum pressure against the abutment comes against the left-hand

abutment under the third condition of loading, when the concentrated load is on the left-hand side of the arch. Although the first condition of loading does not create so great a pressure against the left-hand abutment, yet the angle of the line of pressure is somewhat flatter, and this causes the resultant pressure on the base of the abutment to be slightly nearer the rear toe of the abutment. It is therefore necessary to consider this case, as well as that of the third condition of loading.

An abutment may fail in three ways: (1) by sliding on its foundations; (2) by tipping over; and (3) by crushing the masonry. The possibility of failure by crushing the masonry at the skewback may be promptly dismissed, provided the quality of the masonry is reasonably good, since the abutment is always made somewhat larger than the arch ring, and the unit-pressure is therefore less. The possibility of failure by the crushing of the masonry at the base, owing to an intensity of pressure near the rear toe of the abutment, will be discussed below. The possibility that the abutment may slide on its foundations is usually so remote that it hardly need be considered. The resultant pressure of the abutment on its subsoil is usually nearer to the perpendicular than the angle of friction; and in such a case, there will be no danger of sliding, even if there is no backing of earth behind the abutment, such as is almost invariably found.

The test for possible tipping over or crushing of the masonry due to an intensity of pressure near the rear toe, must be investigated by determining the resultant pressure on the subsoil of the abutment. This is done graphically by the method illustrated in Fig. 224. This is an extension of the arch problem already considered. The line *bc* gives the angle of the skewback at the abutment, while the lines of force for the pressures induced by the first and third conditions of loading have been drawn at their proper angle. In common with the general method used in designing an arch, it is necessary to design first an abutment which is assumed to fulfil the conditions, and then to test the design to see whether it is actually suitable. The cross-section *abcde* has been assumed as the cross-section of solid masonry for the abutment. The problem therefore consists in finding the amount and line of action of the force representing the weight of the abutment. It will be proved that this force passes through the point o_5, and it therefore intersects the pressure on the abutment for the first condition of loading, at the point *k*. The weight of a section of the abutment one foot thick (parallel with the axis of the arch), is computed (as detailed below) to weigh 19,500 pounds, while the pressure of the arch is scaled from Fig. 223 as 16,350 pounds. Laying off these forces on these two lines at the scale of 5,000 pounds per inch, we have the resultant, which intersects the base at the point *m*, and which scales 31,350 pounds. Similarly, the resultant of the weight of the abutment and the line of pressure for the third condition of loading intersects the base at the point *n*, and scales 33,600 pounds. These pressures on the base will be discussed later.

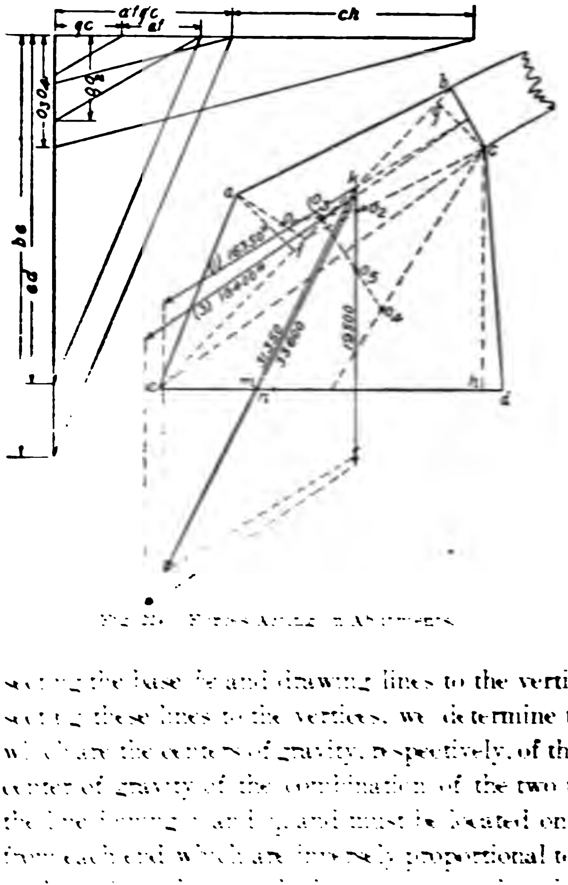


Fig. [illegible] Abutments

The line of action and the amount of the weight of a unit-section of the abutment, are determined as follows: The center of gravity of the pentagon *abcde* is determined by dividing the pentagon into three elementary triangles, *abe*, *bce*, and *cde*. We may consider *be* as a base which is common to the triangles *abe* and *bce*. By bisecting the base *be* and drawing lines to the vertices *a* and *c*, and trisecting these lines to the vertices, we determine the points o_1 and o_2, which are the centers of gravity, respectively, of the two triangles. The center of gravity of the combination of the two triangles must lie on the line joining o_1 and o_2, and must be located on the line at distances from each end which are inversely proportional to the areas of the triangles. Since the triangles have a common base *be*, their areas are proportional [illegible]. In the diagram at the side, we may lay off in succession on the horizontal line the distances *gc* and *af*.

On the vertical line, we lay off a distance equal to o_1o_2. By joining the lower end of this line with the right-hand end of the line *af*, and then drawing a parallel line from the point between *gc* and *af*, we have divided the distance o_1o_2 into two parts which are proportional to the two altitudes *af* and *gc*. Laying off the shorter of these distances toward the triangle *abe* (since its greater altitude shows that it has the greater area), we have the position of o_3, which is the center of gravity of the two triangles combined. The area *abce* is measured by one-half the product of *eb* and the sum of *af* and *gc*. The triangle *cde* is measured by one-half the product of the base *ed* by the altitude *ch*. If we lay off *bc* as a vertical line in the side diagram, and also the line *ed* as a vertical line, and join the lower end of *ed* with the line which represents the sum of *gc* and *af*, and then draw a line from the lower end of *be*, parallel with this other line, we have two similar triangles from which we may write the proportion:

$$ed : (gc + af) :: be : a'f'g'c'.$$

Since the product of the means equals the product of the extremes, we find that $(gc + af) \times be = ed \times a'f'g'c'$; but $\frac{1}{2}(gc + af) \times be =$ the combined area of the two triangles, and therefore the line $a'f'g'c'$ is the height of an *equivalent* triangle whose base equals *ed*; therefore the area of these two combined triangles is to the area of the triangle *cde* as the equivalent altitude $a'f'g'c'$ is to the altitude *ch* of the triangle *cde*. By bisecting the base *ed*, and drawing a line from the bisecting point to the point *c*, and trisecting this line in the point o_4, we have the center of gravity of the triangle *cde*. The center of gravity of the entire area, therefore, lies on the line o_3o_4, and at a distance from o_4 which is inversely proportional to the areas of the two combined triangles and the triangle *cde*. These areas are proportional to the altitudes as determined above; therefore, by laying off in the side diagram the line o_3o_4, and drawing a line from its lower extremity to the right-hand extremity of the line *ch*, and then drawing a parallel line from the point between $a'f'g'c'$ and *ch*, we divide the line o_3o_4 into two parts which are proportional to these altitudes. The line *ch* is the greater altitude, and the triangle *cde* has the greater area; therefore the point o_5 is nearer to the point o_4 than it is to the point o_3, and the shorter of these two sections is laid off from the point o_4. This gives the point o_5, which is the center of gravity of the entire area of the abutment.

The actually computed weight of a unit-section of the abutment is determined by multiplying the sum of $a'f'g'c'$ and ch by the base ed. Since this masonry is assumed to weigh 160 pounds per cubic foot, the product of these scaled distances, measured at the scale of ½ inch equals one foot (which was the scale adopted for the original drawing), shows that the section one foot thick has a weight of 19,500 pounds. Laying off this weight from the point k, and laying off the pressure for the first condition of loading, 16,350 pounds, at the scale of 5,000 pounds per inch, and forming a parallelogram on these two lines, we have the resultant of 31,350 pounds as the pressure on the base of the abutment, that pressure passing through the point m.

The intersection of the weight of the abutment with the line of pressure for the third condition of loading, is a little below the point k; and we similarly form a parallelogram which shows a resulting pressure of 33,600 pounds, passing through the base at the point n. It is usually required that such a line of pressure shall pass through the middle third of the abutment; but there are other conditions which may justify the design, even when the line of pressure passes a little outside of the middle third.

The point n is 2.85 feet from the point e. According to the theory of pressures enunciated in Article 405, it may be considered that the pressure is maximum at the point e, and that it extends backward toward the point d for a distance of three times en, or a distance of 8.55 feet. This would give an average pressure of 3,930 pounds per square foot, or, since the pressure at the toe is twice the average pressure, 7,860 pounds per square foot on the toe. Such a pressure might or might not be greater than the subsoil could endure without yielding. Since this pressure is equivalent to about 55 pounds per square inch, there should be no danger that the masonry itself would fail; and, if the subsoil is rock or even a hard, firm clay, there will be no danger in trusting such a pressure on it.

Another very large item of safety which has been utterly ignored, but which would unquestionably be present, is the pressure of the earth back of the abutment. The effect of the back-pressure of the earth would be to make the line which represents the resultant pressure on the subsoil more nearly vertical, and to make it pass much more nearly through the center of the base ed. This would very much reduce the intensity of pressure near the point e, and would reduce very materially the unit-pressure on the subsoil. Cases, of course, are conceivable, in which there might be no back-pressure of earth against the rear of the abutment. In such cases, the ability of the subsoil to withstand the unit-pressure at the rear toe of the abutment (near the point *e*) must be more carefully considered. In order that the investigation shall be complete, it should be numerically determined whether the lower pressure, 31,350 pounds, passing through the point *m*, might produce a greater intensity of pressure at the point *e* than the larger pressure passing through the point *n*.

416. **Various Forms of Abutments.** The abutment described above is the general form which is adopted very frequently. The front face *cd* is made with a batter of one in twelve. The line *ba* slopes backward from the arch on an angle which is practically the continuation of the extrados of the arch. The total thickness of the abutment *de* must be such that the line of pressure will come nearly, if not quite, within the middle third. The line *ea* generally has a considerable slope, as is illustrated. When the subsoil is very soft, so that the area of the base is necessarily very great, the abutment is sometimes made hollow, with the idea of having an abutment with a very large area of base, but which does not require the full weight of so much masonry to hold it down; and therefore economy is sought in the reduction of the amount of masonry. Since such a hollow abutment would require a better class of masonry than could be used for a solid block of masonry, it is seldom that there is any economy in such methods. Since the abutment of an arch invariably must withstand a very great lateral thrust from the arch, there is never any danger that the resultant pressure of the abutment on the subsoil will approach the front toe of the arch, as is the case in the abutment of a steel bridge, which has little or no lateral pressure from the bridge, but which is usually subjected to the pressure of the earth behind it. These questions have already been taken up under the subject of abutments for truss bridges, in Part II.

VOUSSOIR ARCHES SUBJECTED TO OBLIQUE FORCES

417. **Determination of Load on a Voussoir.** The previous determinations have been confined to arches which are assumed to be acted on solely by vertical forces. For flat segmental arches, or

even for elliptical arches where the arch is very much thickened at each end so that the virtual abutment of the arch is at a considerable distance above the nominal springing line, such a method is sufficiently accurate, and it has the advantage of simplicity of computation; but where the arch has a very considerable rise in comparison with its span, the pressure on the extrados, which is presumably perpendicular to the surface of the extrados, has such a large horizontal component that the horizontal forces cannot be ignored. The method of determining the amount and direction of the force acting on each voussoir, is illustrated in Fig. 225. The reduced load line, found as previously described, is indicated in the figure. A trapezoid represents the loading resting on the voussoir *ac*. The line *df* represents, at some scale, the amount of this vertical loading. Drawing the line *de* perpendicular to the extrados *ac*, we may complete the rectangle on the line *df*, and obtain the horizontal component, while the equivalent normal pressure on the voussoir is represented by *de*.

Drawing a vertical line through the center of gravity of the voussoir, and producing it (if necessary) until it intersects *ed* in the point *v*, we may lay off *vw* to represent, at the same scale, the weight of the voussoir. Making *vs* equal to *de*, we find *vt* as the resultant of the forces; and it therefore measures, according to the scale chosen, the amount and direction of the resultant of the forces acting on that voussoir. Although the figure apparently shows the line *de* as though it passed through the center of gravity of the voussoir, and although it generally will do so very nearly, it should be remembered that *de* does not necessarily pass through the center of gravity of the voussoir.

A practical graphical method of laying off the line *vt* to represent the actual resultant force is as follows: The *reduced load line*, drawn as previously described, gives the line for a loading of solid stone, which would be the equivalent of the actual load line. If this loading has a unit-value of, say, 160 pounds per cubic foot, and if the horizontal distance *ab* is made 2 feet for the load over each voussoir, then each foot of height (at the same scale at which *ab* represents 2 feet) of the line *gd* represents 320 pounds of loading. If the voussoir were actually a rectangle, then its area would be equal to that of the dotted parallelogram vertically under *ac*, and its area would equal *ab* × *dk*; and in such a case, *dk* would represent the weight of that voussoir, and the force *vw* could be scaled directly equal to *dk*, without further computation. The accuracy of this method, of course, depends on the equality of the dotted triangle below *c* and that below *a*. For voussoirs which are near the crown of the arch, the error involved by this method is probably within the general accuracy of other determinations of weight; but near the abutment of a full-centered arch, the inaccuracy would be too great to be tolerated, and the area of the voussoir should be actually computed. Dividing the area by 2 (or the width *ab*), we have the equivalent height in the same terms at which *gd* represents the external load, and its equivalent height would be laid off as *vw*.

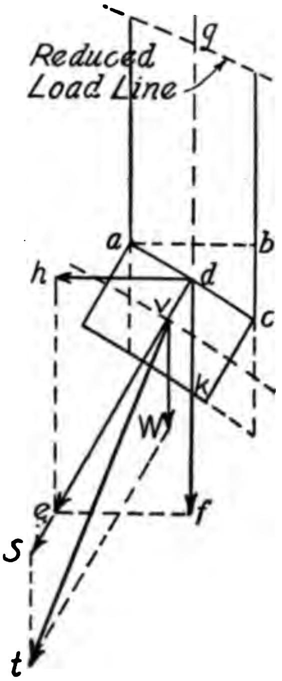


Fig. 225. Resultant of Oblique Pressures.

418. **Application to a Definite Problem.** We shall assume for this case a full-centered circular arch whose intrados has a radius of 15 feet. The depth of the keystone computed according to the rule given in Equation **47**, would be 1.57 feet, which is practically 19 inches. By drawing first the intrados of the arch as a full semicircle (see Fig. 226), and then laying off the crown thickness of 19 inches, we find by trial that a radius of 20 feet for the extrados will make the arch increase to a thickness of about 2½ feet at a point 45 degrees from the center, which is usually a critical point in such arches. We shall therefore draw the extrados with a radius of 20 feet, the center point being determined by measuring 20 feet down from the top of the keystone. We shall likewise assume that this arch is one of a series resting on piers which are 4 feet thick at the springing line.

By drawing a portion of the adjoining arch, we find that its extrados intersects the extrados of the arch considered at a point about 7 feet 6 inches above the pier. By drawing a line from this point toward the *center for joints*, which is about midway between the center for the extrados and the center for the intrados, we have the line for the joint which is virtually the skewback joint and the abutment of the arch.

The center of the pier is precisely 17 feet from the center of the arch. We shall assume that the arch is overlaid with a filling of earth or cinders which is 18 inches thick at the crown, and that it is level. Drawing a horizontal line to represent the top of this earth filling, we may divide this line into sections which are 2 feet wide, commencing at the vertical line through the center of the pier. Extending this similarly to the other side of the arch, we have eight sections of loading on each side of the keystone section. Drawing lines from the points where these verticals between the sections intersect the extrados, toward the center for joints, previously determined, we have the various joints of the voussoirs. Assuming, as in the previous numerical problem that the cinder fill weighs 100 pounds per cubic foot, and that the stone weighs 160 pounds per cubic feet, we determine the reduced load line for the top of the earth fill over the entire arch.

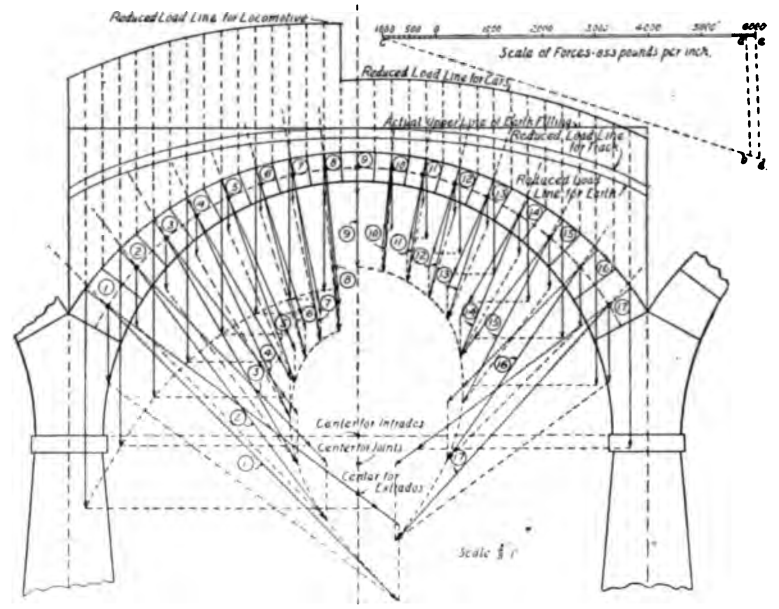

Reduced Load Line for Locomotive
1000 500 0 1000 2000 3000 4000 5000 6000
Scale of Forces-855 pounds per inch.
Reduced Load Line for Cars
Actual Upper Line of Earth Filling
Reduced Load Line for Track
Reduced Load Line for Earth
Center for Intrados
Center for Joints
Center for Extrados
Scale

Fig. 226. Resultant Forces Acting on Voussoirs of a Full-Centered Arch.

We shall assume that the arch carries a railroad track and a heavy class of traffic. The weight of roadbed and track may be computed as follows: The ties are to be 8 feet long; the weight of the roadbed and track (and also the live load) is assumed to be distributed over an area 8 feet wide.

Two rails at 100 pounds per yard will weigh, per square foot of surface	8.4 lbs.
Oak ties, weighing 150 pounds per tie, will weigh, per square foot of surface	9.4 "
Weight of ballast at 100 pounds per cubic foot, average depth 9 inches	75.0 "
Total weight	92.8 lbs.

This is the equivalent of 0.58 foot depth of stone, and we therefore add this uniform depth to the reduced load line for the earth.

A 50-ton freight-car, fully loaded, will weigh 134,000 pounds; with a length between bumpers of 37 feet, this will exert a pressure of about 450 pounds per square foot on a strip 8 feet wide. This is equivalent to 2.8 feet of masonry. We shall therefore consider this as a requirement for uniform loading over the whole arch.

It would be more precise to consider the actual wheel loads for the end trucks of two such cars which are immediately following each other; but since the effect of this would be even less than that of the calculation for a locomotive, which will be given later, and since the deep cushion of earth filling will largely obliterate the effect of concentrated loads, the method of considering the loading as uniformly distributed will be used. We therefore add the uniform ordinate equal to 2.8 feet over the whole arch. We shall call this the *first condition of loading.*

We shall assume for the concentrated loading, a consolidation locomotive with 40,000 pounds on each of the four driving axles, spaced 5 feet apart. This means a wheel base 15 feet long; and we shall assume that this extends over voussoirs 1 to 8 inclusive, while the loading of 450 pounds per square foot is on the other portion of the arch. A weight of 40,000 pounds on an axle, which is supposed to be distributed over an area 5 feet long and 8 feet wide, gives a pressure of 1,000 pounds per square foot, or it would add an ordinate of 6.33 feet of stone; these ordinates are added above the load line representing the load of the roadbed and track. We shall call this the *second condition of loading.*

The load for each voussoir is determined by the method given in Article 417. The direction of the pressure on the voussoir is determined by drawing a line toward the extrados center from the intersection of the vertical through the trapezoid of loading with the extrados. The length of that vertical is laid off below that point of intersection; then a horizontal line drawn from the lower end of the vertical intersects the line of force at a point which measures the amount of that pressure on the voussoir. The area of the voussoir is determined as described in Article 417; and the resultant of the loading and the weight of the voussoir is obtained. This is indicated as force No. 1 in Fig. 226. In this case, it includes the locomotive loading on the left-hand side of the arch. The forces acting on voussoirs Nos. 2, 3, 4, 5, 6, 7, and 8 are similarly determined. The forces on voussoirs Nos. 9 to 17 inclusive, on the basis of the uniformly distributed load equal to 450 pounds per square foot, are also similarly determined. The loads on voussoirs Nos. 10 to 17 inclusive will be considered to measure the loads on voussoirs Nos. 8 to 1 inclusive, for the *first* condition of loading. The loading with the locomotive over voussoirs Nos. 1 to 8, and cars over voussoirs Nos. 9 to 17, constitutes the *second* condition of loading.

As described above, the arrows representing the forces in Fig. 226 are drawn at a scale such that each ⅜ of an inch represents 2 cubic feet of masonry, or 320 pounds; therefore every inch will represent the quotient of 320 divided by ⅜, or 853 pounds per linear inch. The practical method of making a scale for this use is illustrated in the diagram in the upper right-hand corner of Fig. 226. We may draw a horizontal line as a scale line, and lay off on it, with a decimal scale, a length *ca* which represents, at some convenient scale, a length of 800. Drawing the line *ab* at any convenient angle, we lay off from the point *c* the length *cb* to represent 853 at the *same* scale as that used for *ca*. The line *cd* is then laid off to represent 7,000 units at the scale of 800 units per inch. By drawing a line from *d* parallel to *ba*, we have the distance *ce*, which represents 7,000 units at the scale of 853 units per inch. By trial, a pair of dividers may be so spaced that it steps off precisely seven equal parts for the distance *ce;* or the line *ce* may also be divided into equal parts by laying off on *cd* to the decimal scale, the seven equal parts of 1,000 each which are at the scale of 800 units per inch; and then lines may be drawn from these points parallel to *ba* and *de*. The last division may be similarly divided into 10 equal parts, which will represent 100 pounds each.

Using dividers, the resultant force on each voussoir from No. 1 to No. 17 may be scaled off as follows:

1	7,825	10	1,910
2	5,970	11	2,040
3	4,940	12	2,200
4	4,190	13	2,400
5	3,725	14	2,905
6	3,380	15	3,570
7	3,170	16	4,420
8	3,040	17	6,005
9.	1,880		

The student should note the three dotted curves in the lower part of the figure, which have been drawn through the extremities of the forces. The only object in drawing these three curves is merely to note the uniformity with which the ends of these arrows form a regular curve. If it was found that one of the forces did not pass through this curve, it would probably imply a blunder in the method of determining that particular force. Even if such curves are not actually drawn in, it is well to observe that the points do come on a regular curve, as this constitutes one of the checks on the graphical solution of problems.

Fig. 226 is merely the beginning of the problem of determining the stresses in the arch. In order to save the complication of the figure, the arch itself and the resultant forces (1 to 17) are repeated in Fig. 227, the direction, intensity, and point of application of these forces being copied from one figure to the other.

Forces Nos. 1 to 17 are drawn in the force diagram of Fig. 227 at the scale of 4,000 pounds per inch. Forces 1 to 8, inclusive, have a resultant whose direction is given by the line marked R_1'' which joins the extremities of forces 1 to 8. Similarly, the direction of the resultant (R_1' or R_2') of forces 9 to 17, inclusive, is given by the line which joins the extremities of this group. The direction of the resultant of all the forces Nos. 1 to 17, is given by the line joining the extremities of these forces in the force diagram, this resultant being marked R_2. By choosing a pole at random (the point o_2' in the force diagram), drawing rays to the forces, and beginning at the left-hand abutment, we may draw the trial equilibrium polygon, which passes through the point *a* on force No. 17. The line through *a* parallel to the last ray, has the direction *ab*. Producing the section of the polygon which is between forces 8 and 9 (and which is parallel to the ray which reaches the load line between forces 8 and 9), it intersects the first and last lines of the trial equilibrium polygon at the points *b* and *d*. The point *b* is therefore a point on the resultant R_2' of forces Nos. 9 to 17 inclusive; and by drawing a line parallel to the force R_2' in the force diagram, we have the actual line of action of the resultant.

Similarly, the line of action of the force R_2'' is determined by drawing from the point *d* a line parallel to R_2'' in the force diagram. Their intersection at the point *e* gives a point in the line of action of the resultant of the whole system of forces, R_2; and by drawing from the point *e* a line parallel to R_2 of the force diagram, we have the line of action of R_2. We select a point (*f*) at random on the resultant R_2, and join the point *f* with the center of each abutment. By drawing lines from the extremities of the load line parallel to these two lines from *f*, they intersect at the point o_2''. A horizontal line through o_2'' is therefore the locus of the pole of the true equilibrium polygon passing

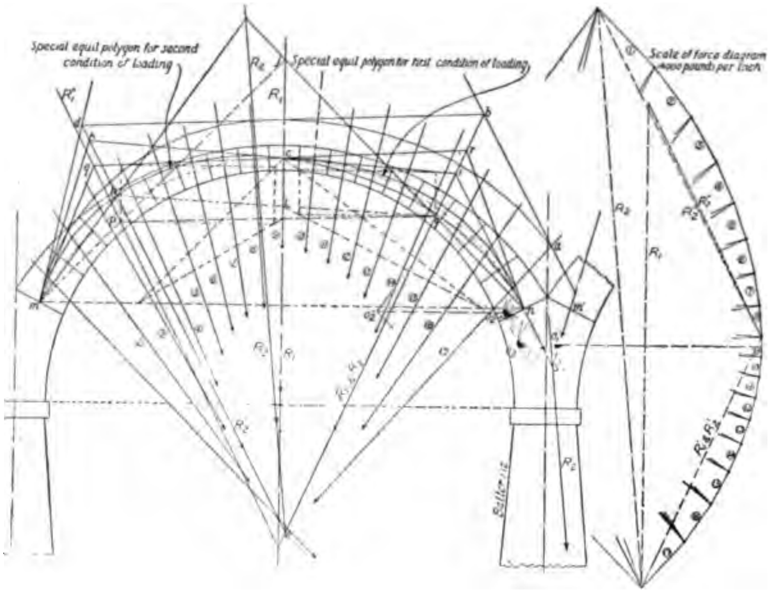

Special equil polygon for second condition of loading
R_1'
R_2
R_1
Special equil polygon for first condition of loading
b
Scale of force diagram
pounds per inch
R_2
R_1
R_2'
R_2
Batter

Fig. 227. Pressures on Voussoirs of a Full-Centered Arch.

through the center of both abutments. The line *fn* intersects R_2' in the point *g*, and the line *fm* intersects the force R_2'' in the point *h*. The intersection of *gh* with the vertical through the center (the point *i*) is the trial point which must be raised up to the point *c*, which is done by the method illustrated in Article 401. The application of this method gives the line *kl*, passing through *c*; and the line *ln* is therefore the first line of the special equilibrium polygon for the complete system of forces from No. 1 to No. 17; and the line *km* is similarly the last line of that polygon. By drawing lines from the extremities of the load line, parallel to *ln* and *km*, we find that they intersect at the point o_2''', which is the pole of the special equilibrium polygon passing through *n*, *c*, and *m*, for the complete system of forces Nos. 1 to 17.

As a check on the work, the intersection of these lines from the ends of the load line, parallel to *ln* and *km*, must be on the horizontal line passing through o_2''. By drawing rays from the new pole o_2''' to the load line, and completing the special equilibrium polygon, we should find as a double check on the work, that both of these partial polygons starting from *m* and *n* should pass through the point *c*; and also that the section of the polygon between forces Nos. 8 and 9 lies on the line *kl*. This gives the special equilibrium polygon for the system of forces Nos. 1 to 17, which corresponds with the *second* condition of loading, as specified above.

The first condition of loading is given by duplicating about the center, in the force diagram, the system of forces from No. 17 to No. 9 inclusive. Since this system of forces is symmetrical about the center, we know that its resultant R_1 passes through the center of the arch, and that it must be a vertical force. We may draw from the *middle* of force No. 9 a horizontal line, and also draw a vertical from the lower end of the load line. Their intersection is evidently at the center of the resultant R_1, which is therefore carried above this horizontal line for an equal amount. Joining the upper end of R_1 with the upper end of force No. 9, we have the direction and amount of the force R_1''. The intersection of *ng* with the force R_1 at the point *j*, gives a point which, when joined with the point *m*, gives one line of a trial equilibrium polygon passing through the required points *m* and *n*, but which does not pass through the required point *c*. The intersection of *jm* with the force R_1'' at the point *p*, gives us the line *pg*, which is the same kind of line for this trial polygon as the line *hg* was for the other.

By a similar method to that used before and as described in detail in Article 401, we obtain the line *qr* passing through *c*, which gives us also the section of our true equilibrium polygon between forces Nos. 8 and 9. The line *rn* also gives us that portion of the true equilibrium polygon for this system of loading, from the point *n* up to the force No. 17.

By drawing a line from the lower end of the load line, parallel to *nr*, until it intersects the horizontal line through the middle of force No. 9 at the point o_1', we have the pole of the special equilibrium polygon for this system of loading, which is the *first* condition of loading. The rays are drawn from o_1' only to the forces from No. 9 to No. 17 inclusive, and the special equilibrium polygon is completed between *n* and *c* by drawing them parallel to these rays.

On account of the symmetry of loading, we know that the equilibrium polygon would be exactly similar on the left-hand side of the arch. In discussing these equilibrium polygons, we must therefore remember that of the two equilibrium polygons lying between the extrados and intrados on the right-hand side of the arch, the upper line represents the line of pressure for a uniform loading over the whole arch (the first condition of loading), while the lower line on the right-hand side, and also the one equilibrium polygon which is shown on the left-hand side of the arch, represent the special equilibrium polygon for the second condition of loading.

419. **Intensity of Pressures on the Voussoirs of the Arch.** An inspection of the equilibrium polygon for the first condition of loading, shows that it passes everywhere within the middle third. The maximum total pressure on a joint, of course, occurs at the abutment, where the pressure equals 24,750 pounds. Since the joint is here about 42 inches thick, and a section one foot wide has an area of 504 square inches, the pressure on the joint is at the rate of 49 pounds per square inch. At the keystone, the actual pressure is 19,750 pounds; and since the keystone has an area of 228 square inches, the pressure is at the rate of 87 pounds per square inch.

At the joint between forces Nos. 13 and 14, the line of force passes just inside the edge of the middle third. The ray from the pole o_1' to the joint between voussoirs Nos. 13 and 14 of the force diagram, has a scaled length of 20,250 pounds. The joint has a total thickness of about 24 inches, and therefore an area of 288 square inches. This gives an average pressure of 70 pounds per square inch; but since the line of pressure passes near the edge of the middle third, we may double it, and say that the maximum pressure at the upper edge of the joint is 140 pounds per square inch. All of these pressures for the first condition of loading are so small a proportion of the crushing strength of any stone such as would be used for an arch, or even of the good quality of mortar which would of course be used in such a structure, that we may consider the arch as designed, to be perfectly safe for the first condition of loading.

The special equilibrium polygon for the second condition of loading shows that the stability of the arch is far more questionable under this condition, since the special equilibrium polygon passes outside the middle third, especially on the left-hand haunch of the arch. The critical joint appears to be between voussoirs Nos. 4 and 5. The pressure at this joint, as determined by scaling the distance from the point o_2''' to the load line between forces Nos. 4 and 5, is approximately 24,500 pounds. The section of the equilibrium polygon parallel to this ray passes through the joint at a distance of a little over three inches from the edge. On the basis of the distribution of pressure at a joint, the compression at this joint would be confined to a width of 9 inches from the upper edge, the pressure being zero at a distance of 9 inches from the edge. This gives an area of pressure of 108 square inches, and an average pressure of 227 pounds per square inch. At the upper edge of the joint, there would therefore be a pressure of double this, or 454 pounds per square inch. This pressure approaches the extreme limit of intensity of pressure which should be used in arch work; and even this should not be used unless the voussoirs were cut and dressed in a strictly first-class manner, and the joints were laid with a first-class quality of mortar.

The propriety of leaving the dimensions as first assumed for trial figures, depends, therefore, on the following considerations:

First—The loading assumed above for the uniformly distributed load is as great a loading as that produced by ordinary locomotives such as are used on the majority of railroads; while the locomotive requirements as assumed above are excessive, and are used on only a comparatively few railroads.

Second—If an equilibrium polygon had been started from a point nearer the intrados than the point *m* (using the same pole o_2'''), it would have passed a little below the point *c*, and likewise a little nearer the intrados than the point *n*. Although this would have brought the equilibrium polygon a little nearer to the intrados on the right-hand haunch of the arch, it would likewise have drawn it away from the extrados on the left-hand haunch. Although it is uncertain just which equilibrium polygon, among the infinite number which may mathematically be drawn, will actually represent the true equilibrium polygon, there is reason to believe that the true equilibrium polygon is the one of which the summation of the intensity of pressures at the various joints is a *minimum;* and it is evident from mere inspection, that an equilibrium polygon drawn a *little* nearer the center (as described above) will have a slightly less summation of intensity of pressure, although the intensity of pressure on the joints on the right-hand haunch will rapidly increase as the polygon approaches the intrados. It is therefore quite possible that the true equilibrium polygon would have a less intensity of pressure at the joint between voussoirs Nos. 4 and 5.

If it is still desired to increase the thickness of the arch so that the line of pressure will pass further from the extrados, it may be done approximately as indicated for a similar problem in Article 414. Evidently the keystone is sufficiently thick, and the voussoirs at the abutments also have ample thickness. The extrados must evidently be changed from an arc of a circle to some form of curve which shall pass through the same three points at the crown and the two abutments. This may be either an ellipse or a three-centered or five-centered curve. Although it will cause an extra loading on the haunches of the arch to increase the thickness of the arch on the haunches, and although this will cause the equilibrium polygon to rise somewhat, the rise of the equilibrium polygon will not be nearly so rapid as the increase in the thickness of the arch; and therefore the added thickness will add very nearly that same amount to the distance from the extrados to the equilibrium polygon. For example, by adding a *little over* three inches to the thickness of the arch at voussoirs Nos. 4 and 5, the distance from the equilibrium polygon to the extrados would be increased from three inches to six inches, and the maximum intensity of pressure on the joint would be approximately half of the previous figure. To be perfectly sure of the results, of course, the problem should be again worked out on the basis of the new dimensions for the arch.

The required radii for a multi-centered arch which should have this required extrados, or the axes of an arc of an ellipse which should pass through the required points, are best determined by trial. The effect of the added thickness on the load line for the right-hand side of the arch, will be to bring the load line nearer to the center of the voussoirs, and therefore will actually improve the conditions on that side of the arch. Of course, when the concentrated load is over the right-hand side of the arch instead of the left, the form of the equilibrium polygon will be exactly reversed. It is quite probable that, for mere considerations of architectural effect, the revised extrados would be made the same kind of a curve as the intrados. This would practically be done by selecting a radius which would leave the same thickness at the crown, allow the required thickness on the haunches, and let the thickness come what it will at the abutments, even though it is needlessly thick.

420. **Stability of the Pier between the Arches.** The stability of the pier on the right-hand side of the arch in Fig. 227, is determined on the assumption of the concentrated locomotive loading on the left-hand end of the next arch which is at the right of the given arch, and the uniform loading over the right-hand end of the given arch. We therefore draw through the point m' a line of force parallel to mk, and also produce the line ln until it intersects the other line of force in the point s. A line from s parallel to R_2, therefore, gives the line of action of the resultant of the forces passing down the pier, for this system of loading. Since this system of loading will give the most unfavorable condition, or the condition which will give a resultant with the greatest variation from the perpendicular, we shall consider this as the criterion for the stability of the pier. The piers were drawn with a batter of 1 in 12, and it should be noted that the resultant R_2 is practically parallel to the batter line. If the slope of R_2 were greater than it is, the batter should then be increased. The value of R_2 is scaled from the force diagram as 55,650 pounds. The force R_2 is about 14 inches from the face of the pier, and this would indicate a maximum intensity of pressure of 221 pounds per square inch. This is a safe pressure for a good class of masonry work. The

actual pressure on the top of the pier is somewhat in excess of this, on account of the weight of that portion of the arch between the virtual abutment at *n* and the top of the pier; and the total pressure at any lower horizontal section, of course, gradually increases; but on the other hand, the weight of the pier combines with the resultant thrust of the two arches to form a resultant which is more nearly vertical than R_2, and the center of pressure therefore approaches more nearly to the axis of the pier. The effect of this is to reduce the intensity of pressure on the outer edge of the pier; and since the numerical result obtained above is a safe value, the actual maximum intensity of pressure is certainly safe.

ELASTIC ARCHES

421. Technical Meaning. All of the previous demonstrations in arches have been made on the basis that the arch is made up of voussoirs, which are acted on only by compressive forces. The demonstration would still remain the same, even if the arches were monolithic rather than composed of voussoirs; but in the case of an arch composed of voussoirs, it is essential that the line of pressure shall pass within the middle third of each joint, in order to avoid a tendency for the joint to open. If the line of pressure passes very far outside of the middle third of the joint, the arch will certainly collapse. An elastic arch is one which is capable of withstanding tension, which practically means that the line of pressure *may* pass outside of the middle third and even outside of the arch rib itself. In such a case, transverse stresses will be developed in the arch at such a section, and the stability of the arch will depend upon the ability of the arch rib to withstand the transverse stresses developed at that section. A voussoir arch is, of course, incapable of withstanding any such stresses. A monolithic arch of plain concrete could withstand a considerable variation of the line of pressure from the middle third of the arch rib; but since its tensile strength is comparatively low, this variation is very small compared with the variation that would be possible with a steel arch rib. A reinforced-concrete arch rib can be designed to stand a very considerable variation of the line of pressure from the center of the arch rib.

422. Advantages and Economy. The durability of concrete, and the perfect protection that it affords to the reinforcing steel which is buried in it, give a great advantage to these materials in the construction of arch ribs. Although the theoretical economy is not so great as might be expected, there are some very practical features which render the method economical. It is always found that, before any considerable transverse stresses can be developed in a reinforced-concrete arch bridge, the concrete will be compressed to the maximum safe limit while the unit-stress in the steel is still comparatively low. Since a variation in the dead load often changes the line of pressure from one side of the arch rib to the other, and thus changes the direction of the transverse bending, it becomes necessary to place steel near *both* faces of the arch rib, in order to withstand the tension which will be alternately on either side of the rib. Of course the steel which is (for the moment) on the compressive side of the rib will assist the concrete in withstanding compression, but this is not an economical use of the steel. There is, however, the practical economy and advantage, that the reinforcement of the concrete makes it far more reliable, even from the compression standpoint. It prevents cracks in the concrete, and it also permits the use of a much higher unit-pressure than would be considered good practice in the use of plain concrete. This advantage becomes especially great in the construction of arches of long span, since in such a case the dead load is generally several times as great as the live load. Therefore the maximum variation in the line of pressure produced by any possible change in loading is not very great; and any method which will permit the use of a higher unit-pressure in the concrete is fully justified by the use of such an amount of steel as is required in this case.

423. **Elements of Integral Calculus.** It has been found impracticable to develop the theory of elastic arches without employing some of the fundamental principles of integral calculus; but an effort will be made to explain each one of the equations which are used, in such a way that the application of calculus to this particular case may be understood. To facilitate this demonstration, a few of the fundamental principles of integral calculus will be briefly demonstrated. All of the calculus equations which are used are similar to Equation 48. In this, a character $\int$, somewhat similar to the letter S (which may be considered to stand for the word summation), is placed in front of some mathematical quantities. The equation generally reads that this summation equals zero. The general

meaning of the equation is that there is a group of quantities all of which are in general similar, but which have a variation in magnitude. In general, some of these quantities are positive, and some are negative, and the equation reads that the *summation* or the algebraic addition of all these positive and negative quantities just equals zero; or, in other words, that the sum of all the positive quantities is just equal to the sum of all the negative quantities.

For example, the first one of the equations marked **48** may be interpreted as follows: M represents the transverse moment of the arch rib at *any* point of the arch rib. M is a variable, being sometimes positive, sometimes negative, and sometimes zero; E is the modulus of elasticity, and we shall here assume that this is also constant; ds represents the distance between any two consecutive sections of the arch rib. Theoretically, ds is assumed to be infinitely small, which means that we consider an infinite number of sections of the arch rib. I represents the moment of inertia of the arch rib at any section. In some cases this may be considered a constant; and it is a constant, provided the arch rib is of a uniform cross-section throughout its length. If, as is frequently the case, the arch rib is of variable cross-section, then the value of I is variable for each section. It is assumed that the moment at each section is multiplied by the distance ds between the consecutive sections, and divided by the product of the modulus of elasticity and the moment of inertia at that section. All these quantities are positive, except M, which is sometimes positive, sometimes negative, and occasionally zero. Whenever any term has a constant value for each one of these small products, it may be placed outside of the summation sign, since the summation of a constant quantity times a variable is, of course, equal to that same constant quantity multiplied by the summation of the variables. As a corollary of this, we may also say that if the summation equals zero, we may even take the constant term out altogether; since, if a constant times a summation of positive and negative terms equals zero, then the summation of those positive and negative terms must of itself equal zero. There will be an illustration in the following sections, of the dropping of constant terms, and therefore the simplification of the mathematics. If such a product were obtained for each one of a very large number of cross-sections of the rib, we should have a series of products, some of which would be positive, some negative, and probably two of which would be zero. The algebraic sum of these terms would equal zero. The letters O and B near the top and bottom of the summation sign represent that sections are made all the way from O to B in Fig. 228. If the sections had been taken between two other points (as, for example, between O and C), the letter C would take the place of the letter B in the equation.

The three equations of Equation 48 are given without demonstration. The student must accept the equations as being mathematically true, since their demonstration involves work in integral calculus which cannot be here given; but it should also be realized that the equations are only precisely true when the number of terms is infinitely large, and the distance ds is therefore infinitely small. When the sections are taken at a finite distance apart, as it is practically necessary to do, then there may be theoretically a slight error; but when the number of sections of an arch rib is made from 12 to 20 in the length of the span, the inaccuracy involved because the number of terms is not infinite is so very small that it is of no practical importance.

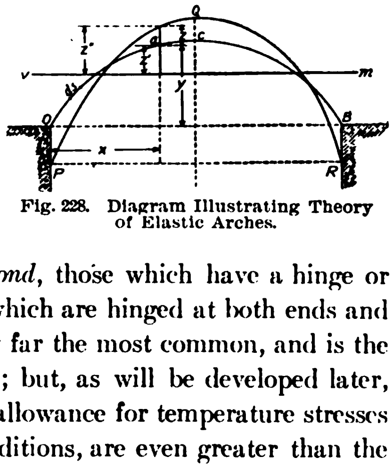


Fig. 228. Diagram Illustrating Theory of Elastic Arches.

424. Classification of Arch Ribs. Arch ribs may be classified in three ways: *first*, those which have fixed ends and no hinges; *second*, those which have a hinge or joint at each end; and *third* those which are hinged at both ends and in the center. The first class is by far the most common, and is the simplest and cheapest to construct; but, as will be developed later, it necessitates a very considerable allowance for temperature stresses which, under very unfavorable conditions, are even greater than the maximum stresses due to loading. The temperature stresses of a two-hinged arch are less severe, while those for a three-hinged arch may be neglected; but the construction of hinges in arch ribs adds considerably to the cost.

425. Mathematical Principles. In the following demonstration, the arch rib is considered as a single line OCB (Fig. 228), which is assumed to have the properties of an arch rib—namely, the moment of inertia, modulus of elasticity of the material, and the consequent resisting moment. The curved line PQR represents the special equilibrium polygon corresponding to some one condition of loading. Although this line is drawn as a curved line, it is assumed to be a curve which is made up of a large number of correspondingly short lines, each of which corresponds to a section of an equilibrium polygon similar to those described under "Voussoir Arches." This equilibrium polygon is yet to be determined.

In Church's "Mechanics of Engineering," Chapter XI, is given the mathematical proof of three general equations which apply to this problem. No demonstration will here be made of these three equations, which are as follows;

$$\int_{O}^{B} \frac{M\,ds}{EI} = 0\,; \quad \int_{O}^{B} \frac{M\,x\,ds}{EI} = 0\,; \quad \int_{O}^{B} \frac{M\,y\,ds}{EI} = 0\,; \quad \ldots . (48)$$

The practical meaning of the first of these equations may be described as follows (see Fig. 228): *ds* represents one of an *even* number of very short sections into which the length *OCB* of the arch rib has been divided. *M* represents the transverse moment acting on the arch rib at that section under the particular condition of loading which is being considered. *E* is the modulus of elasticity of the material, and *I* is the moment of inertia of the section. At some of the sections the moment is positive, and at some it is negative. The product of *M* and *ds*, divided by the product of *E* and *I*, is therefore sometimes positive and sometimes negative. According to this equation, the *summation* of these various products for each short section (*ds*) of the rib equals zero; or, in other words, the summation of the positive products will exactly equal numerically the summation of the negative products.

The other two parts of Equation 48 must be interpreted similarly, the only difference being that in each case the term $\frac{Mds}{EI}$ is multiplied by the corresponding value of y for one of the equations, and by x for the other. This group of three equations (48) has nothing to do with the form of the special equilibrium polygon *PQR*.

It may also be proved by analytical mechanics, that if the curve *PQR* represents the special equilibrium polygon corresponding to some system of loading, and z represents the vertical distance between the arch rib and the special equilibrium polygon at any section, then the moment M at that section a of the rib, equals Hz, in which H is a constant which may be determined from the force diagram. The curve PQR represents a typical special equilibrium polygon which crosses the arch rib at two points. These points of intersection indicate points of contraflexure, where the transverse moment changes its direction of rotation, and where it is therefore zero. When the special equilibrium polygon is above the rib curve, we call the moment *positive*; and when it is below, we call it *negative*. When it is positive, it means that there is tension in the lower part of the rib, and compression in the upper part. The conditions are, of course, the reverse of this when the curve is below the rib. We may therefore substitute Hz for the value of M in the group of Equations **48**; and since H and E are both constant for all points, from the principle enunciated in Article 423, we may not only place them outside of the sign of summation, but may even drop them altogether, since the summation equals zero; and we may therefore transform Equations **48** to the following:

$$\int_{O}^{B} \frac{z\,ds}{I} = 0\,; \qquad \int_{O}^{B} \frac{z\,x\,ds}{I} = 0\,; \qquad \int_{O}^{B} \frac{z\,y\,ds}{I} = 0\,. \quad \ldots\ldots \textbf{(49)}$$

Whenever we are investigating the mechanics of an arch rib which has a *constant* moment of inertia, we may simplify Equations **49** by dropping out altogether the I of the denominators of those equations; but since arch ribs are usually made with deeper sections near the abutments, the I will be greater near the abutments. Calling the I at the center I_c, then I equals nI_c, in which n is a variable. If we substitute this value of I in the denominators of Equations **49**, then, since I_c is a constant quantity, it may be placed outside of the summation sign, and even dropped altogether, which practically means that we substitute n for I in Equations **49**. We shall also substitute for z its value $z'' - z'$ (see Fig. 228), and shall rewrite Equations **49** as follows, by making the substitutions:

$$\int_{O}^{B} \frac{z'' - z'}{n}\,ds = 0\,; \qquad \int_{O}^{B} \frac{(z'' - z')\,x}{n}\,ds = 0\,; \qquad \int_{O}^{B} \frac{(z'' - z')\,y}{n}\,ds = 0\,; \quad \ldots \textbf{(50)}$$

It will later be shown how we can draw a line (marked *vm* in Fig. 228) which will satisfy the following equations:

$$\int_{O}^{B} \frac{z'\,ds}{n} = 0, \quad \text{and} \quad \int_{O}^{B} \frac{x\,z'\,ds}{n} = 0 \quad \ldots\ldots (51)$$

Since the arch rib (represented by the curve *OCB*) is assumed to be symmetrical about its center *C*, and since *vm* is horizontal, any position of *vm* which will satisfy the first of Equations **51** will also satisfy the second.

It is another principle of the science of summations, that if we have a series of terms whose summation equals zero, and also have another series of terms whose summation equals zero, but whose terms are made up of the difference of two terms, one of which corresponds in each case to the terms of the first summation, then we may say that the summation of the other corresponding terms is likewise zero. For example, the first one of Equations **50** consists of a series of terms which may be rewritten:

$$\frac{z''}{n}ds - \frac{z'}{n}ds.$$

The first one of Equations **51** is the summation of a series of terms, each with the form $\frac{z'}{n}ds$. In each of these summations the different terms corresponding to the variable values of z' exactly correspond. We may therefore say that the summation of a series of corresponding terms, each one of the form $\frac{z''}{n}ds$, will exactly equal zero; and we may therefore write Equation **52** as given below. We may also combine the second part of Equation **50** with the second part of Equation **51** in a similar manner, and obtain Equation **53** as given below It will be found more convenient to separate the third part of Equation **50** into two summations, one of which consists of a series of terms $\frac{z''}{n}y\,ds$, and the other of a series of terms consisting of $\frac{z'}{n}y\,ds$; and since the difference of these summations equals zero, then the summations must equal each other, and we may therefore write Equation **54**:

half of the previous figure. To be perfectly sure of the results, of course, the problem should be again worked out on the basis of the new dimensions for the arch.

The required radii for a multi-centered arch which should have this required extrados, or the axes of an arc of an ellipse which should pass through the required points, are best determined by trial. The effect of the added thickness on the load line for the right-hand side of the arch, will be to bring the load line nearer to the center of the voussoirs, and therefore will actually improve the conditions on that side of the arch. Of course, when the concentrated load is over the right-hand side of the arch instead of the left, the form of the equilibrium polygon will be exactly reversed. It is quite probable that, for mere considerations of architectural effect, the revised extrados would be made the same kind of a curve as the intrados. This would practically be done by selecting a radius which would leave the same thickness at the crown, allow the required thickness on the haunches, and let the thickness come what it will at the abutments, even though it is needlessly thick.

420. **Stability of the Pier between the Arches.** The stability of the pier on the right-hand side of the arch in Fig. 227, is determined on the assumption of the concentrated locomotive loading on the left-hand end of the next arch which is at the right of the given arch, and the uniform loading over the right-hand end of the given arch. We therefore draw through the point *m'* a line of force parallel to *mk*, and also produce the line *ln* until it intersects the other line of force in the point *s*. A line from *s* parallel to R_2, therefore, gives the line of action of the resultant of the forces passing down the pier, for this system of loading. Since this system of loading will give the most unfavorable condition, or the condition which will give a resultant with the greatest variation from the perpendicular, we shall consider this as the criterion for the stability of the pier. The piers were drawn with a batter of 1 in 12, and it should be noted that the resultant R_2 is practically parallel to the batter line. If the slope of R_2 were greater than it is, the batter should then be increased. The value of R_2 is scaled from the force diagram as 55,650 pounds. The force R_2 is about 14 inches from the face of the pier, and this would indicate a maximum intensity of pressure of 221 pounds per square inch. This is a safe pressure for a good class of masonry work. The

429. **Moment of Inertia of any Section.** Assume that Fig. 229 represents a portion of the cross-section of an arch at any point, the particular portion having a total depth h equal to the thickness of the arch at that point, and a unit-width b, which is presumably less than the total length of the arch parallel to its axis. Assume that there is reinforcement in both the top and bottom of this section, and that the reinforcement is placed at a distance of $\frac{1}{10}$ of the thickness of the arch from the top and the bottom, or from the extrados and the intrados. The moment of inertia of the plain concrete without any reinforcement would evidently equal $\frac{1}{12}\,bh^3$. A transverse stress on such a section will cause the bars on one side of the section (say the bottom) to be in tension, while those in the top will be in compression. As already developed in the treatment of columns (Part III, Article 310), the steel which is in compression will develop a compressive stress which is in proportion to the ratio of the moduli of elasticity of the steel and the concrete; and we may therefore consider that the area of steel in compression at the top (calling its area A) is the equivalent of an area of concrete equal to Ar, in which r is as usual $E_s \div E_c$. The exact position of the neutral axis in a section which is reinforced both in compression and in tension, depends upon the percentage of steel which is used; but when the percentage is as large as it usually is, the neutral axis is not far from the center of the section; and since it very much simplifies the computations to consider it at the center of the section, it will be so considered, and the moment of inertia of the steel and concrete combined may be expressed by the equation:

$$I = \frac{1}{12}\,bh^3 + 2Ar(.4h)^2 \dots\dots\dots\dots \quad (55)$$

If, in any numerical problem, it is considered preferable to place the steel so that the distance of its center of gravity from the surface of the concrete is greater or less than 0.1 h, a corresponding change must be made in the second term of the right-hand side of Equation 55.

Example. Assume that $p = .015$, and that the thickness of the arch h equals 15 inches. For a unit-section 12 inches wide, the area of the concrete would be bh, which equals 180 square inches. Then $180 \times .015 = 2.70 = 2A$, since A is the area at either top or bottom. Therefore $A = 1.35$.

Assume that $r = 12$; $.4h = .4 \times 15 = 6$; then,

$$\begin{aligned} I &= \tfrac{1}{12} \times 12 \times 15^3 + 2 \times 1.35 \times 12 \times 6^2 \\ &= 3{,}375 + 1{,}166 \\ &= 4{,}541. \end{aligned}$$

It should be noted from Equation **55**, that when, as is usual, the area of the steel in the extrados and intrados remains constant, while the thickness of the arch varies, the increase in the moment of inertia is not strictly according to the cube of the depth, but increases in accordance with two terms, one of which varies as the cube of the depth, and the other as the square of the depth. To illustrate the discrepancy, let us assume that the depth of the arch at the abutment is 10 per cent greater than the depth at the crown; or that, applying it to the above numerical case, the depth at the crown is 15 inches, and at the abutment the depth is 16.5 inches. Then, since b, A, and r remain the same in Equation **55**, the value of the moment of inertia for the abutment would be:

$$I = \tfrac{1}{12} \times 12 \times 16.5^3 + 2 \times 1.35 \times 12 \times 6.6^2 = 5{,}903.$$

Using the approximate rule that I varies as the cube of h, we find that:

$$I = 4.541 \times (1.10)^3 = 6{,}044,$$

which is about two per cent in excess of the value found from Equation **55**. Computing the moment of inertia similarly on the assumption that the depth at the abutment is increased 50 per cent, so that it equals 22.5 inches, we find that the approximate rule will give a moment of inertia which is nearly 8 per cent in excess of the actual. Therefore, when the increase in the depth of the rib from the crown to the abutment is comparatively small, we may adopt the approximation that the moment of inertia increases as the cube of the depth. When the variation is greater, the inaccuracy will not permit the utilization of the simplified forms which this approximation allows.

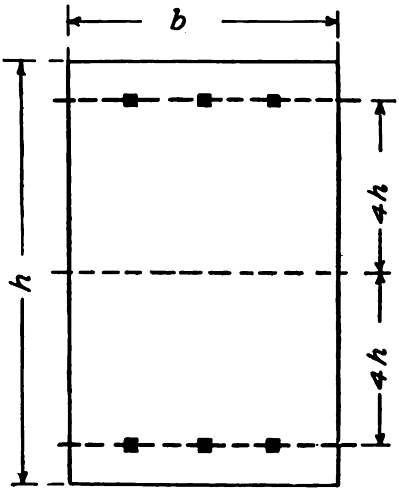


Fig. 229. Arch Cross-Section.

427. Value of n. Still another simplification may be made, on the assumption that the moment of inertia varies as the cube of the depth, and also that we may increase the depth of the rib as desired. Assume that the depth of the rib is increased so that at any point $n=$

$ds \div dx$ (see Fig. 230); ds is always greater than dx, and n is a ratio varying from *one* upward. Then, on the assumption that:

$$n = \frac{I}{I_c} = \frac{h^3}{h_c^3},$$

we may compute a series of values for h in terms of the height at the center h_c which will correspond to various angles a. For each angle, we find the ratio between h and h_c that will correspond to the value which n has for that particular angle on the basis that $n = \frac{ds}{dx}$. If we substitute this value of $\frac{ds}{dx}$ in Equations **52, 53,** and **54**, we shall have the following Equations:

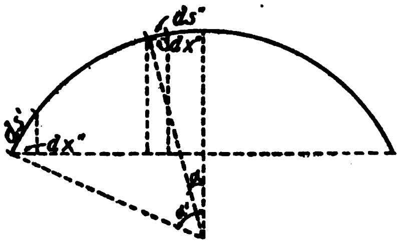


Fig. 230. Determination of Value of *n*.

$$\int_{O}^{B} z'' \, dx = 0 \quad \ldots\ldots\ldots\ldots (56)$$

$$\int_{O}^{B} z'' \, x \, dx = 0 \quad \ldots\ldots\ldots\ldots (57)$$

$$\int_{O}^{B} z'' \, y \, dx = \int_{O}^{B} z' \, y \, dx \quad \ldots\ldots (58)$$

The values of h which cause n to vary in this way, are as given in the tabular form:

a	$n = \frac{ds}{dx}$	$h = h_c \sqrt[3]{n}$	a	$n = \frac{ds}{dx}$	$h = h_c \sqrt[3]{n}$
0°	1.000	1.00h_c	60°	2.000	1.26h_c
15°	1.035	1.01	65°	2.366	1.33
30°	1.155	1.05	70°	2.924	1.43
40°	1.305	1.09	75°	3.864	1.57
45°	1.414	1.12	80°	5.759	1.79
50°	1.556	1.16	85°	11.474	2.25
55°	1.743	1.20	90°	Infinity	Infinity

Therefore, when all sections have the same moment of inertia, and n is uniformly 1, use Equations **52, 53,** and **54**, ignoring the n. When an increase in depth of section, as indicated above, will fulfil the ultimate requirements, there is an advantage of simplicity in making the sections accordingly, and using the Equations **56, 57,** and **58.** When it proves necessary to vary the sections according to some different law, n must be determined at frequent intervals, spaced by a uniform ds, and the summations of Equations **52**, **53**, and **54** determined. The remainder of this method follows out the assumption that n varies as $\frac{ds}{dx}$, or that $dx = \frac{ds}{n}$.

428. **Position of** ***vm.*** We may locate vm by satisfying Equation **51**, which may be written $\int_O^B z'\,dxz = 0$. But this integral is represented by the shaded area (Fig. 231), which is the equivalent of saying that the segment OCB = the rectangle $OK \times OB$. If OCB were a parabola, OK would exactly equal $\frac{2}{3}$ CD. Even with circular arcs, the ratio $\frac{2}{3}$ is approximately correct if the angle is small. Therefore, for flat circular arcs, draw vm at $\frac{2}{3}$ the height of the arc. If necessary, increase the height according to the figures given in the accompanying tabular form:

$2a$	RATIO	EXCESS ERROR
10°	.667	0.04 per cent
15°	.667	0.09 " "
20°	.668	0.15 " "
25°	.668	0.24 " "
30°	.669	0.35 " "
40°	.671	0.62 " "
60°	.676	1.42 " "
90°	.689	3.35 " "
180	.785	17.8 " "

Of course, for full-centered arches in which $2a$ = 180°, the error of the $\frac{2}{3}$ rule is very great, but the tabular values are correct.

Since an elliptic arc may be considered as a circle in which the vertical ordinates have all been shortened by some constant ratio, the same law and same percentage of error will hold true.

For any other curve, particularly multi-centered curves, the position of vm may be found by determining by trial a position such that the summation of equally spaced ordinates is zero.

429. **Weight and Thickness of Arch.** Theoretically, this should be known before any calculations are made; but since the weight of filling and pavement are always large, and their unit-weight

is but little less than that of the concrete, it is possible to estimate from experience on the required crown thickness, and to make the thickness at other points in the required ratio. If this should prove too thin (or too thick), all sections can be changed in the same ratio. If the outline of the intrados is determined (as in the case of an arch spanning railroad tracks), and the upper surface line (of earthwork or pavement) is also known, the change in the arch ring will mean only a change in weight due to the *difference* of unit-weight of concrete and earth filling. If the original assumption is even reasonably close, this difference will hardly exceed the uncertainties in the loading.

430. Intrados. The span and rise are frequently predetermined. Fortunately this method is applicable to almost any form of curve, if the change in curvature is not too extreme. Even if the arch is very flat and the curvature very sharp near the abutments, it only means that the virtual abutment is somewhere on the haunches. Therefore, draw the intrados; assume and lay off a reasonable crown thickness; multiply this thickness by the factors given in the tabular form in Article 427 for the angles with vertical lines made by the various normals to the curve. These thicknesses can be laid off, and the extrados can be drawn through the points.

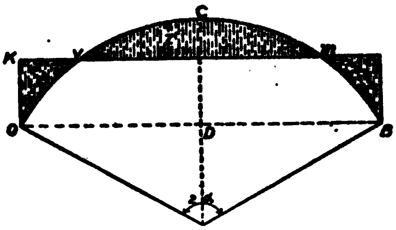


Fig. 231. Determination of Position of *vm*.

But since the curve *OCB* of Fig. 228 does not represent either the intrados or extrados, but the center line of the rib, we should draw a line midway between the intrados and extrados which will represent the center line of the rib, and which corresponds to the line *OCB* in the figures which refer to the theoretical demonstrations. This also means that the span of the rib, measured between the centers of the skewbacks, will be slightly greater than the nominal clear span. The rise of the center of the rib above the line joining the abutment points *O* and *B* will in general be slightly different from the nominal rise of the arch.

COMPLETE SOLUTION OF NUMERICAL PROBLEM ELASTIC ARCH

431. **Dimensions of Arch.** We shall apply the above principles to the design of a segmental arch having a span of 60 feet and a rise of 15 feet. To find the radius for the intrados which will fulfil these conditions, we may note from Fig. 232 that the angle $O'B'C'$ is measured by one-half of the arc $O'C'$, and therefore $O'B'C'$ is one-half of the angle a; but the angle $O'B'C'$ is an angle whose natural tangent equals 15 ÷ 30, or precisely 0.5. The angle whose tangent has this value is 26° 34′, and therefore a equals 53° 8′. To find the radius, we must divide the half-span (30) by the sine of 53° 8′, and we find that the radius equals 37.50 feet.

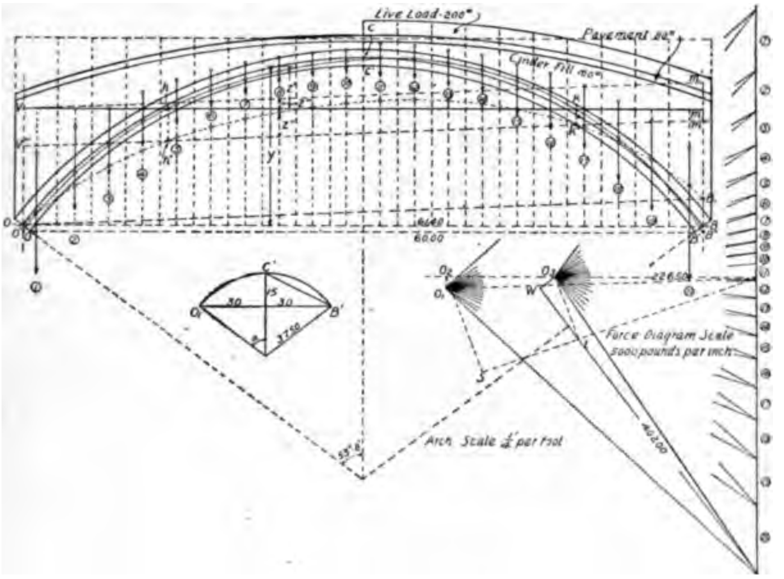


Fig. 232. Reinforced-Concrete Arch Rib, Fixed Ends.

For the depth of the keystone we can employ only empirical rules. The depth as computed from Equation **47** would call for a keystone depth of about 27 inches, which would be proper for an ordinary masonry arch; but considering recent successful practice in reinforced-concrete arches, and the far greater reliability and higher permissible unit-stresses which may be adopted, we may select about ⅔ of this—or, say, 18 inches—as the depth of the arch ring at the crown.

We shall compute the depth the arch ring should have at various points, according to the tabular form in Article 427, so that the moment of inertia will vary in the ratio $\frac{ds}{dx}$, which will make Equations 56 to 58 applicable. An arc of 1 degree equals .0175 of the radius, and therefore an arc of 1 degree on a circle with a radius of 37.5 feet will have a length of .6545 foot. At a distance of 15 degrees from the center, or at a distance of 9.82 feet, the depth is one per cent greater than the depth at the center, or it is 18.18 inches deep.

At 30 degrees from the center, or at a distance of 19.63 feet measured on the arc, the depth is 5 per cent greater, or it is 18.90 inches deep. At a distance of 40 degrees from the center, or 26.18 feet measured on the arc, the value of h is 9 per cent greater, or it is 19.62 inches. At 45 degrees from the center, or 29.45 feet measured on the arc, h is 12 per cent greater than the center depth, or the depth is 20.16 inches. At 50 degrees from the center, or 32.72 feet measured on the arc, h is 16 per cent greater, or its thickness is 20.88 inches. At the abutment, which is 53° 8′ from the center, the thickness should be (by interpolation) about 18 per cent greater than at the center, or it should be 21.24 inches, or say $21\frac{1}{4}$ inches.

Laying off these various distances from the center on the intrados, and measuring radial distances at each point to represent the proper thickness at the several points, we may join the various points and obtain the curve of the extrados. Bisecting each one of these several arch thicknesses will give us a series of points which are points on the *center line* of the arch rib. We thus find that the actual span may be considered as 61.40 feet, and that the rise is scaled at 15.25 feet.

432. **Position of *vm*.** When the center line of the rib is a parabola, we may lay off vm by drawing it at a height $\frac{2}{3}$ of the rise above the line OB. Even when it is a circle, it is comparatively easy to compute with mathematical accuracy the height of OK, by computing the area of the segment OCB, and dividing it by the length of the line OB. As previously explained in Article 428, the two-thirds rule may be used even for circular arcs, when the arch is very flat. In this particular case, the two-thirds rule is far from applicable; and since for multi-centered curves no such rule is applicable, the general method will be here given. We divide the span (61.40) into 20 equal parts.

Practically this is most easily done by setting a pair of dividers by trial so that 10 equal spaces may be stepped off in the length of the half-span. From these division points on the line OB, erect perpendiculars to the center line of the arch rib OCB. The area of a curve bounded by a straight line at the bottom, and which has vertical and equally spaced ordinates, may be computed with very close accuracy by the adoption of *Simpson's rule.* If y_0 represents the ordinate at the beginning of the curve (and in this case $y_0 = 0$), while y_1 up to y_n represent the lengths of the several ordinates (y_n being the last ordinate, and, in this case, also equal to 0; and n being equal to the *even* number of divisions, in this case 20), then the area may be expressed by the formula:

$$\text{Area} = \frac{OB}{3n}\left\{ y_0 + 4\,(y_1 + y_3 + \ldots y_{(n-1)}) + 2(y_2 + y_4 + \ldots\ldots y_{(n-2)}) + y_n \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (59)$$

Applying this rule, we find that the area will be 640.50 square feet. Dividing this by the span, 61.40, we find that the height of vm above the line OB will be 10.59 feet. The approximate two-thirds rule would give us 10.17. Making a rough interpolation in the tabular form of Article 428, we could say that for an angle $2\,a$ equal to 106° 16′, the quantity to be added to the result by the two-thirds rule would be approximately 5 per cent. Adding 5 per cent to 10.17, we would have 10.68, which gives a rough check with the far more accurate value just found.

433. **Laying Off the Load Line.** We shall assume that the arch carries a filling of earth or cinders weighing 100 pounds per cubic foot, that the top of this filling is level, and that it has a thickness of one foot above the crown. Since concrete weighs about 150 pounds per cubic foot, we shall assume this weight of 150 pounds as the unit of measurement, and therefore reduce the ordinates of earthwork to the load line for the top of the earth, as shown in Fig. 232. We shall assume as an additional dead load a pavement weighing 80 pounds per square foot, and shall therefore lay off an ordinate of $\frac{80}{150}$ of a foot above the ordinates for the earth-filling load. For this particular problem, we shall only investigate a live load of 200 pounds per square foot, extending over one-half of the span from the abutment to the center. From our previous work in arches, we know that such a

loading will test the arch more severely than a similar unit live load extending over the entire arch; and therefore, if the arch proves safe for this eccentric load, we may certainly assume that it will be safe for a full load. These load lines are laid off similarly to the method elaborated in Article 409. The arch has already been laid off in equal horizontal sections, each having a width of 3.07 feet. The two end sections are slightly longer if we consider the entire load which is vertically over the extreme ends of the extrados of the arch. The 18 sections lying between the end sections, have a width of 3.07 feet, and a variable height which may be considered as extending from the top of the load line down to the intrados. We may therefore multiply the widths of these sections by their various heights, and by 150, and obtain the number of pounds weight on each section, and we find the loads as follows:

No. of Section	Loading	No. of Section	Loading
1	6,860	11	1,889
2	4,589	12	1,957
3	3,860	13	2,095
4	3,070	14	2,173
5	2,526	15	2,716
6	2,102	16	3,140
7	1,757	17	3,684
8	1,481	18	4,474
9	1,343	19	5,203
10	1,275	20	7,620

The sum total of this loading, which represents the total dead and live load on a section of the arch one foot wide (in the direction of the axis of the arch), is 63,814 pounds. We lay off these various loads on the right-hand side of the drawing in a vertical line, using a scale of 5,000 pounds per inch. Selecting a pole o_1 at random, we draw rays to the various points in the load line.

434. **Trial Equilibrium Polygon.** Commencing at the point O, we draw the segments of the trial equilibrium polygon parallel with the rays in the force diagram which run from the point O_1, to the load line, and obtain the trial equilibrium polygon OB_1. By drawing from o_1 the line o_1n parallel to the line OB_1, we obtain the point n on the load line, from which we draw an indefinite horizontal line which

will be the locus of the pole of the true equilibrium polygon. A vertical from the point o_1 intersects the horizontal from n in the point o_2, and this would be the pole of a trial equilibrium polygon whose closing line is horizontal, and whose vertical ordinates are equal to those of the corresponding vertical ordinates of the trial equilibrium polygon OB_1. It is only necessary to find the proper ratio by which these several ordinates should be multiplied, in order to find the corresponding ordinates of the special equilibrium polygon. It is also necessary to shift the entire trial equilibrium polygon up or down, so that the line $v'''m'''$ which corresponds to it shall coincide with the line vm which has already been drawn. The special line $v'''m'''$ corresponding to this trial equilibrium polygon, is found by satisfying Equation 56; but since dx is in this case a constant, it is found by determining the average value of z''', which is the distance from any point in the trial equilibrium polygon to the proper position of the line $v'''m'''$ corresponding to this equilibrium polygon. If the trial equilibrium polygon had been again redrawn with O_2 as a pole, it should terminate in the point B; but it is practically unnecessary to do this, since we may draw the line vm' parallel to OB_1, and measure the distance from vm' down to the various points of the trial equilibrium polygon OB_1. These various distances in the column headed z''' are as given in the accompanying tabular form (page 414).

We have here the rather unusual case that the trial equilibrium polygon is entirely below the line vm'; and the ordinates are all negative, instead of being partially positive and partially negative. In any case, the *algebraic* sum should be taken, which should be divided by the number of ordinates. In this case we find that the mean value of z''' is -3.44. Drawing the line $v'''m'''$ parallel to vm', and at a vertical distance of 3.44 feet below it, we find the position of the vm line for the trial equilibrium polygon OB_1. This line intersects the trial equilibrium polygon at the points h'' and k''. The student should note that it is a mere accidental coincidence that the point k'' comes almost exactly on the intrados. By drawing verticals from h'' and k'' to the line vm, we obtain the points h and k, which will be two points in the true equilibrium polygon.

435. **Pole Distance of the True Equilibrium Polygon.** It is necessary to satisfy Equation 58. We shall consider in this case that the various points in the arch-rib curve and in the special equilibrium

	y	z'	yz'	z'''	z''	yz''
1	1.55	−9.04	− 14.01	− 9.50	−6.06	− 9.39
2	5.15	−5.44	− 28.00	− 7.15	−3.71	− 19.11
3	7.65	−2.94	− 22.50	− 5.50	−2.06	− 15.76
4	9.70	−0.89	− 8.64	− 4.20	−0.76	− 7.38
5	11.35	+0.76	+ 8.64	− 3.20	+0.24	+ 2.72
6	12.70	+2.11	+ 26.80	− 2.35	+1.09	+ 13.84
7	13.75	+3.16	+ 43.45	− 1.70	+1.74	+ 23.93
8	14.45	+3.86	+ 55.80	− 1.25	+2.19	+ 31.65
9	14.95	+4.36	+ 64.45	− 0.90	+2.54	+ 37.95
10	15.20	+4.61	+ 70.07	− 0.65	+2.79	+ 42.40
11	15.20			− 0.55	+2.89	+ 43.90
12	14.95		− 73.15	− 0.60	+2.84	+ 42.45
13	14.45		+269.21	− 0.80	+2.64	+ 38.15
14	13.75			− 1.20	+2.24	+ 30.81
15	12.70		+196.06	− 1.80	+1.44	+ 18.27
16	11.35			− 2.60	+0.84	+ 9.53
17	9.70			− 3.65	−0.21	− 2.04
18	7.65			− 5.05	−1.61	− 12.32
19	5.15			− 6.85	−3.41	− 17.56
20	1.55			− 9.35	−5.91	− 9.16
	212.90		+392.12	−68.85 − 3.44		+335.60 − 82.72 +252.88

polygon are the points where these two lines are intersected by the load lines through the centers of the 20 sections. Therefore the points where these verticals intersect the center of the arch rib, give the points from which we measure down to the line *OB*, and obtain various values for *y* which are given in the tabular form above (Article 434). From Fig. 228 we may observe that *z'* equals the numerical difference between the value of *y* and the distance from *OB* up to the line *vm*. This distance has already been computed at 10.95 feet. Therefore, having scaled off as accurately as practicable the various values of *y*, it is unnecessary to scale off the values of *z'*, but merely to take the numerical difference (carefully observing the algebraic sign) between 10.59 and the various values of *y*. We thus obtain the values of *z'* as given in the tabular form. Since *z'* measures the ordinates to points in the curve, and since the curve is symmetrical about its center, it is unnecessary, in this case, to set down the values of *z'* on both sides of the center; and therefore only the values from 1 to 10, inclusive, are written down in the tabular form. Multiplying the corresponding values of y and z', we find the products as given under the heading yz'. Adding these products for the half-span of the arch, we find an algebraic summation of + 196.06; multiplying this by 2, we find that the algebraic summation for the entire arch is + 392.12.

The ordinates z'' are measured from vm to the trial equilibrium polygon when it has been shifted not only so that its closing line is horizontal, but also shifted vertically (up or down) so that its line $v'''m'''$ corresponds with vm; but it is unnecessary to draw it in that way, since we may measure the ordinates from the transposed line $v\,m'$, because we know that they are in each case the equal of the ordinates as they would be if the transposition had been actually made. But the lengths of these ordinates below vm' have already been determined; and since we know that $v'''\ m'''$ is 3.44 feet vertically below vm', we need only change the ordinates z''' by 3.44 (taking care of the algebraic sign), and we then obtain the values of z'' as given in the tabular form. Multiplying each value of z'' by the corresponding value of y, we obtain the various values (plus and minus) for yz'' as given in the last column of the tabular form. The algebraic sum of these quantities is + 252.88.

Since this value is less than the value of the summation of yz', we must select a smaller value of H, so that the values of z'' will be proportionately larger. We must therefore use a pole distance which shall be smaller than O_2n in the ratio of 252.88 to 392.12. To solve this graphically, we must draw an indefinite line ns, and lay off the distance ns equal to 392.12, at any convenient scale. Laying off a distance nt at the same scale so that it equals 252.88; we may then join s and O_2, and draw a line from t parallel to sO_2, obtaining the point O_3 which is the required pole of the special equilibrium polygon.

436. **Locating the True Equilibrium Polygon.** We know that the segments of the true equilibrium polygon must be parallel with the rays of the force diagram which has its pole at O_3, and also that it must pass through the points h and k on the line vm. The point h lies between loads 4 and 5; therefore we draw through the point h a line parallel to the ray of the force diagram from O_3 to the point on the load line between load 4 and load 5. Similarly, since the point k

lies between loads 16 and 17, we may draw through k a segment of the equilibrium polygon which is parallel to the ray from O_3 to the point on the load line which is between load 16 and load 17. In order to avoid inaccuracy, the segments of the equilibrium polygon should be drawn from these two segments each way toward the crown of the arch and each way toward the abutments. As a check on the work, these separate sections of the equilibrium polygons should accurately meet at the top of the arch, and they should also reach the last verticals through O and B at points which are on *the same horizontal line.* It is merely a coincidence that these points O'' and B'' are almost exactly at the same level as the lowest point of the skewbacks.

437. Maximum Moment under this Loading. An inspection of the diagram shows what might be expected, that the maximum moment occurs on the right-hand side of the arch, nearly under the center of the live load and very near to load 16. At this point the vertical distance z, between the equilibrium polygon and the center of the arch rib, is 0.55 foot, or 6.6 inches. The pole distance O_3n, scaled at 5,000 pounds per inch, indicates 22,650 pounds; therefore the moment at that point equals 22,650 × 6.6 = 149,490 inch-pounds. It may be observed, also, that the moment at the abutment scales exactly the same quantity, as nearly as it can be measured, but the moment is of contrary sign. In other words, the intrados will be in tension under load 16, and in compression at the two abutments. If this arch were reinforced with ¾-inch bars spaced 12 inches apart, in both the top and the bottom, there would then be two such bars in each section of the arch one foot wide; and the area A (see Equation 55) would be the area of *one* such bar, or 0.56 square inch. Since the depth of the arch at the abutment (h) is 21.25 inches, then $.4h$ equals 8.5; and, by substituting these quantities in Equation 55, we find that the moment of inertia is:

$$\begin{aligned} I &= \tfrac{1}{12} \times 12 \times 21.25^3 + 2 \times 0.56 \times 12 \times 8.5^2 \\ &= 9{,}593 + 971 \\ &= 10{,}564 \text{ biquadratic inches} \end{aligned}$$

Then, transposing the equation $M = \frac{pI}{c}$ into the equation $p = \frac{Mc}{I}$, we may substitute for M the value 149,490; for c the half-depth of the beam, 10.625; and for I the value found above, 10,564; and find that the unit-stress in the concrete at the abutment equals about 151 pounds per square inch.

For this particular case of loading, the moment at the crown is almost zero, since it may be observed from the drawing, that the special equilibrium polygon crosses the center line of the rib about 18 inches at the right of the center. This crossing indicates a point of contraflexure where there is no moment. Also, since the equilibrium polygon is below the center line of the rib at the crown, it indicates that such moment as there is for this loading is negative; or, in other words, the tension is on the upper side of the rib. This same kind of moment exists on the entire left-hand side of the arch for this loading. It should also be observed that there is another point of contraflexure a few feet from the right-hand abutment. It will be shown later that the thrust at the abutment has a greater intensity per square inch than the maximum compression or tension due to moment. This practically means that the compression side of the arch is subjected to the combined compressions due to thrust and moment; while, on the other side, the thrust more than neutralizes the tension, and actually relieves it altogether.

Near the crown of the arch, the thrust is not so great, and will not wholly neutralize the tension due to moment. In order to withstand the various stresses, the rib must have a larger cross-section than would be required for moment alone. This means that the equation developed in Article 271, Part III, $M = 62\,bd^2$, can be utilized only in a roundabout way. For example, the moment at the abutment was computed above as 149,490 inch-pounds. But a section 12 inches wide and 19.125 inches deep to the reinforcement, should withstand a moment of 272,300 inch-pounds with 0.43 per cent of steel. The above moment is only 55 per cent of this. Therefore 55 per cent of the steel, or $.55 \times .0043bd = .00236bd$ of steel, could safely carry the tension. But the actual ratio of steel adopted was .00245. As shown above, the steel at the abutment is unnecessary for transverse moment and for this condition of loading. Nearer the crown, the moment is less; but the relief to tension afforded by thrust is very much less, and the steel has much more to do. With other conditions of loading, it will also be different; and therefore the same amount of steel ¾-inch bars spaced 12 inches, in both top and bottom, is used throughout.

438. **Maximum Thrust Due to this Loading.** The thrust at any point of the arch is measured by the projection onto the tangent to the arch at that point, of the corresponding ray of the force diagram. Since the rays of the force diagram which are parallel to the segments of the equilibrium polygon are approximately tangent to the arch rib, it is approximately true to measure the thrust at any point by measuring the corresponding rays of the force diagram; but a more precise value may be found by drawing a line from one end of a ray parallel to the corresponding tangent, and projecting the ray on it. This method is particularly useful, since it measures at the same time the amount of the shear at that point, as will be explained in the next article. Since the increase in the thickness of the arch is comparatively slight from the crown to the abutment, in this particular problem, and since the amount of the thrust evidently increases very rapidly toward the abutment, the critical point, so far as the thrust is concerned, is evidently at the abutment. Therefore we may draw from the lower end of load 20, a line parallel to the tangent to the curve at the abutment, and obtain the line *wx*, which scales 40,200 pounds, and which therefore measures the thrust at the abutment. Since the abutment is 21.25 inches thick, the area of a section one foot wide is 255 square inches. Dividing this into 40,200, we have a quotient of 158 as the unit-compression per square inch due to thrust. Adding the value of the compression at the intrados which is due to moment (151), to the compression just found for thrust, we have a total of 309 pounds per square inch at the abutment. This compression does not allow for temperature stresses, which will be computed later, and which may exist simultaneously with the stresses due to moment and to thrust.

439. **Shear at Any Section.** The shear at any section is measured by the projection onto the normal to the arch rib, of the corresponding ray of the force diagram. It is seldom that the shear is a serious factor in the design of an arch. Whenever (as in the case just being worked out) the equilibrium polygon coincides approximately with the arch rib, the shear is very small. When the amount of the thrust is definitely computed, as determined above, the amount of the shear at the same point may be readily determined at the same time.

For example, the shear at the abutment is the projection of the ray O_3x onto the line O_3w, which is parallel to the normal to the curve at B. This line, scaled at the rate of 5,000 pounds per inch, indicates a shear of about 2,200 pounds. Dividing this by 225, the area of the section of the arch at that point, the unit-shear is less than 10 pounds per square inch, which of course may be neglected. The shear in any arch may be very easily tested by noting the portion of the special equilibrium polygon which makes the largest angle with the direction of the arch rib at any point; the larger the angle, the greater the shear. If the arch is tested at that point, and the shear is found to be insignificant, or well within the power even of plain concrete to carry, there is no need for further investigation.

On the other hand, there are minor stresses which occur in arches as well as other concrete structures, which must be provided for. These are caused by a possible excessive concentration of loading; possible structural weakness due to a poor quality of concrete in comparatively limited areas; the effect of slight settlement of the foundations, etc. On account of these various stresses, which are more or less non-computable, it is the usual practice to insert bars, which are not only useful in taking up shear, but also tend to bind the whole structure together, make it act more nearly as a unit, and permit the structural weakness in local spots to be made up by the strength of the sounder portions of concrete around it; and therefore shear bars are put in, such as are illustrated in Fig. 235. These bars are *laced* between the upper and lower sets of bars that run parallel with the axis of the arch. By this means, not only are the upper and lower sets of reinforcing bars tied together, but the bars have such a direction that they can take up any shearing force which may, by any chance, be developed.

440. **Temperature Stresses.** The provision which should be made for temperature stresses in a concrete arch, is often a very serious matter, for the double reason that the stresses are sometimes very great, and also because the whole subject is frequently neglected. It will be shown later that the stresses due to certain assumed changes of temperature may be greater than those due to loading. There is much uncertainty regarding the actual temperature which will be assumed by a large mass of concrete. The practice which is common and proper with metal structures, is not applicable to masonry arches.

A steel bridge, with its high thermal conductivity, will readily absorb or discharge heat; and it is usually assumed that it will readily acquire the temperature of the surrounding air. On the other hand, concrete is relatively a non-conductor. No matter what changes of temperature may take place in the outer air, the interior of the concrete will change its temperature very slowly. One test bearing on this subject was conducted by burying some electrically recording thermometers in the interior of a large mass of concrete, and recording the temperatures as they varied for a period of ten months, which included a winter season. It was found that the total variation of temperature was but a few degrees.

It is probably safe to assume that even during the coldest of winter weather the temperature of the interior of a large mass of concrete will not fall below that of the mean temperature for the month. Since the Weather Bureau records for temperate climates show that the *mean* temperature for a month, even during the winter months, is but little if any below freezing, it may usually be assumed that for concrete a fall of 30 degrees below the temperature of construction (say 60°) will be a sufficient allowance. A rise of temperature to 90° F. is probably much greater than would ever be found in an arch of concrete. The earth and pavement covering protect the arch from the direct action of the sun. Even in the hottest day, the space under a masonry arch seems cool, and the real temperature of the masonry probably does not exceed 70°, even if the outer air registers 95°. If we therefore calculate the stress produced by a change of temperature of 30 degrees from the temperature of construction, we are probably exceeding the real stresses produced. Even if this extreme limit should be sometimes exceeded, it simply lowers temporarily the factor of safety by a slight percentage. The following demonstration, which has been adapted from that in Church's "Mechanics" Section [illegible], will be described, but without demonstrating the fundamental equations, which depend on elaborate mathematical reasoning. Assume that:

l = The length of the arch [illegible];
t = Temperature in degrees Fahrenheit [illegible] $t - t'$ represents the [illegible] of temperature expressed in degrees Fahrenheit;
c = Coefficient of expansion for concrete, which is here assumed to be .0000055;

H_t = Imaginary horizontal forces which would produce the same stress on the arch as that produced by any assumed change of temperature;
d = Height of the line of action of H_t above the abutment level.

The other symbols are as previously used. In Fig. 233, the heavy full line between O and B represents the arch rib in its normal and unstrained condition. If it were free to expand, it would assume some such form as indicated by the dotted line $O'C''B'$; but since the arch is fixed at the abutments, the arch rib is forced to preserve the same distance between the abutments, and the tangents to the rib at the abutments remain fixed in direction. The rib therefore is forced to the form $O\ C'''\ B$. Of course the relative distortion is enormously exaggerated. These two requirements furnish the data for the equations given below. If the arch rib were assumed to be extended by the addition of the imaginary arms as shown, and the two equal and opposite forces (H_t) were acting as indicated, these forces would hold the arch rib rigid against the tendency to expand or to change its direction at the abutments. Considered as an example of the general case of an arch rib acted on by forces, we may consider that the arch rib is acted on only by these two equal and opposite forces H_t. Their line of action is the *vm* of the problem, as previously explained; and d is therefore the distance from the abutment up to this line *vm*. The equation which is based on the fact that the span of the arch is not changed by the temperature, or that it does *not* expand from OB to $O'B'$, is as follows:

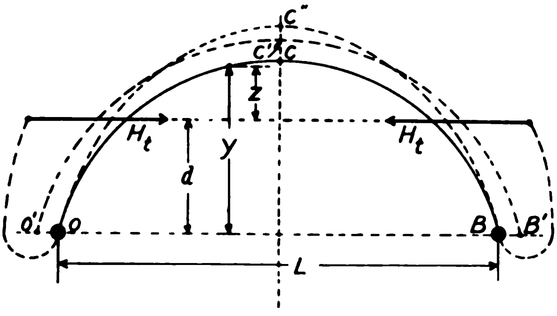


Fig. 233. Temperature Stresses in Arch Rib.

$$L(t-t_0)c = \frac{H_t}{E}\int_O^B \frac{(y-d)\,y\,ds}{I} \quad \ldots\ldots (60)$$

The fact that there was no change in direction of the tangents, gives rise to the equation:

$$\frac{H_t}{E}\int_O^B (y-d)\frac{ds}{I} = O \quad \ldots\ldots\ldots\ldots (61)$$

As before, we must allow for the variable moment of inertia, I; but since $I = nI_c$, and $ds = ndx$, then $\frac{ds}{I} = \frac{dx}{I_c}$, and Equation 61 becomes:

$$\frac{H_t}{EI_c}\int_0^B (y-d)\,dx = 0.$$

But since H_t, E, and I_c are all constants, we may drop them in this case, and write the equation:

$$\int_0^B (y-d)\,dx = 0.$$

Since d is a constant, this equation virtually means that the summation of all the y's between B and O equals d times the number of sections. But the summation of the y's, as shown in the tabular form in Article 434, is 212.90 feet, or 2,544.8 inches. Dividing this by 20, we have 127.24 inches, which is the value of d, or the height of *vm* above the abutment, and is the distance above the abutment of the line of action of the pair of imaginary forces H_t which will produce the same stress in the arch as the assumed change in temperature. Equation 60 may be transposed so as to solve for H_t, and we may write (substituting $\frac{dx}{I}$ for $\frac{ds}{I}$):

$$H_t = \frac{L\,(t - t_0)\,c\,E\,I_c}{\int_0^B (y^2 - yd)\,dx}$$

For practical use, we transform the denominator of the above expression into the summation of $y^2 - yd$ for each point, times the constant distance between these points (= 3.07 feet, or 36.84 inches).

The various values of y are obtained by taking the figures in the first column of the tabular form in Article 434, and multiplying each by 12 to reduce to inches. Squaring the several values of $12y$ for [illegible] and adding the squares, we may multiply the sum [illegible] as the sum of the squares; but the sum of all [illegible] and we may multiply this by 127.24 [illegible] summation of yd. Subtracting this from [illegible] which multiplied by the [illegible] 36.84, which is the denominator [illegible] the moment of inertia of the arch rib at the center, in a similar manner to the computation in Article 437, we find a value of 6,529 biquadratic inches.

Substituting all the known values in the above equation, we have:

$$H_t = \frac{720 \times 30 \times .0000065 \times 2{,}400{,}000 \times 6.529}{2{,}495{,}984} = 881.$$

$(y - d)$ for center $= 183 - 127.2 = 55.8$
$(y - d)$ for abutments $= 0 - 127.2 = - 127.2.$

The moment produced by the assumed change of temperature of 30 degrees, is therefore as follows:

At the center, $881 \times 55.8 = 49{,}160$ inch-pounds.
At the abutments, $881 \times (- 127.2) = - 112{,}063$ inch-pounds.

It was computed above, that a moment of 149,490 inch-pounds at the abutment would produce a unit-stress in the concrete of 151 pounds per square inch; therefore a unit-stress produced in the abutment by a moment of 112,063 pounds would be $\frac{112{,}063}{149{,}490} \times 151 = 113$ pounds per square inch. In cold weather the effect of the moment due to temperature would be to produce compression at the intrados at the abutment, and tension at the intrados at the crown; but it was found above, that the compression at the intrados at the abutment due to transverse moment and to thrust totaled 309 pounds per square inch. Adding the stress due to temperature, we have a total of 422 pounds per square inch. If the reduction of temperature below the temperature of construction was more than 30 degrees, the stresses would be increased in direct proportion. If the reduction of temperature was, say, 40 degrees, instead of 30 degrees, the added stress due to temperature would be about 151 pounds per square inch. On the other hand, during warm weather, the effect of a rise in temperature will be to relieve the strain; and the net compression or tension will be less than that which would be due to direct loading and to thrust. For the loading which has been computed, the moment due to loading at the crown is very small, and is in the opposite direction to the moment usually produced by a load over the entire arch. It is generally true that in cold weather the arch is stressed by the *sum* of the stresses due to transverse moment, thrust, and temperature; in warm weather the stresses usually tend to counteract each other. Cold weather is therefore a critical time for an arch, and the time when an excess of live load would be particularly dangerous.

441. **Stresses Due to Rib Shortening.** The compression in a rib results in shortening the arch rib very slightly; and this produces precisely the same effect in altering the moment as an equivalent fall in temperature. For example, in the above case, we have at the abutment a thrust of 296 pounds per square inch; dividing this by E, the modulus of elasticity, 2,400,000, we have .0001233, the proportional shortening; dividing this by .0000065, the coefficient of expansion, we find that the thrust due to this rib shortening is the equivalent of a reduction of temperature of 19 degrees. Since we have found that a reduction of temperature of 30 degrees produced a unit-stress of 113 pounds per square inch, a virtual reduction of 19 degrees would produce a unit-stress of 72 pounds per square inch. Since such a stress is always the same as that due to a *reduction* of temperature, and since this always has the effect of increasing the stresses for usual loading, such unit-stress must be added to the value found above; therefore, adding this 72 pounds per square inch to the total previously found (422), we have a unit-compression at the intrados at the abutment, of 494 pounds per square inch.

442. **Testing this Arch for Other Loading.** A live load of 200 pounds per square foot over the entire arch would unquestionably increase the thrust over the entire arch, especially at the abutments. The stress due to shortening will of course be increased in proportion to the increase in the thrust. The stress due to moment cannot be accurately predicted. Of course such an examination and test for full loading should be made in the case of any arch to be constructed, and should be worked out precisely on the same principles and in general by identically the same method as was used above.

To test the arch for a concentrated loading such as would be produced by the passage of a road roller, or, in the case of a railroad bridge, by an especially heavy locomotive, the test must be made by assuming the position of that concentrated load which will test the arch most severely. Ordinarily this will be found when the concentrated load is at or near one of the quarter points of the arch. The only modification of this test over that given above in detail, is in the drawing of the load line, but the general method is identical.

443. **Testing an Arch with Variable Moments of Inertia.** It has already been indicated how the equations on which the arch theory is based may be simplified when the moment of inertia is constant.

The above problem was worked out on the basis that the moment of inertia varied in the ratio of $\frac{ds}{dx}$. In either case the solution is considerably simplified. Arches are frequently designed where the moment of inertia varies according to some other law. The very frequent practice is to increase the thickness of the arch toward the abutment much more rapidly than the $\frac{ds}{dx}$ rule would call for, and thus increase the moment of inertia of the arch much more rapidly. In such a case, Equations 49 must be used; and the summations must be made up by computing for each unit-section the value of the moment of inertia for that point, and by measuring ds along the length of the arch rib. This means also that the sections of dead and live load, instead of having a constant width (as in the above problem), have a variable width, and the loads must be separately computed. While there is nothing especially difficult about such a solution, it involves considerably more work.

HINGED ARCH RIBS

444. **General Principles.** The construction of hinged arches of reinforced concrete is very rare, but is not unknown, and will probably come into greater use when their advantages are more fully realized. We may consider that structurally they consist of curved ribs which have hinges at each abutment, and which may or may not have a hinge at the center of the arch. The advantage of the three-hinged arch lies in the fact that it is not subject to temperature stresses. The two-hinged arch is partially subject to temperature stresses, but not to so great an extent as the fixed arch, since the arch rib is not held rigid at the abutments as in the case of the fixed arch. Practically the *hinges* are made by having at each hinge a pair of large cast-iron plates which are a little larger than the size of the rib, and which have at their centers a bearing for a *pin* of due proportionate size. The bearings are so made that one may turn, with respect to the other, about the axis of the pin through an angle of a very few degrees.

445. **Arch of Two Hinges.** The third of Equations 48 must be satisfied, which practically means that Equation 54 must be satisfied. This means that we must find a trial equilibrium polygon, and increase or decrease its pole distance so that the summation of the

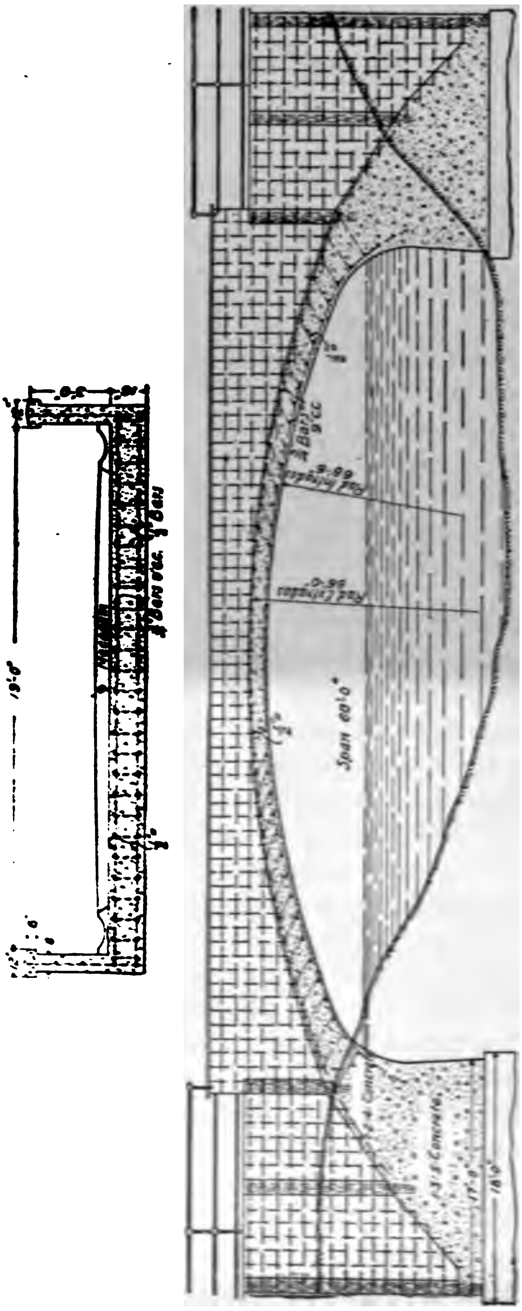


Fig. 234. Berkley Bridge, Berks County, Pennsylvania.—Reinforced-Concrete Arch Rib.

products based on x'' shall equal the summation of similar products based on x'. But in this case the special equilibrium polygon passes through the abutment points, and there is no moment at the abutment. Therefore, after having found the pole distance of the special equilibrium polygon, we may draw the special equilibrium polygon by commencing at one abutment point; and, as a check on the work, we should find that it passes through the other abutment point. The maximum moment due to temperature will be at the center of the arch rib, and will be based on an equation similar to Equation 60, which may be used by calling $d = o$.

Equation **61** does not apply, since the ends of the arch rib are free to turn at each abutment.

446. Arch of Three Hinges. A three-hinged arch is a still more simple case, since none of the three fundamental equations (Equation **48**) which are used for fixed arches needs to be satisfied. It is only necessary to find the special equilibrium polygon which will pass through the two abutment hinges and the center hinge. There are no temperature stresses and no stresses due to the shortening of the rib. It may thus be said that a three-hinged arch is much more simple to calculate, and its stresses are more definite. The construction of the hinges will of course add somewhat to the cost, and probably add more than any saving which might be made by a reduction in the cross-section of the arch. Probably the greatest advantage of three-hinged arches lies in their immunity from damage which may result from a settlement of the foundations. It has been assumed, in considering the theory of fixed arches, that the foundations are absolutely immovable. A settlement of either abutment of a fixed arch with reference to the other abutment, will inevitably result in stresses in the arch rib which might easily be greater than any stresses to which the arch rib would be subjected either on account of the loading or through change in temperature. The failure of many arches is

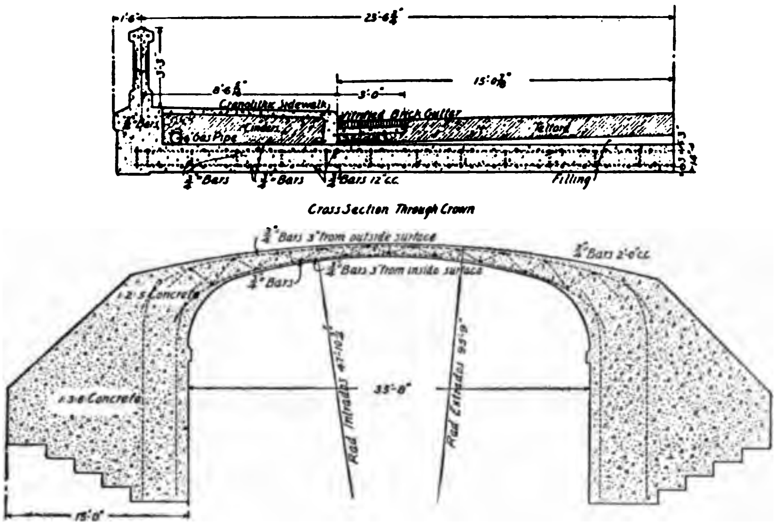


Fig. 235. Reinforced-Concrete Oblique Arch. Graver's Lane Bridge, Philadelphia, Pa.

unquestionably due to this cause. An arch rib with either two or three hinges is absolutely immune from any such danger; and there is therefore a strong argument for the use of hinged arches when the arch must be placed on foundations which are so uncertain that a settlement of either foundation is quite possible. Of course an equal settlement of both foundations would do no damage, but the equality of such a settlement could never be counted on.

447. Description of Two Reinforced-Concrete Arches. In Figs. 234 and 235 are shown the details and sections of two reinforced-concrete arches having fixed abutments, which have been recently erected. The first bridge has a nominal span of 60 feet between the two faces of the abutments. On account of the great thickening of the arch rib near the abutment, the virtual abutments are practically at points which are approximately 26 feet on each side of the center. The method of reinforcing the spandrel and parapet walls is clearly shown in the figure. The side view also gives an indication of some buttresses which were used on the inside of the retaining walls above the abutments in order to reinforce them against a tendency to burst outward.

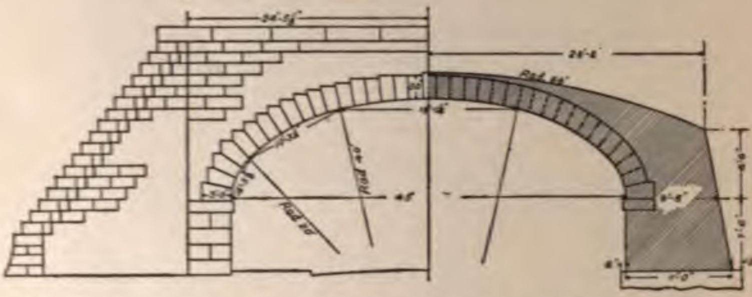


Fig. 236. Stone Arch on Line of New York, New Haven & Hartford Railroad.

Fig. 235 shows a bridge which is slightly oblique, and which spans a double-track railroad. The perpendicular span between the abutments is 34 feet, but the span measured on the oblique face walls is 35 feet 8 inches. In this case, similarly, the arch is very rapidly thickened near the abutment, so that the virtual abutment on each side is at some little distance out from the vertical face of the abutment wall. In both of these cases, the arch rib was made of a better quality of concrete than the abutments.

The arch of Fig. 234 was designed for the loading of a country highway bridge; that of Fig. 235 was designed for the traffic of a city street, including that of heavy electric cars.

448. **Stone Arch.** In Fig. 236 is shown a stone arch on the New York, New Haven & Hartford Railroad at Pelhamville, N. Y. This arch was constructed over a highway, and the length of its axis is sufficient for four overhead tracks. The span is 40 feet, and the rise is 10 feet above the springing line, the latter being 7 feet 6 inches above the roadway. The length of the barrel of the arch is 76 feet.

The arch is a five-centered arch, the intrados corresponding closely to an ellipse, the greatest variation from a true ellipse being 1 inch. The theoretical line of pressure is well within the middle third, with the full dead load and partial live load, until the short radius is reached, where it passes to the outer edge of the ring-stone, and thence down through the abutment. There is a joint at the points where the radii change, to simplify the construction.

The stone is a gneiss found near Yonkers, N. Y., except the keystone, which is Connecticut granite, and the coping, which is bluestone from Palatine Bridge, N. Y.

REVIEW QUESTIONS.

PRACTICAL TEST QUESTIONS.

In the foregoing sections of this Cyclopedia numerous illustrative examples are worked out in detail in order to show the application of the various methods and principles. Accompanying these are examples for practice which will aid the reader in fixing the principles in mind.

In the following pages are given a large number of test questions and problems which afford a valuable means of testing the reader's knowledge of the subjects treated. They will be found excellent practice for those preparing for Civil Service Examinations. In some cases numerical answers are given as a further aid in this work.

REVIEW QUESTIONS

ON THE SUBJECT OF

MASONRY AND REINFORCED CONCRETE

PART I

1. Discuss the qualities of good building stone.

2. Describe the tests that you would apply to determine the qualities of a building stone.

3. Describe the distinguishing characteristics of limestone, sandstone, and granite; and the uses for which these characteristics make them especially suitable.

4. What is meant by the seasoning of stone; and why is it of importance in building construction?

5. What are the distinguishing characteristics of a good quality of brick?

6. Discuss the crushing strength of various kinds of brick.

7. Describe briefly the characteristics and method of manufacture of Sand-Lime brick.

8. Describe the essential features in the manufacture of concrete building blocks.

9. Describe the various changes that take place in transforming the original limestone into lime, and from that into the hardened mortar.

10. What is the essential characteristic of hydraulic lime?

11. What is the essential characteristic of slag cement, and for what kind of use is it especially suited?

12. What is the essential difference between Natural cement and Portland cement?

13. How would you obtain testing samples from a carload of cement?

14. Why must kerosene or benzine be used instead of water in determining the specific gravity of cement?

15. While determining the specific gravity of cement by Le Chatelier's apparatus, it was observed that the introduction of 64 grams of cement increased the volume by 20.3 cubic centimeters. What was the specific gravity of the cement?

16. How many holes should there be in each square inch of a No. 100 sieve?

17. If a certain brand of cement requires 30 per cent of water to produce a paste of standard consistency, how much water should be used in a 1:3 mortar?

18. What is "initial set?" How soon should it develop, and what is the standard test for the time?

19. What general precautions should be taken in handling cement, to prevent deterioration?

20. How much tensile strength should be developed by briquettes of neat Natural cement, and also by those of neat Portland cement, in 7 days? Also in 28 days?

21. What are the desirable characteristics of sand for use in mortar?

22. Why does sand with grains of variable size produce a stronger concrete?

23. What are the characteristics of various kinds of broken stone and gravel which have an influence on their value in concrete?

24. What practical method would you adopt to mix a large amount of lime mortar in the proper proportions?

25. What is the effect of using lime in cement mortar?

26. Describe in your own words the principles underlying the mixing of concrete so as to obtain the best possible product.

27. Discuss the compressive strength of concrete, and its increase with the richness of the concrete.

28. What general principles must be followed to obtain the densest concrete when using sand and stone of definite sizes? How would you determine the required proportions?

29. Assume that the voids in the sand are measured to be approximately 40 per cent, and that the voids in the stone are approximately 45 per cent. Using barrels containing 3.8 cubic feet of

cement, how much cement, sand, and stone will be required for 100 cubic yards of 1:3:6 concrete?

30. With cement at $1.25 per barrel, sand at $1.00 per cubic yard, and broken stone at $1.40 per cubic yard, the cost including delivery on the site of the work, what will be the cost on the mixing board, per cubic yard of 1 : 3 : 6 concrete?

31. Under what conditions is it proper to use *dry* concrete?

32. What is the danger in the excessive ramming of very wet concrete?

33. Why is there any practical difficulty in bonding old and new concrete?

34. What is the effect of the freezing of concrete before it is set? How can concrete be safely placed in freezing weather?

35. Describe in detail how you would make concrete watertight by varying the proportions or by the use of cement grout.

36. Describe in detail the fundamental principle and the method of application of the Sylvester process.

37. Describe the method of waterproofing by the use of felt and asphalt, or by the use of asphalt alone.

38. What form of bitumen should be used for waterproofing purposes?

39. Discuss the effectiveness of concrete in preserving imbedded steel from corrosion.

40. Discuss the protection afforded to imbedded steel by the concrete, against fire.

41. What precautions should be taken to insure that hand-mixed concrete is properly mixed?

42. Discuss the relative strength of machine-mixed and hand-mixed concrete.

43. What tests should a high-carbon steel satisfy in order to be suitable for reinforcing concrete?

REVIEW QUESTIONS

ON THE SUBJECT OF

MASONRY AND REINFORCED CONCRETE

PART II

---

1. Define the different classes of masonry with respect to the dressing of the stones.

2. Give an outline of the method of dressing a stone which shall have a warped surface.

3. What is the purpose of bonding? Describe several ways in which it is accomplished.

4. A square pier in a building is to carry a load of 420,000 pounds; the pier is to be made of squared-stone masonry. What are the proper dimensions of the pier?

5. What are the elements affecting the cost of stone masonry?

6. Describe the various kinds of bonds used in brick masonry.

7. What tools are used, and how are they employed in the operation of quarrying and dressing stone for ashlar masonry?

8. Describe the various methods used in measuring brickwork.

9. A brick pier is 20 feet high; it is required to carry a load of 400,000 pounds, and is to be laid in a 1 to 2 natural cement mortar. Assume that the pier is to be square, what should be its cross-sectional dimensions?

10. Assuming that two-men stone is to be used in making rubble concrete, what will be the proper proportions of cement, sand, small broken stone, and rubble in such a concrete?

11. Describe the method of depositing concrete under water, using buckets.

12. What precautions must be taken when depositing concrete under water through a tube?

13. Describe the tests for determining the suitability of clay for use as clay puddle.

14. How would you test the bearing power of a soft soil?

15. Discuss the bearing power of various kinds of soil.

16. Describe some of the methods of improving a compressible soil.

17. Describe some of the methods of preparing the bed for foundations on various kinds of soil.

18. What is the purpose of a footing?

19. The wall of a building has a thickness of 2 feet; the total load on the wall has been computed as 16,000 pounds per running foot of the wall; the soil is estimated to carry safely a load of 3,000 pounds per square foot. What should be the thickness and width of limestone footings to support this wall on such a soil?

20. Classify the various kinds of piles, describing their uses.

21. Under what conditions do timber piles rapidly decay?

22. What are the most necessary specifications for timber piles?

23. A wall having a weight of 15,000 pounds per running foot is to be built on two lines of piles placed $2\frac{1}{2}$ feet apart transversely. It is found that piles driven 20 feet into such a soil have an average penetration for the last five blows of 1.5 inches, when a 2,500-pound hammer is dropped 24 feet. What is the bearing power of such piles, and how far apart must they be placed longitudinally in order to carry that wall?

24. Discuss the advantages and disadvantages of drop-hammer and steam-hammer pile-drivers, and the use of the water-jet.

25. What are the relative advantages and disadvantages of concrete piles compared with wooden piles?

26. What is a grillage, and what is its purpose?

27. What combination of circumstances justifies the use of a cofferdam?

28. What is the essential disadvantage involved in the use of a crib as a foundation for a pier?

29. What general constructive principle is involved in the sinking of a hollow crib through a soft soil?

30. What is the fundamental principle involved in the use of pneumatic caissons, and what is the practical limitation as to the depth below the surface?

31. Describe the several ways in which a retaining wall may fail.

32. A retaining wall whose height is 24 feet is surcharged with earth at the natural slope; assuming 35° as the angle of repose for that material, and that the wall has a batter of 1 in 4 on the front face and a vertical rear face, what should be the dimensions of the wall?

33. What are abutment piers; and under what circumstances is their use desirable?

34. What are the general principles governing the design of culverts?

35. What precautions are necessary in the design of a stone box culvert, to prevent its being washed out?

36. What precaution should be taken to prevent a concrete sidewalk from being broken up by frost?

37. Describe the method of making and finishing the top surface of a concrete sidewalk.

REVIEW QUESTIONS

ON THE SUBJECT OF

MASONRY AND REINFORCED CONCRETE

PART III

---

1. Why is there but little if any structural value to a beam made of plain concrete?

2. State briefly the fundamental reasons for the economy of using concrete for compressive stresses, and steel for tensile stresses.

3. What is meant by the *neutral axis* of a beam?

4. What is the essential difference between the elasticity of concrete under compression and that of steel or wood?

5. Develop the formula (Equation **11**) for the summation of the compressive forces in a concrete beam, employing your own language altogether, and elaborating in detail every step in the line of argument.

6. What is the practical effect of using a lower percentage of steel than that called for by the theory (Equation **18**)? Is there any economy in using less steel?

7. What is the practical effect of using more steel than the theory calls for? Does it make the structure any stronger?

8. Develop a series of equations (similar to Equation **23**) on the basis of 1:2½:5 concrete whose modulus of elasticity (E_c) is assumed at 2,650,000, and whose ultimate crushing strength (c') is assumed at 2,200 lbs.

9. Using a factor of 2 for dead load and a factor of 4 for live load, what is the maximum permissible live load which may be carried on a slab of 1:2½:5 concrete with a total actual thickness of 6 inches and a span of 8 feet?

10. If a roof slab is to be made of 1:3:5 concrete and designed to carry a live load of 40 pounds per square foot on a span of 10 feet, what should be the thickness of the slab, and the spacing of ⅜-inch square bars?

11. A beam having a span of 18 feet is required to carry a live load of 12,000 pounds uniformly distributed. Using 1:3:5 concrete and a factor of 4, what should be the dimensions of the beam whose depth is approximately twice its width?

12. What will be the intensity per square inch of the maximum vertical shear in the above beam?

13. What are the two general methods of providing for diagonal shear near the ends of the beam?

14. Make a drawing of the beam designed in Question 13, showing especially the reinforcement and the method of providing for the diagonal shear.

15. Discuss the advantage of using steel with a high elastic limit, and also the possible danger in such use.

16. Make a design for a slab of 1:3:5 concrete, reinforced in both directions, which is laid on I-beams spaced 10 feet apart in each direction.

17. What is the general structural principle which makes T-beams more economical and efficient than plain rectangular beams having the same volume of concrete?

18. What assumption is made regarding the distribution of compressive stress in a T-beam?

19. How is the width of the flange of a T-beam usually determined?

20. What principles govern the determination of the proper width of the rib of a T-beam?

21. Recompute the numerical problem, Example 1, Article 291, on the basis that the beams are to be spaced 6 feet apart?

22. Make complete drawings of the reinforcement of the floor-slabs and beams (Question 21), making due provision for shear, and making all necessary checks on the design as called for by the theory?

23. What will be the bursting stress per inch of height at the bottom of a concrete tank having an inside diameter of 10 feet, designed to hold water with a depth of 40 feet? What size and spacing of bars will furnish such a reinforcement?

24. With a nominal wind pressure of 50 pounds per square foot, on a flat surface, what will be the intensity of the compression on the leeward side of the tank, allowing also for the weight of the concrete, and assuming a thickness of 12 inches?

25. On the basis of the approximate theory given in the text, what would be the required steel vertical reinforcement for the above described tank?

26. Design a retaining wall to hold up an embankment 30 feet high, making a cross-sectional drawing and plan drawing similar to Fig. 113, assuming that the buttresses are to be 12 feet apart.

27. Compute the required detail dimensions and the reinforcement for the box culvert illustrated in Fig. 115, on the basis that the culvert is to be 10 feet wide, 12 feet high, supporting an embankment 15 feet deep, and also a railroad loading of 1,500 pounds per square foot.

28. A column is to be supported on a soil on which the safe load is estimated at 6,000 pounds per square foot; the column carries a total load of 210,000 pounds; the column is 22 inches square; what should be the dimensions of the footing, and how should it be reinforced?

29. A pair of columns which are 12 feet apart, one of which carries a load of 210,000 pounds and the other a load of 150,000 pounds, are to be supported on the same soil as described above. What should be the dimensions of the compound footing which will carry both of these columns, and what should be its reinforcement? Make a detail drawing similar to Fig. 112, but showing all the dimensions.

30. Discuss the various ways in which steel may be used to reinforce a concrete column.

31. What should be the steel reinforcement of the column described in Question 28, on the basis that the compressive stress in the concrete shall not exceed 400 pounds per square inch?

32. In case the line of pressure on the column of Question 31 should be 3 inches away from the center of the column, what would be the maximum intensity of the pressure per square inch?

33. A column which is to carry a working load of 230,000 pounds is to be reinforced with spiral reinforcement, the spiral having a diameter of 18 inches. Assuming high-carbon steel, what should be the pitch and size of the spiral rod?

REVIEW QUESTIONS

ON THE SUBJECT OF

MASONRY AND REINFORCED CONCRETE

PART IV

1. What are the difficulties in obtaining a satisfactory outer surface of concrete?

2. Describe two successful methods of obtaining a good outer surface.

3. When and how can acid be properly used in treating a concrete surface?

4. What pigments should (and should not) be used for coloring concrete?

5. Describe the various methods for finishing concrete floors.

6. How may efflorescence be removed from masonry surfaces?

7. What are the advantages and disadvantages of continuous mixers and batch mixers?

8. What are the practical difficulties and disadvantages, in the operation of automatic measuring machines, of measuring the materials of concrete?

9. Discuss the various engines and motors which are used to operate a concrete plant.

10. Describe the various methods of charging mixers.

11. Describe some of the methods of economically transporting concrete.

12. Make a sketch and plan for the concrete plant for a 6-story building, 40 feet by 100 feet; or, describe, with comments and sketch, the plant of some similar building actually being erected.

13. What are the advantages of using a portable concrete plant for laying the concrete foundations for pavements?

14. How would you wash sand when the magnitude of the work will not justify the employment of special machinery?

15. What methods are used to prevent the forms or centering from adhering to the concrete?

16. What precautions are taken to prevent the lumber in the forms from swelling or buckling?

17. How long should the forms and centering for reinforced concrete remain in place under various conditions?

18. What general principle must be followed to obtain the maximum economy in designing the forms for reinforced-concrete work?

19. Describe various devices for holding column forms together.

20. How are I-beams utilized to support the forms for concrete slabs laid on them?

21. Make an estimate of the cost of the forms for the building described in Article 384.

22. Make a sketch design for the forms for a vertical wall ten feet high, six inches thick, and twenty feet long.

23. Describe the methods of lowering the centering under arches.

24. What are the proper dimensions for a hemlock beam carrying a distributed load of 6,000 pounds on a span of 12 feet, so that the deflection shall not exceed $\frac{1}{8}$ inch?

25. What should be the dimensions of a column of hemlock 12 feet high, to support safely a load of 15,000 pounds?

26. Make up a bar list and sketches, similar to those of Figs. 175 to 180, with dimensions, for the bending of the bars in one panel as sketched in Fig. 109.

27. What are the several methods of bonding old and new concrete in floor construction?

REVIEW QUESTIONS

ON THE SUBJECT OF

MASONRY AND REINFORCED CONCRETE

PART V.

1. Draw on a sheet of drawing paper four lines which are *not* parallel, and whose directions and lengths represent the directions and intensities of the forces. Then determine graphically the resultant of the four forces.

2. Draw four parallel vertical lines which represent the location, direction, and intensities of four parallel forces. Select two points, one on either side of the group of forces, which are to be considered as abutment points. Find, first, the position and intensity of the resultant of the four forces; and second, the amount of the vertical reactions at the two abutments.

3. Select some point within the equilibrium polygon of Answer No. 2, and find the pole, the force diagram, and the equilibrium polygon which shall pass through this chosen point.

4. Draw two lines which are *not* parallel, which represent the locations, intensities, and directions of two forces. Select two points in the same horizontal line as abutments, and find an equilibrium polygon which will hold these forces in equilibrium.

5. Select another point in the diagram of Answer No. 4, and draw an equilibrium polygon through it.

6. What is the distinction between a *right arch* and a *skew arch?*

7. The pressure on a masonry joint 50 inches wide by 12 inches thick is 120,000 pounds. The center of pressure is 15 inches from one edge. What is the maximum unit-pressure on any part of the joint?

8. Draw the intrados for a segmental arch with a span of 40 feet and a rise of 10 feet. Compute the proper depth of keystone; make the thickness at the abutment ¼ greater, and draw the extrados. Use scale of ½ inch = 1 foot.

9. On the basis of Question 8, draw the load line, allowing for a level cinder fill, a 7-inch pavement, and a live load of 200 pounds per square foot. Use scale of 3,000 pounds per inch for load line.

10. Assuming 15 voussoirs in the above arch, compute the vertical loads on each voussoir, and draw a *half* load line for *full loading* over the whole arch.

11. Determine the special equilibrium polygon for the above loading, and the maximum unit-intensity of pressure at any joint.

12. Determine the load line for a concentrated load of 20,000 pounds on an area of 25 square feet at the quarter-point of the arch, and a load of 200 pounds per square foot over the remainder of the *half-span*.

13. Draw the special equilibrium polygon for the loading of Question 12, and determine the maximum unit-intensity of pressure at any joint.

14. Design an abutment for the above arch which shall be stable under either of the above conditions of loading.

15. Draw the load line for the above arch on the basis of the loading of Question 12, but on the assumption that the pressures on the arch are perpendicular to the extrados.

16. What is the essential distinction between a voussoir arch and an arch rib?

17. What are the three kinds of arch ribs, and what are their distinctive characteristics?

18. Assuming that the special equilibrium polygon for any loading has been determined, how do you determine the moment, thrust, and shear at any point of an arch rib?

19. How do you determine the moment of inertia of a rib section which is reinforced both top and bottom?

20. Redraw the extrados and intrados of Fig. 234 on the scale of ¼ inch = 1 foot; and then, by scaling the various thicknesses at every two-foot section, for 26 feet on each side of the center, compute the moment of inertia for each section.

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